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It is with deep regret that we record the death at Stockholm on November 10, 1914, in his seventy-sixth year, of

NILS CHRISTOFFER DUNÉR

formerly Professor of Astronomy and Director of the Observatory of Upsala, Sweden.

An appropriate sketch of our associate, who had been an editorial collaborator of this journal from its foundation, will appear in a later issue.

THE
ASTROPHYSICAL JOURNAL
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RADIAL MOTION IN SUN-SPOTS?

By W. H. JULIUS

In two elaborate and interesting papers, bearing the above title without the interrogation point, St. John[1] has very skilfully discussed his observations on line displacements in sun-spot spectra from the point of view that the efficient cause of those displacements is motion in the line of sight. Although at the end of the second paper he also devotes a few pages to criticizing the theory that attempts to explain such phenomena on the basis of anomalous dispersion, he could not, of course, do full justice to a point of view differing so radically from his own. I therefore thought it my duty to defend the attacked position which is by no means so weak as he represents it.

THE INTERPRETATION BASED ON THE DOPPLER EFFECT

St. John's investigation relates to eleven spots in positions between 25° and 60° from the center of the disk. The slit of the spectrograph was parallel to the radius of the solar image, passing through the center of the spot umbra. Nearly all of the 506 lines included in the measurements are displaced to the red on the edge of the penumbra nearest the limb, and to the violet on the opposite edge; only 13 lines show the reverse effect. In agreement with

[1] *Mt. Wilson Contr.*, Nos. 69 and 74; *Astrophysical Journal*, **37**, 322, and **38**, 341, 1913.

Evershed, who discovered the phenomenon in 1909, St. John concludes that the displacements are due to radial flow of matter tangential to the solar surface, generally directed from the center of the spot outward, but, on the contrary, inward in the case of the 13 exceptional lines.

If this interpretation of the displacements were free from serious difficulties it would be unnecessary to propose other views of the subject. It is a fact, however, that there are great difficulties; hence the pro and con of rival theories may well be submitted to the consideration of astrophysicists.

The displacements on which St. John's conclusions depend are of the order of magnitude 0.020 A, only very few exceeding 0.040 A. It certainly testifies to that author's experimental proficiency as well as to the power of the Mount Wilson apparatus that on such minute quantities a regular investigation can be based; but at the same time the conditions of the research warn us against putting too much confidence in the individual observations. On p. 4 of *Contribution No. 69*, St. John explains why the mean deviations (given in the sixth column of his Table I) from the mean displacements calculated for every line are so considerable. Trustworthy results can therefore be expected only from those considerations in which the statistical treatment includes a sufficiently great number of observed displacements, whereas the reliability of apparent relations rapidly decreases with the number of measurements from which they are deduced.

A survey of the whole series of measurements led St. John to the discovery of two very important and striking laws: (1) the displacements increase with increasing wave-length; (2) the displacements progressively decrease with the increase of line intensity.

Tables II and III,[1] illustrating these laws as they appear in the measurements on 193 iron lines of intensity 1 to 8, are reproduced here for convenience of reference. In the upper section of Table II the displacements are as measured; in the lower section they are reduced to a common wave-length, λ 5000. This reduction has been applied by St. John on the ground that, if the displacements

[1] *Mt. Wilson Contr.*, No. 69, pp. 16–17; *Astrophysical Journal*, **37**, 337–338, 1913. I keep the numbers II and III for these tables, so that in the present paper there is no Table I.

are due to the Doppler effect, they should be proportional to the wave-length. He therefore divides every displacement by the fraction $\lambda/5000$; the values thus obtained form the lower section of the table; they should be equal for lines of equal intensity.

TABLE II

DISPLACEMENTS AND WAVE-LENGTH

Region	Mean λ	Intensity								Mean
		1	2	3	4	5	6	7	8	
Violet..........	4017	0.022	0.020	0.014	0.014	0.013	0.011	0.008	0.006	0.014 A
Yellow-red......	6121	.032	.030	.032	.024	.028	.024	.019	.016	.026
Violet..........	4017	.026	.024	.019	.018	.017	.013	.011	.008	.017
Yellow-red......	6121	0.027	0.024	0.026	0.021	0.023	0.020	0.015	0.013	0.021

TABLE III

DISPLACEMENTS AND LINE INTENSITIES REDUCED TO λ 5000

Mean λ	Intensity								No Lines	Mean Interval
	1	2	3	4	5	6	7	8		
4017............	0.026	0.024	0.019	0.018	0.017	0.013	0.011	0.008	81	0.003 A
4992............	0.030	0.026	0.026	0.025	0.018	0.016	0.006	0.007	69	0.003
6121............	0.027	0.024	0.026	0.021	0.023	0.020	0.015	0.013	43	0.002
Wtd. mean......	0.028	0.025	0.023	0.021	0.019	0.016	0.012	0.009	......	0.003
Vel. km/sec......	1.68	1.50	1.38	1.26	1.14	0.96	0.72	0.54	......	0.18

Now, I think it impossible to agree with St. John when he concludes that the results collected in Table II "seem decisively in favor of an effect varying as the wave-length" and that "the decrease from 0.012 A to 0.004 A in the mean difference [between the average displacements in the violet and in the yellow-red regions] when the displacements are reduced to a common wave-length indicates that the observed differences are due to the Doppler effect, and that we are dealing with real movements of gases in the reversing layer." The table gives plenty of evidence, on the contrary, that the displacements are by no means proportional to the wave-length. Indeed, in every intensity class they appear to increase more rapidly than the wave-length. Moreover, the rate

of increase with wave-length is far from regular, for if we transfer the series of displacements relating to mean λ 4992 from Table III to the upper section of Table II and put it between violet and yellow-red, we see that for lines of low intensity the rate of increase of the displacements is in general more rapid between mean λ 4017 and λ 5000 than between λ 5000 and mean λ 6121, whereas for lines of high intensity the reverse seems to obtain.

There would not be any objection, of course, to advancing the *hypothesis* that the Doppler effect is involved in the phenomenon, and to try additional hypotheses in order to account for the systematic and the irregular deviations[1] from proportionality between displacement and wave-length. I deny only that the data included in the above tables give any decisive indication in favor of the view which considers the Doppler effect as the principal cause of the diplacements. Neither do the same data contain a positive proof that the rival explanation which is based on anomalous dispersion is the right one; but, as will be shown in the second section of this paper, the latter interpretation deserves notice and has more features that recommend it than the former if the value of both be judged by the criteria so well formulated by St. John at the beginning of the section on p. 14 of *Contribution No. 74*.

Reverting to St. John's discussion of the above tables, we remark that Doppler's principle alone has nothing in it to explain the remarkable and well-established progressive decrease of the displacements with the increase of line intensity. A new hypothesis, therefore, had to be introduced. St. John assumes that the weaker iron lines originate at the lower level, so that the displacements would increase with depth. By this assumption he intends to insure a *progressive* change in the velocity of outflow of matter with change of level, and at the same time opens a way to account for the fact that the displacements do not exhibit the otherwise expected proportionality with the wave-length, but increase more rapidly. For it might be that among the lines of the same intensity those belonging to the yellow-red region of the spectrum originate at a lower level than those belonging to the violet region, a difference which the author calls a natural consequence of the scattering of light by small particles.

[1] Cf. Fig. 6, p. 23.

Many spectroscopists, however, will shrink from accepting this bold hypothesis of the "Iron Scale" which supposes the various intensities of the solar lines of an element to indicate the levels where they originate. In the laboratory, the absorption spectrum of a gas (for instance, of NO_2) shows strong and weak lines at the same time; it is not easy to admit with St. John[1] that in the sun "the absorbing centers effective for lines of different intensities of a given element are apparently not identical and appear to be allocated in successive spherical shells." What are we to think, e.g., about the theories of series lines from this point of view?

But let us pass this difficulty, and see what the Doppler interpretation of these line-shifts results in.

All elements included in the investigation, except hydrogen, are moving from the axis of the spot vortex outward with velocities varying between 1.1 and 0 km per second. Hydrogen flows inward at a higher level, and so do special forms of calcium, magnesium, sodium, iron, and strontium, respectively represented by 3, 2, 2, 2, and 1 lines.

In general the horizontal velocities seem to be greatest near the outer edge of the penumbra. Multiplying the circumference of the spot by the distance between the lowest and the highest level of outflowing matter, we get the transverse section of the stream, in which the average velocity of the gases may be estimated at 0.7 km per second. The stream must be fed by a rising current in the center of the spot, the stream lines of which will progressively pass from a vertical to a horizontal direction. The vertical component of the motion would become the more conspicuous, the more a spot approaches the center of the solar disk; and as very probably the transverse section of the vertical current in the umbra is not much greater than that of the horizontal current near the outer edge of the penumbra, we should expect to find considerable displacements of most umbral lines toward the violet, especially in the spectra of spots located in the central parts of the disk. And in the penumbral spectra of eccentrically located spots the vertical component of the motion would have the effect of increasing the displacements to the violet and diminishing those toward the red.[2]

[1] *Mt. Wilson Contr.*, No. 74, p. 13; *Astrophysical Journal*, **38**, 353, 1913.

[2] The 13 exceptional lines are not considered here.

Definite indications of such vertical velocities are not mentioned, however. This, I think, is one of the most serious objections against the assumption that radial motion in sun-spots is the effective cause of the displacements in question. St. John suggests[1] that the levels where the vertical components of the velocities prevail are too low to be accessible to spectroscopic investigation. If this were true for the lines of intensity 00, it would not hold good for the lines belonging to higher levels, because their absorbing centers rise a long way through optically accessible layers. So the difficulty is not removed.

The opposite displacements of the 13 above-mentioned chromospheric lines are explained as indicating a radial inflow of chromospheric material into the spot. From the displacements which the lines H and K of calcium show in the spectrum of the umbra[2] it would follow that this gas is moving downward in the center of the spot with a velocity of 1.3 km per second. The other chromospheric gases are supposed to share the downward motion, although their lines appear not sufficiently displaced in the spectrum of the umbra to make the assumption plausible. The question where this chromospheric material goes after having reached the reversing layer finds no satisfactory solution in the results of the measurements; and it seems very difficult to reconcile the necessary consequences of an impact between the downward-rushing chromospheric matter and the upward-rushing matter of the reversing layer with the assumed relatively low temperature of the umbral region.

St. John attempts to corroborate his intensity-and-level hypothesis by considering the atomic weight of the elements in connection with the displacements of their lines in the spot spectrum. We may doubt whether the data suffice for the purpose. It is stated, e.g., that the lines of the heavy elements, such as barium, lanthanum, neodymium, cadmium, cerium, lead, and ytterbium, originate at lower levels than the lines of like intensity of iron;[3] but if we refer to Table I of the first paper we find that

[1] *Mt. Wilson Contr.*, No. 69, p. 24; *Astrophysical Journal*, **37**, 345, 1913.

[2] Cf. *Mt. Wilson Contr.*, No. 54, pp. 28–29; No. 69, pp. 24–25 and 27–28; *Astrophysical Journal*, **34**, 136–137, 1911; **37**, 345–346, 348–349, 1913.

[3] *Mt. Wilson Contr.*, No. 74, p. 6; *Astrophysical Journal*, **38**, 346, 1913.

the evidence is not so very strong. Lead, ytterbium, and cadmium are represented by only one line each, barium by two lines; these five lines really show larger displacements than the mean iron lines of like intensities. With lanthanum the conclusion has somewhat greater weight, because nine lines have been measured. Four of them (intensity 1) give the mean displacement 0.029 instead of 0.028 which would correspond to the iron scale; four lines of intensity 2 give 0.026 instead of 0.025, and one of intensity 4 gives 0.025 instead of 0.021. The differences are small (considering the great atomic weight of lanthanum), but in the required direction. With cerium, on the other hand, one of the two lines by which it is represented in the table shows a displacement 0.026 instead of 0.023 but the other one (intensity 1) gives 0.017 instead of 0.028 and thus points to a *higher* level. Of neodymium, finally, three lines have been measured; two of them (intensity 1) give the mean value 0.025 instead of 0.028 and one (intensity 2) gives 0.022 instead of 0.025, so that the lines of this element (atomic weight 144) would seem to originate on the average at a higher level than the iron lines of like intensity. An equally unfavorable account must be given of zirconium (atomic weight 91), for the displacements of six of its seven lines in the list would also decidedly, from the point of view of St. John's hypothesis, indicate higher levels than those corresponding to the iron scale.

It is possible, though not probable, that the large deviations which the mean displacements of individual lines often show with respect to the mean value corresponding to their intensity class and spectral region find a sufficient explanation in the extreme difficulty of the measurements. If, however, the accuracy attained would permit of regarding such deviations as genuine, the interpretation of the displacements on the basis of the Doppler effect would be condemned. One could not reasonably admit the various absorption centers present in a gas-current at a certain level to have different proper velocities of outflow from the spot vortex.

THE INTERPRETATION BASED ON ANOMALOUS DISPERSION

As would appear from the preceding discussion of the radial-motion hypothesis, it is not superfluous to look for other ways of

explaining the relative displacements of the Fraunhofer lines at the limb- and center-edges of the penumbrae of eccentrically located spots.

In a previous paper[1] I suggested an explanation of the sun's edge and of the general distribution of the brightness on the solar disk, assuming the non-existence of a photosphere in the sense of the surface of a body or of a layer of clouds. The term "photosphere" is preserved; but in the new interpretation it only means a mathematical sphere constructed round the sun's center and having for its radius the distance between the center and the apparent edge of the disk. The gaseous condition of the solar atmosphere continues below the photosphere without any abrupt change in the physical or chemical properties of the mixture. The increase of the mean density and the variation of the mean composition are progressive.

Our sun-spot hypothesis advanced in 1909[2] considered the distribution of the light in a spot as chiefly produced by the refraction which photospheric light suffers when traversing a region where the optical density passes through a minimum (e.g., a solar vortex).

It has been doubted whether in the solar *atmosphere* differences of density, sufficient for imparting to the rays the important deviations required by the theory, really could exist. This objection will disappear if the new interpretation of the photosphere be accepted, because the present point of view permits of locating that region of minimum density somewhere below the photospheric level, in layers where sufficient gradients of optical density are sure to be found.

A rough estimate of the values which optical density gradients must have in order to produce an observable incurvation of average rays (for which the medium possesses small refracting power) has been given in a previous paper.[3] It should be remembered that in the present paper we are dealing with R-light and V-light,[4]

[1] *Astrophysical Journal*, 38, 129, 1913.

[2] *Proc. Roy. Acad. Amst.*, 12, 266, 1909; *Physikalische Zeitschrift*, 11, 56, 1910.

[3] *Astrophysical Journal*, 38, 135, 1913.

[4] Defined as waves lying respectively on the red-facing and the violet-facing sides of absorption lines and very near to them; cf. *Proc. Roy. Acad. Amst.*, 12, 275, 1909; or *Physikalische Zeitschrift*, 11, 63, 1910.

that is, with waves generally more strongly refracted than the average light of the spectrum.

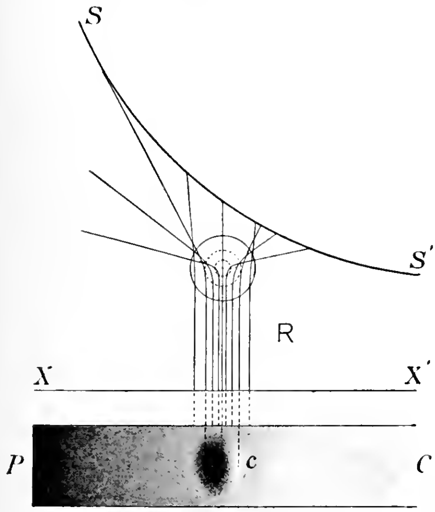


FIG. 1

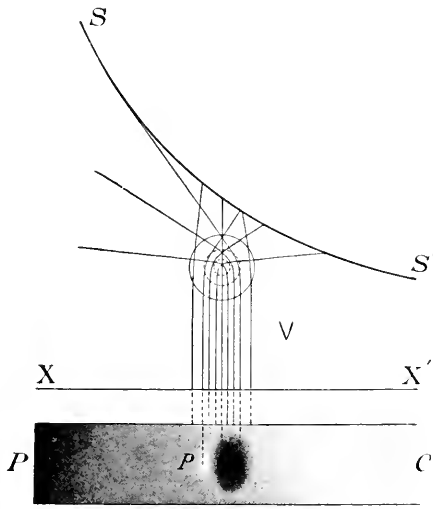


FIG. 2

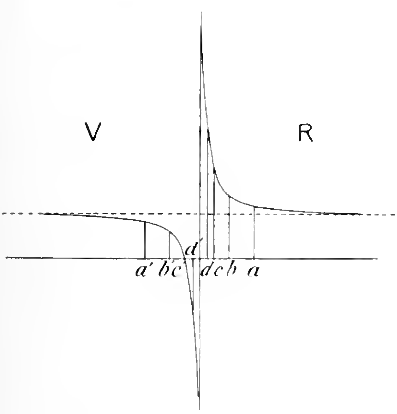


FIG. 3

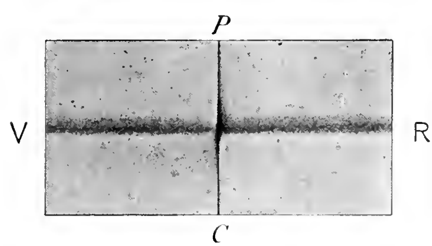


FIG. 4

The influence of an eccentrically located density minimum on the course of R-light and V-light, and the resulting effect on the spectrum, may be illustrated by means of Figs. 1, 2, 3, and 4. These are taken from the above-mentioned publication of 1909,

to which I refer for a more complete explanation of the purely *schematic* case represented by them.[1] It should be noted that the drawings were only intended to bring out the essential feature of the effect of anomalous refraction in an ideal depression. In a real spot the conditions are of course far more complicated; the shape of the depression will not be spherical as assumed, and irregular gradients superposed upon the systematic gradients of the vortex region will especially influence the course of the most refrangible rays, thus confusing the schematic results for the central parts of strong lines. But provisionally abstracting from details—to which we shall revert farther on—we see that anomalous refraction causes just the kind of wryness observed with all the lines of the spot spectrum, except with a few very strong lines that correspond to prominent chromospheric lines. A discussion of the behavior of those 13 exceptional lines of the table must be postponed; in this section we confine our attention to the normal case of the 493 lines.

Besides explaining the general character of the displacements in question, the anomalous-dispersion theory should account for the following quantitative results of the observations:

1. The *mean* displacements calculated for each intensity class decrease with increasing line intensity. (For explaining this rule St. John had to introduce his new hypothesis connecting line intensity with level.)

2. Large deviations from the mean values occur in every intensity class. (St. John does not consider such deviations as purely accidental, for he bases on them various conclusions on relative levels of different elements. But if the large residuals often found when deducting the mean from the individual displacements, for lines of one element and one intensity, are real, this

[1] Some experiments on the refraction of light in whirling gases were shown on the occasion of the fifth meeting of the Solar Union in August, 1913, at Bonn, and are described in *Physikalische Zeitschrift*, **15**, 48, 1913. As remarked in a footnote of that paper, our recent interpretation of the photosphere made it possible to improve the sun-spot hypothesis of 1909 by admitting that the spot vortex or region of minimum density might be located *below* the photospheric level. The arc SS' of our figures, therefore, does not now represent a part of the photosphere, but a part of a lower level.

would seem to present, as already remarked, an insuperable difficulty for the interpretation of the phenomenon as a radial-motion effect.)

3. The displacements increase with the wave-length, but not proportionally. The observations do not indicate a simple law, and the rate of increase seems to vary differently for different intensity classes. (It is impossible to explain these facts on the basis of the Doppler effect without calling in additional hypotheses in order to account for the large deviations from proportionality with wave-length.)

From the point of view of the anomalous-dispersion theory it is at once clear that there must be a direct connection between the intensity of the lines and the magnitude of their displacements in spots, as both phenomena depend on the "dispersion bands" enveloping the absorption lines. It would be rash and erroneous, however, to conclude that strong anomalous dispersion, because it produces Fraunhofer lines of great intensity, must also give rise to large relative displacements in the spot spectrum. Indeed, the reverse is true. This will come out clearly in the course of the discussion; but before settling this point we had better first consider the possible cause of the second rule: the great disparity of the displacements.

Disparity of the displacements. Mutual influence of Fraunhofer lines.—If the displacement of a certain line A depends on the refracting power of the medium for the adjacent waves, it must be influenced by the presence of a strong neighboring line B. Let us discuss the nature of that influence.

Let n_0 be the value which the refractive index would have in the part of the spectrum under observation if this were free from absorption lines, and suppose n_0 to be >1. The effect of a line B (Fig. 5) is to reduce the indices on its violet side and to raise them on its red side, as indicated by the partly broken curves. The line A, if isolated, would produce its own anomaly in the dispersion-curve as shown in A_1. If A were situated near B, in one of the positions A_2 or A_3, that anomaly would have a somewhat different shape in consequence of its being superposed upon one of the branches of the dispersion-curve due to B. The refracting

properties of the medium, being determined by the values of $n-1$, will be different in the three cases represented by A_1, A_2, and A_3.

Only those waves for which the absolute values of $\pm(n-1)$ exceed a certain minimum value will become sufficiently curved in the outer parts of the vortex region to give rise to sensible refraction effects in the spectrum of the penumbra. This is indicated in the figure by means of the two broken lines drawn at equal distances above and below the line $n=1$. We may assume that only the

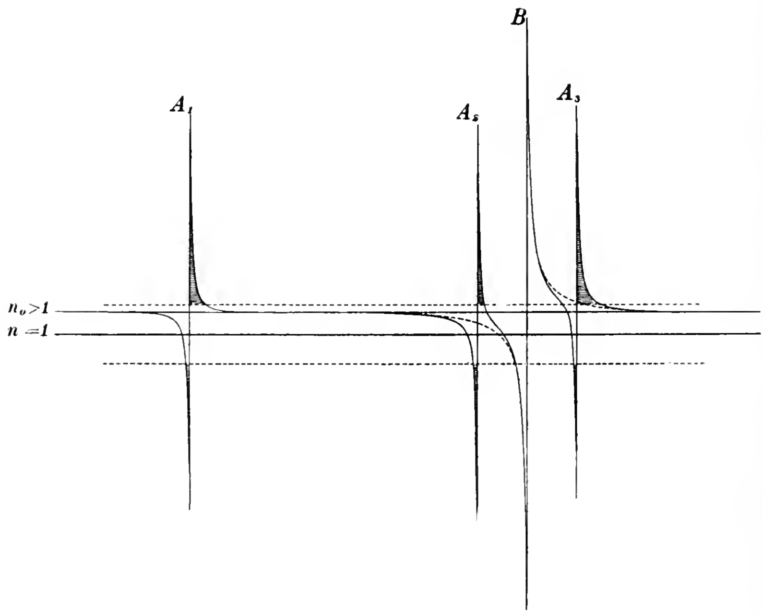


FIG. 5.—Mutual influence of Fraunhofer lines

parts of the dispersion-curve lying outside the zone between these broken lines are material to the formation of the dispersion bands enveloping the absorption lines of the penumbral spectrum. The R-light corresponding to the shaded area above the zone is responsible for the displacements toward the red observed at the peripheral edge of the penumbra; the V-light corresponding to the shaded area below the zone causes the displacements toward the violet at the central edge.

Now comparing for the lines A_1, A_2, and A_3 the horizontal distances between the "centers of gravity"[1] of their R-area and V-area, we at once realize that A_2 will show a smaller displacement than A_1, and A_3 a greater displacement than A_1; but it is also evident from the figure that the difference between the cases A_3 and A_1 is not so marked as that between A_2 and A_1.

This deduction from the theory can be put to the test by means of St. John's observations.

After having marked out on Higgs's atlas of the normal solar spectrum the 506 lines of Table I, I selected *all* cases in which a measured line A of intensity 3 or lower was on the *violet* side of a stronger line B (generally of intensity 4 or higher) at a distance of about 0.5 A or less. A few cases in which the line A had another strong companion B' equally near but on the other side were of course discarded. Forty-three pairs answering the conditions were found; they are united in the first column of Table IV. The wave-lengths of lines B not appearing in St. John's Table I were read on Higgs's atlas. Most of these lines could be identified with lines occurring in Mitchell's table of wave-lengths of the chromosphere.[2] In the second and third columns of our Table IV are the elements and intensities; the elements in brackets are taken from Mitchell's table. The fourth column contains the corrected values Δ' of the observed relative displacements as given by St. John.[3] In the fifth column are indicated "normal" values of the displacements. These were obtained from the data appearing in the upper section of Table II after having supplemented them by inserting between the numbers for violet and yellow-red those given for mean λ 4992 in Table III. Thus, on the basis of the two rules found by St. John that connect the displacements with line intensity and with wave-length, it was possible to indicate a "normal" displacement peculiar

[1] The term "center of gravity" is used for convenience' sake. Properly speaking, the estimated location of the lines will of course depend on considerations somewhat different from those involved in the determination of the center of gravity of the said area.

[2] *Astrophysical Journal*, **38**, 407, 1913.

[3] I have not used the values Δ' reduced to λ 5000, because that reduction derived its direct meaning from the assumption that the relative displacements are due to the Doppler effect.

TABLE IV

THE *Decrease* OF THE AMOUNT OF THE EVERSHED EFFECT IN THE SPECTRUM OF ECCENTRICALLY LOCATED SUN-SPOTS, OBSERVED WITH THE *Violet*-AND-WEAKER MEMBERS OF PAIRS OF LINES.

λ	Element	Intensity	Observed Displacement Δ′	Normal Displacement	Difference	Remarks
3649.137	Cr	1	0.014	0.022	−0.008	
3649.4	....	5	....	....	....	
3662.096	Ni	3	.015	.015	.000	
3662.38	(Ti)	5	....	....	....	
3686.926	Cr	1	.016	.022	− .006	
3687.2	....	3	....	....	....	
3687.234	Fe	3	.010	.015	− .005	
3687.610	Fe	6	....	....	....	
3688.210	V	1	.018	.022	− .004	
3688.5	....	4	....	....	....	
3690.599	Fe	2	.017	.020	− .003	
3690.8	....	3	....	....	....	
3707.702	Ti	2	.013	.020	− .007	
3708.07	(Fe)	6	....	....	....	
3708.964	Co	1	.015	.022	− .007	
3709.389	Fe	6	....	....	....	
3895.119	Co	3	.012	.015	− .003	
blend	....	4?	....	....	....	
3895.583	Mn	3	.008	.015	− .007	
3895.803	Fe	7	....	....	....	
3898.032	Fe	3	.007	.015	− .008	
3898.2	....	4	....	....	....	
3899.171	Fe	3	.013	.015	− .002	
3899.21	(V–Fe)	3	....	....	....	
3906.438	Co	2	.010	.020	− .010	
3906.628	Fe	10	....	....	....	
3913.123	Ni	2	.016	.020	− .004	
3913.609	Ti	5	....	....	....	
3916.879	Fe	5	.009	.014	− .005	
3917.32	(Fe)	6	....	....	....	
3947.522	?	2	.014	.021	− .007	
3947.675	Fe	4	....	....	....	
3956.603	Fe	4	.010	.014	− .004	
3956.879	Fe	6	....	....	....	
3958.073	Co	2	.018	.022	− .004	
3958.36	(Zr–Ti)	4	....	....	....	
3962.995	Ti	3	.012	.016	− .004	
3963.2	....	4	....	....	....	
3995.899	La	1	.014	.023	− .009	
3996.14	....	3	....	....	....	
3997.115	Fe	2	.019	.022	− .003	
3997.547	Fe	4	....	....	....	
4035.752	Co	2	.016	.022	− .006	
4035.883	Mn	4	....	....	....	
4109.609	Nd	1	.016	.024	− .008	
4109.95	....	4	....	....	....	
4132.100	V	2	0.011	0.022	−0.011	
4132.235	Fe	10	....	....	....	

TABLE IV—*Continued*

λ	Element	Intensity	Observed Displacement Δ′	Normal Displacement	Difference	Remarks
4133.755	Fe	2	0.022	0.022	0.000	
4133.95	(Fe–Ce)	4	....	....	....	
4149.360	Zr	2	.016	.022	− .006	
4149.5	....	4	....	....	....	
4216.136	CN	1	.022	.024	− .002	
4216.35	(Fe)	4	....	....	....	
4233.328	Mn–Fe	4	.017	.017	.000	
4233.772	Fe	6	....	....	....	
4271.325	Fe	6	.009	.014	− .005	Distance >0.6 A
4271.934	Fe	15	....	....	....	
4274.746	Ti	2	.016	.023	− .007	
4274.958	Cr	7	....	....	....	
4289.525	Ca	4	.015	.019	− .004	
4289.885	Cr	5	....	....	....	
4294.936	Zr	2	.018	.023	− .005	
4295.29	Dy	6	....	....	....	
4302.353	Fe	2	.019	.023	− .004	
4302.692	Ca	4	....	....	....	
4315.138	Ti	3	.010	.021	− .011	
4315.262	Fe	4	....	....	....	
4408.364	V	2	.027	.024	+ .003	Influenced by λ 4407.8, int. 6
4408.54	(V)	4	....	....	....	
5168.832	Ni	1	.026	.028	− .002	
5169.16	(Fe)	7	....	....	....	
5188.863	Ti	2	.015	.026	− .011	
5189.0	....	3	....	....	....	
5226.707	Ti	2	.020	.026	− .006	
5227.0	....	4	....	....	....	
5250.385	Fe	2	.028	.027	+ .001	
5250.82	(Fe)	3	....	....	....	
5298.194	Cr	1	.021	.029	− .008	
5298.455	Cr	4	....	....	....	
5598.524	Fe	1	.019	.030	− .011	
5598.711	Ca	4	....	....	....	
5615.520	Fe	2	.025	.027	− .002	
5615.877	Fe	6	....	....	....	
5624.245	Fe	1	0.027	0.031	−0.004	
5624.77	(Fe)	4	....	....	....	
					Mean diff. −0.0051	

to the spectral region and the intensity of each measured line. With these normal values the observed values had to be compared.

As expected, the differences shown in the sixth column are negative. There is one distinct exception: λ 4408.364, for which the difference is +0.003. This line, however, has also a companion *B′* on the violet side (λ 4407.8 of intensity 6) that would work the

opposite way, so that perhaps the case ought to have been discarded although the distance between A and B' is a little greater than 0.5 A. On the average, the measured displacements of these forty-three violet members of pairs are as much as 0.0051 A smaller than the normal values. This result is in perfect harmony with our assumption that the displacements depend on the refracting power of the medium.

Additional evidence is obtained from Table V, which contains data similar to those of Table IV, but now relative to thirty-nine pairs, the weaker line A of which is on the *red* side of the stronger line B. In these cases the observed displacements of A should generally exceed the normal values, but the effect is expected to be less conspicuous than the reduction of the displacements on the violet side of lines B.

As a matter of fact, the differences in the sixth column are for the greater part positive. And examining on Higgs's atlas the environment of the 12 lines that gave negative deviations from the normal displacements, we find some cases where an additional strong neighboring line B' on the wrong side may be responsible for the discrepancy (e.g., λ 3947.918 might be influenced by λ 3948.25 of intensity 5, and λ 3949.039 by λ 3949.25 of intensity 3). The average increase of the relative displacements above their normal values, attributed to the violet companions of our thirty-nine lines, amounts to nearly +0.0015 A. Omitting the dubious cases we should have found the mean residual +0.0019 A.

It must be granted that the difference between the absolute values of the negative mean residual 0.0051 and the positive mean residual 0.0015 appears too great to be entirely accounted for by the inequality of the configuration of the shaded areas on the two sides of B, as represented in Fig. 5. The difference, however, may be partly due to a systematic observational error; for it is not improbable that the proximity of a strong line B causes the displacements of lines A to be underestimated. Allowing for this error—which of course would have the same sign on either side of B—we must reduce the observed negative and enlarge the positive mean residual by the same amount. Their absolute values thus

TABLE V

THE *Increase* OF THE AMOUNT OF THE EVERSHED EFFECT IN THE SPECTRUM OF ECCENTRICALLY LOCATED SUN-SPOTS, OBSERVED WITH THE *Red*-AND-WEAKER MEMBERS OF PAIRS OF LINES

λ	Element	Intensity	Observed Displacement Δ′	Normal Displacement	Difference	Remarks
3694.24	(Fe-Ni)	8	....	....	....	
3694.344	Yt	3	0.020	0.015	+0.005	
3694.344	Yt	3	....	....	....	
3694.576	La	1	.027	.022	+ .005	
3704.603	Fe	4	....	....	....	
3704.840	V	1	.016	.022	− .006	
3706.24	(Mn-Ti-Ca)	7	....	....	....	
3706.363	Fe	3	.017	.015	+ .002	
3711.364	Fe	4	....	....	....	
3711.552	Fe	3	.015	.015	.000	
3898.2	....	4	....	....	....	
3898.531	Mn	2	.014	.020	− .006	
3947.675	Fe	4	....	....	....	
3947.918	Ti	2	.013	.021	− .008	Influenced by
3948.925	Fe	4	....	....	....	λ 3948.25, int. 5
3949.039	Ca	1	.018	.022	− .004	Influenced by
3950.102	Fe	15(?)	....	....	....	λ 3949.25, int. 3
3950.497	Y	5	.013	.014	− .001	
3984.17	(Fe-Mn)	6	....	....	....	
3984.294	Mn	2	.021	.020	+ .001	
3989.912	Ti	4	....	....	....	
3990.011	Fe	3	.012	.015	− .003	
4018.25	(Mn)	7	....	....	....	
4018.420	Fe	3	.022	.015	+ .007	
4078.49	?	4	....	....	....	
4078.631	Ti	3	.017	.016	+ .001	
4079.4	....	5	....	....	....	
4079.570	Mn	3	.018	.016	+ .002	
4134.54	(V-Fe)	6	....	....	....	
4134.840	Fe	5	.014	.014	.000	
4161.68	(Ti)	5	....	....	....	
4161.961	Sr	1	.025	.024	+ .001	
4184.32	(Ti-Gd)	5	....	....	....	
4184.472	Ti	2	.022	.022	.000	
4196.35	?	4	....	....	....	
4196.699	La	2	.024	.022	+ .002	
4236.112	Fe	8	....	....	....	
4236.429	Ni	1	.024	.024	.000	
4240.64	(Zr-Ce-Fe)	4	....	....	....	
4240.872	Cr	1	.022	.024	− .002	
4338.084	Ti	4	....	....	....	
4338.430	Fe	1	.025	.024	+ .001	
4637.685	Fe	5	....	....	....	
4638.193	Fe	4	.027	.021	+ .006	
4667.626	Fe	4	....	....	....	
4667.768	Ti	3	.027	.023	+ .004	
4679.027	Fe	6	....	....	....	
4679.409	Ni	2	0.037	0.024	+0.013	

TABLE V—*Continued*

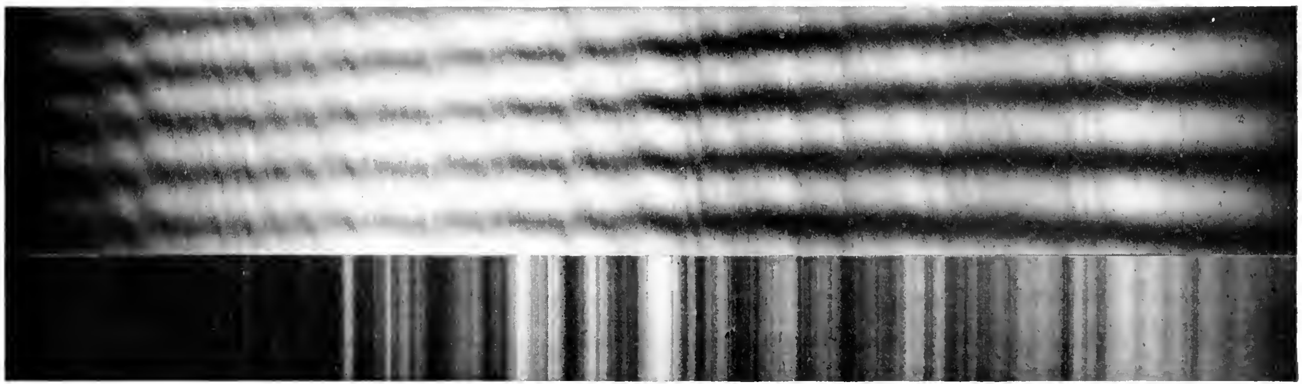
λ	Element	Intensity	Observed Displacement Δ′	Normal Displacement	Difference	Remarks
4703.177	Mg	10	....	....	....	
4703.994	Ni	3	0.035	0.023	+0.012	Distance >0.6 A
4731.65	(Fe)	4	....	....	....	
4731.984	Ni	1	.030	.026	+ .004	
4736.96	(Fe)	6	....	....	....	
4737.540	Cr	2	.034	.024	+ .010	
4762.567	Mn	5	....	....	....	
4762.820	Ni	1	.039	.026	+ .013	
5129.42	(Ti–Ni)	5	....	....	....	
5129.546	Ni	2	.026	.024	+ .002	
5129.546	Ni	2	....	....	....	
5129.805	Fe	1	.033	.028	+ .005	
5131.642	(Fe–C)	3	....	....	....	
5131.942	Ni	1	.029	.028	+ .001	
5152.087	(Fe–C)	3	....	....	....	
5152.361	Ti	0	.031	.031	.000	
5192.523	(Fe–Nd)	5	....	....	....	
5193.139	Ti	2	.021	.026	− .005	Distance >0.6 A
5283.802	(Fe)	6	....	....	....	
5284.281	Ti	1	.026	.028	− .002	
5298.455	Cr	4	....	....	....	
5298.672	Ti	1	.022	.028	− .006	
5349.652	Ca	4	....	....	....	
5349.928	Fe	1	.027	.028	− .001	
5857.674	Ca	8	....	....	....	
5857.976	Ni	3	.030	.027	+ .003	
5953.0	(Ti–Fe)	5	....	....	....	
5953.386	Ti	1	.038	.032	+ .006	
6400.217	Fe	8	....	....	....	
6400.528	Fe	2	0.026	0.031	−0.005	
					Mean diff. +0.00146	

approach each other, whereas the characteristic difference between the cases A_2 and A_3 remains unaffected.

Taken all in all, the evidence is very strong in favor of the view that the displacements here considered are entirely due to anomalous refraction.

The interpretation of the phenomena on this basis easily accounts for the great and frequent deviations of individual displacements from the normal values, and thus increases our confidence in the accuracy of St. John's measurements. Indeed, if we take it for granted that every Fraunhofer line influences the refractive index of the gaseous mixture in a way similar to that in

PLATE I



Record obtained with a Jamin interferential refractometer and a Hilger spectrograph, showing how the refractive index of a selectively absorbing medium (NO_2) fluctuates along the spectrum under the influence of the absorption lines.

which our lines B have been proved to do, the value of n_0 must oscillate sensibly along the whole spectrum, especially in regions where the lines are crowded. The refracting power of the solar atmosphere would then be analogous to that of a terrestrial gas giving an absorption spectrum with a great number of lines. Plate I is intended to illustrate such a case. It shows the refracting power of *nitrogen peroxide* as recorded by means of a Jamin interferential refractometer and a small Hilger spectrograph.[1] Both the rapid variations of the refractive index near prominent lines, and the gradual fluctuations of the mean index caused by groups of lines are well shown. Now, from our Fig. 5 (p. 12) it is clear that for a line of given intensity the magnitude of the relative displacement in the spot spectrum will depend on the value which n_0 has in the part of the spectrum under consideration, as well as on the presence of direct neighboring lines. Hence one cannot be astonished at finding very unequal displacements with lines of the same intensity, the same element, and about the same region of the spectrum. This is an important inference, in respect to which our point of view has the advantage over the radial-motion hypothesis.

Unless some other plausible explanation of this peculiar mutual influence of neighboring lines on the magnitude of their displacements be found, we are forced to consider the foregoing results as a direct proof of the efficiency of anomalous dispersion in producing solar phenomena.

Displacements and line intensity.—Our next aim must be to explain the fact that, on the average, the displacements decrease with the increase of line intensity.

This problem brings us in contact with a characteristic feature of our theory that has given rise to some misapprehension and opposition.

Line displacements caused by Doppler effect, Humphreys effect, and Zeeman effect will increase in proportion as the velocity, the pressure, and the strength of the magnetic field increase; but the analogous inference that solar line displacements caused by

[1] *Proc. Roy. Acad. Amsterdam*, **13**, 1088, 1911; *Zeitschrift für wissenschaftliche Photographie*, **10**, 62, 1911.

anomalous dispersion should always increase in proportion as the degree of anomalous dispersion in the sun increases, or that they should even be proportional to the refraction effects observed with the corresponding lines in the laboratory, is entirely erroneous.

This, of course, does not involve the assertion that results on anomalous dispersion obtained from terrestrial sources would be without any value for the interpretation of solar phenomena. On the contrary, in the theory advanced, as in any other interpreting system, observation aided by experimental research is the only reliable basis; but in criticizing the conclusions of the rival theories it should be noted that the relations of anomalous dispersion are very different from those of other causes of line displacements, and require special study.

In order to compare the refraction effects associated with weak and with strong lines we once more refer to the schematic Figs. 1–4 on p. 9, and to their interpretation given in the paper mentioned in the footnote of p. 10. The case represented is an ideal one, not only because we gave the depression a spherical shape, but also on account of the assumed smoothness of the gradients.

We shall now consider the effect of slight irregularities of optical density superposed upon the systematic gradients due to the vortex.

Waves for which $\pm(n-1)$ is not very much greater than n_0-1 (n_0 being the mean refractive index for the spectral region under consideration) will essentially behave in accordance with the ideal case, although their paths will appear somewhat sinuous on account of the small fluctuations of the density. Such are the conditions obtaining with the R-light and V-light of lines of low intensity. The aspect of those lines will therefore nearly correspond to the shape represented in Fig. 4. If a wave-length belonging to the R-light of a weak line could be isolated with the spectroheliograph, the solar image thus obtained would show the spot displaced toward the limb; similarly a wave-length belonging to the V-light would show the spot displaced toward the center of the disk. This involves that in the spectrum of the limb-edge the intensity of the dark line falls off sharply on the violet side, pro-

gressively on the red side, and just the reverse in the spectrum of the center-edge of the spot.

The following weak lines, visible on Fig. 1 of the plate[1] of St. John's first paper, show this characteristic of our schematic line (Fig. 4, p. 9, where the effect is exaggerated) unmistakably: $\lambda\lambda$ 4750.1, 4751.28, 4764.5, 4764.72, 4768.85, 4776.26, 4778.4, 4781.9.

The measured values of the displacements of lines of this kind are determined by the difference of wave-length between the "centers of gravity" of their R-light and V-light areas, in so far as the width of the true absorption line, common to both edges of the penumbra, may be neglected.

The peculiar shape which these lines of very low intensity show in the spot spectrum when observed with radial slit corroborates the fundamental hypothesis of our solar theory, viz., that the width of Fraunhofer lines is in the main an effect of anomalous dispersion.

Proceeding to the case of a stronger line, we are concerned with waves for which $\pm(n-1)$ has such high values that even the lesser, parasitical density gradients make those rays deviate very sensibly. The rays will then follow winding paths entirely different from the smooth lines of the drawings, Figs. 1 and 2, p. 9. Thus, e.g., a ray of V-light emerging from the peripheral part of the penumbra, which if only moderately refracted would have carried much energy (according to Fig. 2), will now on account of its frequent curving possibly take the energy from a less favorable direction, and will at all events have suffered more loss by scattering, both molecular and refractional, than have waves for which $\pm(n-1)$ is smaller.

We may also consider the matter thus: very strongly refrangible rays are not so much influenced by the large-scale density configuration of the vortex region. In fact, such rays are refracted by the irregular gradients outside as well as inside the vortex regions; and although equal positive and negative values of $n-1$ determine opposite incurvations, the paths are everywhere so twisted throughout the whole layer corresponding to the levels where sun-spots occur, that the combined effects of those waves blend into a

[1] Cf. *Mt. Wilson Contr.*, No. 69, Plate XXIV; *Astrophysical Journal*, **37**, 324, 1913, Plate XII.

vague, fine-grained structure, and the systematic spot-gradients are scarcely indicated by them.[1]

In the spectrum of the spot, therefore, the strongly refracted waves will not in general produce any marked asymmetrical phenomenon. By their winding and scattering amid the lesser density fluctuations they get possibly still more weakened inside than outside the vortex region, and thus make the line appear strengthened and widened,[2] but nearly equally so on both edges of spot and line, the average effect being almost the same for R-light and V-light. Waves a little farther from the core of the line, however, are less refracted and behave according to the scheme that holds good for weak lines; hence the shading of the line will be broader on the red than on the violet side in the peripheral penumbra, and broader on the violet than on the red side in the penumbra directed toward the center of the disk. This makes it appear that the line is shifted bodily. When the relative displacement is being measured, the strong central part which the lines of both spot-edges have in common will preponderate in the determination of the "centers of gravity"; in this way the displacement will come out smaller than with lines of low intensity. This diminution of the distance between the estimated centers, progressive with increasing line intensity, explains the law discovered by St. John.

Displacements and wave-length.—We shall now discuss from our point of view the connection that seems to exist between displacements and wave-length.

Together with the data given in Tables II and III the contents of Table VI, graphically represented in Fig. 6, may serve to provide us with a survey of the available material.

[1] Spectroheliographic images obtained with the very centers of strong lines really do not show the spots (Deslandres); they give, however, some coarse details corresponding to higher levels, where the smoother gradients suffice to impart to those highly refrangible rays the deviations necessary for producing contrasts.

[2] The exceptionally wide lines H, K, H_α, H_β, H_γ, H_δ, and some other winged lines require special treatment, because with them very probably the middle part of the dispersion-curve uniting the minimum with the maximum will have to be taken into consideration. A discussion of the enhanced lines and those weakened in the spot spectrum must also be postponed until the completion of a laboratory investigation now in progress.

In the first column of Table VI are indicated ten regions of the spectrum, including all the observed lines; the second column contains the numbers of the lines measured in each region; in the

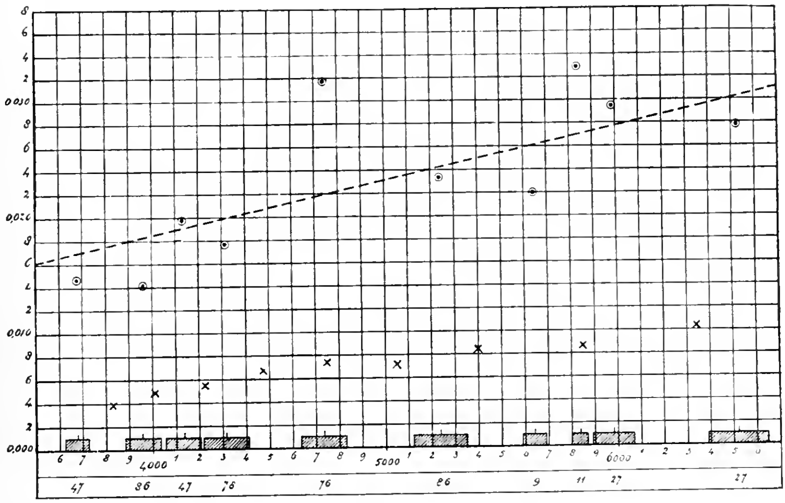


FIG. 6.—Mean displacements in successive regions of the spectrum
⊙ ⊙ ⊙ relative displacements in spot spectra (St. John)
× × × displacements at the sun's limb (Adams)

TABLE VI

DISPLACEMENTS AND WAVE-LENGTH

Region of Spectrum	Number of Lines	Mean λ	Mean Displacement
λ 3625 to 3725	47	3675	0.0147
3880 to 4035	86	3957	.0141
4055 to 4205	47	4130	.0197
4215 to 4410	78	4312	.0176
4635 to 4830	76	4732	.0317
5120 to 5350	86	5235	.0233
5590 to 5690	9	5640	.0219
5800 to 5870	11	5835	.0328
5890 to 6070	27	5980	.0294
6390 to 6650	27	6520	0.0277

third and the fourth columns are the mean wave-length and the mean displacement for each region.

Although on the average the displacements obviously increase with wave-length, the most striking feature of the table and the

figure[1] is the great variety of the *mean* values along the spectrum. One cannot admit such fluctuations to be entirely accidental or due to observational errors. The important retrograde difference between the values for mean λ 4732 and mean λ 5235, e.g., far exceeds the mean error of the result of 76 and 86 observations, and must be genuine.[2] If the Doppler effect were the cause of the displacements, their variation with wave-length would be represented by the straight line shown in the figure. The great deviations from this line prevent us from considering the data as decisively in favor of the hypothesis that we are dealing with a radial motion phenomenon. We suggest under reserve the following explanation, in which the combined effects of scattering and refraction are considered.

Suppose for a moment that the irregular density gradients of the solar gases had all disappeared, only the slow radial gradient being left. We should then look down through a perfectly calm, moderately transparent medium upon an evenly luminous background. The brilliant core of the sun would be seen through a kind of haze or fog caused by molecular scattering. As the scattering coefficient varies inversely as the fourth power of the wave-length, any definite degree of fogginess would be found for violet light at a higher level than for red light. If there were self-luminous or absorbing objects in the medium, they would be visible down to greater depths with red waves than with violet waves.

Now let the density gradients reappear, and with them the sinuating of rays and the resulting irregular distribution of the light. Evidently the beams of red waves will have traveled longer distances through the sun before emerging than the beams of violet waves; at the lower levels the average density gradients

[1] The mean relative displacements in the spot spectrum are indicated by small circles. The crosses in the lower part of the figure give the mean values of the displacements of Fraunhofer lines *at the limb* for successive regions of the spectrum, as deduced from measurements made by Adams.

[2] It is worth noticing that this anomaly occurs at the same place in the spectrum where the curve representing the means of Adams' measurements of limb displacements as a function of wave-length also sinuates (cf. Fig. 6), and where H. C. Vogel's well known table of spectrophotometric observations on the distribution of various kinds of light on the solar disk shows a similar anomaly. We do not venture to explain these remarkable coincidences as yet, although the anomalous-dispersion theory indicates connective points.

probably are steeper; red light therefore has in general more opportunities to get strongly deviated, and thus to produce contrasts, than violet light. This may account for the general tendency of the displacements to increase with increasing wave-length.

As to the fluctuations of the mean displacements along the spectrum, it seems possible to make the oscillating values of $n_0 - 1$ responsible for them, in the manner already discussed on p. 19.

ON ST. JOHN'S APPRECIATION OF ANOMALOUS DISPERSION AS A POSSIBLE CAUSE OF THE RELATIVE DISPLACEMENTS

Although the preceding pages implicitly contain a reply to the greater part of St. John's critical remarks on anomalous dispersion, it would seem proper still to discuss briefly his principal objections seriatim.

In judging of the degree of correspondence between the figure illustrating the way in which anomalous dispersion acts in producing the shifts (Fig. 4, p. 9) and the aspect of the lines found on the plates, St. John confined his attention to the stronger lines. Our schematic figure, however, is not directly applicable to strong lines for the reasons amply discussed on p. 21. In the case of weak lines the correspondence appears to be quite satisfactory.

Of the three facts mentioned by St. John as requiring explanation from the point of view of anomalous dispersion, two have been explained in this paper, viz., the variation of displacement with wave-length, and the variation with the intensity of the lines. The fact that a few lines (corresponding to prominent chromospheric lines) are displaced in the opposite direction has provisionally been left out of consideration in view of a special research now in progress. On the other hand, a fourth and very marked fact not explained by St. John's theory, viz., the large deviations of individual displacements from the means for each intensity class and spectral region, is shown to be in harmony with our interpretation on the basis of anomalous dispersion.

In a note on the interpretation of spectroheliograph results and of line-shifts[1] I explained why Mr. Adams, when comparing

[1] *Astrophysical Journal*, **31**, 428, 1910.

certain laboratory results on anomalous dispersion with the displacements of Fraunhofer lines at the limb,[1] failed to find any clear relationship between the two phenomena. Mr. St. John criticizes this explanation. He quotes from my paper:

> That a simple comparison of Geisler's observations on anomalous dispersion of metallic vapors in the arc with displacements at the limb—as given by Adams on page 28—could not possibly serve the purpose of finding such a relationship is evident. . . . .

Now, this was only a fraction of a sentence; the continuation of it runs thus:

> for the amount of that part of the displacement which is due to anomalous dispersion is determined by the degree of asymmetry of the Fraunhofer line under consideration; and this asymmetry is not a mere property of the corresponding element itself, revealable in laboratory experiments, but depends on the concentration with which that element is represented in the solar atmosphere.

But instead of completing the quotation by adding these explanatory lines, St. John comments as follows:[2]

> In view of the consideration that the basis of all astrophysical investigations rests upon the fundamental postulate that direct comparison is possible between the spectrum results obtained from terrestrial sources and the behavior of the spectrum lines in solar and stellar spectra, the first statement in the quotation is somewhat remarkable.

I am sure Mr. St. John would not have suggested to his readers such a bad opinion of my working method if he had realized the meaning of the part of my argument which he represented by an ellipsis. Indeed, from the point of view that Fraunhofer lines are dispersion bands, their asymmetry is due to the fact that, for the narrow region of wave-length surrounding each separate line, n_0-1 generally differs from zero (n_0 being determined by the composition of the solar gaseous mixture). The degree of asymmetry depends on both n_0 and the anomaly produced by the line itself.[3] As in Geisler's experiments the solar values of n_0 did not enter, nor anything analogous to them, the magnitude of the displacements of

[1] *Astrophysical Journal*, **31**, 57, 1910.

[2] *Mt. Wilson Contr.*, No. 74, p. 43.

[3] *Proc. Roy. Acad. Amsterdam*, **12**, 281, 1909; *Physikalische Zeitschrift*, **11**, 68, 1910.

the Fraunhofer lines at the limb bears no relation at all to the results of those experiments. This assertion does not imply any disregard of the necessity of testing theories by experimental research wherever possible, provided that the test be based on sound reasoning.

St. John's next criticism bears on the following statement which he quotes from the same paper: "A peculiar feature of our explanation is that both very strong and very weak anomalous dispersion make the displacements small, whereas intermediate values give larger displacements."

We were here concerned with displacements at the limb. Relative displacements at opposite edges of the spot spectrum have an entirely different origin. It is clear, indeed, that if a Fraunhofer line happened to be perfectly symmetrical ($n_0-1=0$) in the spectrum of the mean photospheric light, anomalous dispersion would not displace it at the limb, but would nevertheless produce a relative displacement at the opposite edges of the spot spectrum. The sign of $n-1$ is material to the Evershed effect, but almost immaterial to the Adams effect.

This fundamental difference between the two kinds of solar displacements escaped St. John's notice. In Table XXII (*Contribution No. 74*, p. 43) he uses some of his observations on spot lines in order to disprove my contention regarding limb lines! The same confusion runs through pp. 44 and 45 of the paper; this is the cause of St. John's finding so many discrepancies between the results of his observations and what he wrongly believes to be requirements of the dispersion theory.

I can easily show, using Adams' measurements of the displacements of the Fraunhofer lines at the sun's limb,[1] that the above-quoted deduction from the anomalous-dispersion theory is in perfect harmony with the facts.

Adams himself considers pressure as the effective agent in producing these displacements; it therefore did not occur to him to classify the shifts according to line intensity. Now accomplishing this classification, we obtain the synopsis given in Table VII.

[1] Adams, *Mt. Wilson Contr.*, No. 43; *Astrophysical Journal*, 31, 30, 1910.

The result is very striking; the shifts are greatest for lines of intensity 5, 6, and 7, and decrease progressively for the lower as well as for the higher intensities. This is exactly what the theory requires on the view that the intensity of Fraunhofer lines is chiefly determined by anomalous dispersion, and that their apparent displacements toward the red are simply due to the inequality of the average refraction suffered by the R-light and the V-light of

TABLE VII

SYNOPSIS OF ADAMS' MEASUREMENTS OF THE DISPLACEMENTS OF LINES AT THE LIMB, SHOWING THE MEANS FOR EACH LINE INTENSITY

	Intensity										
	0	1	2	3	4	5	6	7	8	9–12	15–40
Number of lines measured.......	7	51	99	106	71	40	41	14	12	11	15
Mean displacement (unit=0.001 A)	3.6	5.5	6.6	6.8	7.1	8.8	8.3	8.8	7.9	5.3	3.0

TABLE VIII

SYNOPSIS OF ADAMS' MEASUREMENTS OF THE DISPLACEMENTS OF LINES AT THE LIMB, SHOWING THE MEANS FOR NINE CONSECUTIVE REGIONS OF THE SPECTRUM

	Region of Spectrum								
	3740–3923	3923–4100	4100–4350	4350–4600	4600–4900	4900–5200	5200–5600	5600–6100	6100–6580
Number of lines.....	52	54	70	76	38	41	54	34	50
Mean displacement (unit=0.001 A)...	3.9	4.9	5.5	6.7	7.4	7.1	8.4	8.6	10.3

each line. Indeed, the apparent displacement must then always be a *fraction* of the width of the line; it therefore decreases with line intensity. The fraction, however, will in general be smaller with wide lines than with narrow lines (for it depends on the variable proportion between $n-1$ and n_0-1), thus making the asymmetry less conspicuous in the case of the wide lines. Lines of moderate intensity therefore show the largest mean displacements. And because the value of n_0-1 fluctuates along the spectrum,

especially in the vicinity of strong lines, we also conceive that in every intensity class the individual displacements may deviate widely from the mean, as they really do.

Table VIII contains the means of Adams' measurements for nine consecutive regions of the spectrum. They are plotted in the lower part of Fig. 6, and give evidence of a progressive increase with wave-length, excepting the anomaly between λ 4500 and λ 5500 already alluded to in the footnote on p. 24. This variation with the wave-length,[1] exhibited by the general shifts of the Fraunhofer lines toward the red, may perhaps be explained on the same basis as the corresponding variation observed in the case of the relative displacements in the spot spectrum (cf. p. 24). Both phenomena would seem to be due to the united influence of refraction and molecular scattering, and the refraction effects would be greater for the longer waves.[2]

Continuing the discussion of St. John's criticism, we arrive at his statement that relative displacements are sometimes observed in the spot spectrum when the slit of the spectrograph is perpendicular to the radius of the solar disk passing through the center of the umbra. It is argued that such displacements are very simply explained as Doppler effects, indicating occasional cyclonic movement, and that, on the other hand, the dispersion theory is unable to account for them. The latter inference, however, supposes the region of minimum density to have an ideally symmetrical shape. As deviations from that condition are quite probable, there is no difficulty in accounting for the occasional displacements

[1] In a paper, "Les Raies de Fraunhofer et la dispersion anomale de la lumière," published in *Le Radium*, **7**, October 1910, I suggested that the variation with wave-length here considered might be due to a general increase of n_0 with the wave-length, but I am now inclined to think that the influence of refractional scattering is more effective.

[2] A general displacement of the Fraunhofer lines toward the red, proportional to the wave-length, and amounting to about 0.010 A for λ 5000, is required by the gravitation theories of Einstein (*Annalen der Physik*, **35**, 905, 1911) and Nordström (*ibid.*, **42**, 549, 1913). Gravitation may thus contribute to the production of the observed shifts, but it certainly is not their main cause, since it does not account for the principal features of the phenomenon: the great variability of the shifts from line to line, and the marked relation between the mean shifts and the intensities of the lines.

in question on the basis of unequal refraction at opposite edges of the spot.

In the case of the *winged lines*, I had originally assumed that the core of the line was a pure absorption effect. In later publications,[1] evidently not considered by St. John in connection with the present subject, I was led to the conclusion that even the cores of those winged lines might be influenced by anomalous refraction and scattering. Probably a thorough treatment of these cases will prove to be difficult because the electronic theory of the dispersion, scattering, and absorption of light seems to require some extension in order to make it applicable to the very centers of wide lines. The subject is reserved for further investigation.

St. John's final remarks on the general question how far it seems probable that refraction and anomalous dispersion would produce any solar phenomena may be passed over, because they essentially refer to a paper, "On the Application of the Laws of Refraction in Interpreting Solar Phenomena," by Mr. Anderson.[2] At the time Mr. St. John wrote his criticism, my refutation[3] of Anderson's argument had not yet been published.

SUMMARY

1. The best established general result deduced by St. John from his measurements of the Evershed effect is that the displacements appear to vary progressively with the intensity of the lines. The regular progression of the mean values calculated for successive intensity classes pleads in favor of the general accuracy of the measurements.

2. Means taken for successive regions of the spectrum (Table VI, Fig. 6), though roughly indicating increase with wave-length, run very irregularly. Their deviations from a line representing proportionality with wave-length are too great to be attributable to accidental errors, and therefore prevent us from considering the results as decisively in favor of the hypothesis that the displacements are due to the Doppler effect.

[1] *Proc. Roy. Acad. Amsterdam*, **13**, 881 and 1263, 1911; *Physikalische Zeitschrift*, **12**, 329 and 674, 1911.

[2] *Astrophysical Journal*, **31**, 166, 1910. [3] *Ibid.*, **38**, 129, 1913.

3. St. John's hypothesis of the "Iron Scale," according to which the lines of an element are supposed to originate at a lower level as their intensity is smaller, meets with difficulties from the point of view of the physicist.

4. The insufficiency of indications of vertical motion in sun-spots is unfavorable to the hypothesis that the displacements considered are due to radial outflow of matter from spots.

5. A characteristic feature of the displacements is their great diversity of magnitude along the spectrum, even if lines of about equal intensity are considered. This peculiarity, which seems to be inexplicable on the basis of the radial-motion hypothesis, follows immediately from the anomalous-dispersion theory, because from that point of view the displacement of a line in the spot spectrum depends on (*a*) the anomaly of the dispersion-curve produced by the line considered, and (*b*) the value of n_0 which is determined by the other lines, and therefore fluctuates along the spectrum.

6. For each line intensity and spectral region a "normal displacement" can be deduced from St. John's measurements. The dispersion theory requires that the amount of the displacement of a line A will be sensibly influenced by a strong neighboring line B. On the assumption that n_0 is > 1, the influence must be such that if B lies on the red side of A, it reduces the displacement of A as compared with the normal value; if B is situated on the violet side, it must have the opposite effect, but to a lesser degree. This inference is perfectly borne out by all the evidence that can be gathered from St. John's Table I. In so far as other theories appear unable to account for this mutual influence of Fraunhofer lines, we may consider the phenomenon as directly proving the efficacy of anomalous dispersion in the sun.

7. The law connecting the Evershed effect with the intensity of the lines is in harmony with the deductions from the dispersion theory.

8. Judging from the behavior of the weakest lines (for which the optical effect of the general spot gradients is not much disturbed by the effect of superposed irregular density gradients), one gets the impression that nearly the whole width of the Fraun-

hofer lines must be due to anomalous dispersion, or that Fraunhofer lines are in the main *dispersion bands*.

9. The apparently intricate connection between displacements and wave-length seems to be explicable if we consider (*a*) that on account of molecular scattering short waves have on the average less opportunity of being refracted than long waves, and (*b*) that the value of n_0 fluctuates along the spectrum.

10. A discussion of St. John's remarks on anomalous dispersion made it necessary to expatiate on the difference in character which from the point of view of the anomalous-dispersion theory exists between the displacements of spot lines (Evershed effect), and the displacements of lines at the limb, as studied by Adams.

11. If the displacements at the limb, measured by Adams, are classified according to line intensity, and averaged, the means are found greatest for intensities 5, 6, and 7, and gradually decrease for greater as well as for smaller intensities. This law was predicted a few years ago by our theory; it will be difficult to explain it on the basis of the current interpretation of those displacements as a pressure effect.

Physical Laboratory
University of Utrecht
March 1914

ON BRIGHTNESS AND CONTRAST IN OPTICAL IMAGES

By P. G. NUTTING

The brightness of an image has been treated theoretically by a number of investigators and quantitative measurements have been attempted by several. However, both the theory and its experimental verification appeared so unsatisfactory that it seemed desirable to obtain a more nearly complete solution of the theoretical problem and to develop methods for making more precise direct measurements of relative illumination.

The theory here given is believed to be complete for portions of object and image near the axis and normal to it, and for an object at any distance with any focal length of lens or mirror. It has not been extended to oblique pencils. On the other hand, experimental methods have been developed to a point where determinations are scarcely more uncertain than photometric settings. The experimental method finally adopted was, in brief, to form an image of an extended plane source, by means of the lens under test, upon a matte, white-reflecting surface of magnesium carbonate. The illumination of this image was compared with that of the same spot with the lens removed.

Precision is attained by (*a*) using light from the *same* source to illuminate both lens and comparison surface of the photometer, thus avoiding errors due to fluctuation in the source, and (*b*) by careful determination of stray light. Calling I_0 the illumination at the source (strictly the normal luminous flux), I_1 that at the receiving screen without the lens, and I_2 that with the lens, the ratio I_1/I_0 is computed from dimensions and distances, while I_1/I_2 is observed with the photometer, hence I_2/I_0, the relative illumination of object and image, is obtained.

The actual source used was a 1500 candle-power tungsten lamp in a cubical white-lined box, the front face of which was a sheet of opal glass. This face was 33.5×35.5 cm and in use emitted about 0.7 candle per sq. cm. At about 5 meters from this face was the receiving block. The photometer used was a modified Beckstein

illuminometer. This was sighted directly on the image spot on the receiving screen, after removal of the receiving screen attached to the instrument. The comparison screen of this instrument is ordinarily illuminated by a glow lamp contained in a white-lined sphere; this was removed and light directly from the testing source

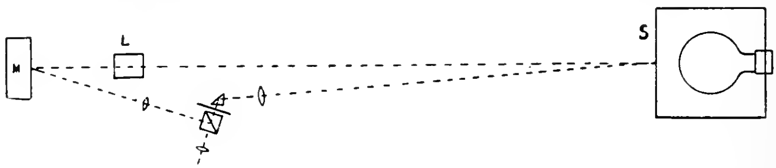


FIG. 1

reflected on the screen, so that fluctuations in the source were of no consequence. The final illumination in the photometer cube was ample for maximum sensibility.

In previous treatments of the theory of image illumination there appears to have been no complete consideration of the foreshortening of zone pencils. The error is not negligible with aperture ratios greater than F/8, and in modern high-speed objectives becomes quite large. Further difficulties have arisen from confusion of terms relating to luminous intensities.

Consider an axial element ds_0 of a plane object normal to the axis. Let the brightness of this object and its diffuseness of emission be such that in the direction of the axis the illumination is dL lumens per unit area of object. Call this intensity I_0. The illumination in the direction of the lens is then such as would be given by a point source of intensity $I_0 dS_0$ at the axial point of the

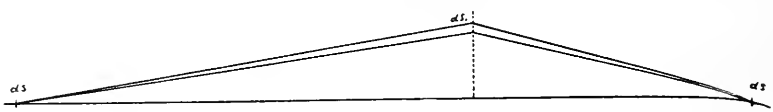


FIG. 2

object. The axial illumination (flux density) at a distance u from the object is then $I_0\, dS/u^2$. The total flux dL on a normal axial element dS at a distance u is $I_0 dS_0 dS/u^2$. The flux from an element dS_0 to an element dS_1 not parallel or coaxial with it, is

$$dL = \frac{I_0 dS_0 \cos \alpha \cos \alpha' \, dS_1}{u^2 + r^2} \tag{1}$$

where α and α' are the angles between the normals to dS_0 and dS_1 and the line joining them.

In the case of a lens, the element dS_1 is a ring-shaped element passing through the lens, normal to its axis at the intersection of the zonal surface with the axis. The zonal surface is the locus of intersection of corresponding image and object zone pencils. The element dS_1 is strictly an element of the zonal surface, but if this is used the light-integral assumes an exceedingly complex, non-integrable form. The assumption that the zonal surface is plane introduces a second order error which will be discussed and evaluated below.

We have then $dS_1 = 2\pi r dr$ and $\alpha' = \alpha$. On the image side of the lens the light within each cone element from dS_0 falling upon the image element dS_1 is constant and equal to that falling on the ring element dS_1 of the zonal surface, corrected twice for foreshortening by the factor $\cos^2 V$. The total flux on dS is then

$$L = I_0 dS_0 \int_0^R \frac{\cos^2 U \cos^2 V}{u^2 + r^2} dS_1 \tag{2}$$

But,

$$\cos^2 U = u^2/(u^2+r^2) \text{ and } \cos^2 V = v^2/(v^2+r^2)\,,$$

hence

$$L = 2\pi I_0 dS_0 \int_0^R \frac{u^2 v^2 r dr}{(u^2+r^2)^2 (v^2+r^2)} \tag{3}$$

The flux density at the image is L/dS, but $dS_0/dS = u^2/v^2$, hence

$$I = I_0 \pi \left[\frac{u^4}{(u^2-v^2)^2} \log \frac{\dfrac{u^2}{u^2+R^2}}{\dfrac{v^2}{v^2+R^2}} - \frac{u^2}{u^2-v^2} \frac{R^2}{u^2+R^2} \right] \tag{4}$$

This is the complete expression for relative illumination I/I_0 of object and image, for any object distance u, image distance v, and any aperture $2R$.

The correction for curvature of zonal surface may be found as follows, making use of the sine condition. The equation of the zonal surface is

$$\frac{\sin V}{\sin U} = \text{const.} = \frac{u}{v}$$

or in Cartesian co-ordinates (see Fig. 3)

$$\frac{(u+x)^2+y^2}{(v-x)^2+y^2}=\frac{u^2}{v^2}$$

$$(u^2-v^2)y^2=v^2(u+x)^2-u^2(v-x)^2$$

Hence for any y the zonal surface makes an angle with the axis whose tangent is

$$\frac{dy}{dx}=\frac{v^2(u+x)-u^2(v-x)}{y}$$

The most unfavorable case is evidently for $U=0$, $u=\infty$. In this case

$$\frac{dy}{dx}=\frac{v-x}{y}$$

or the zonal surface is a circle with v as a radius and the image as center.

Now the error in (2) due to assuming the zonal surface plane is an error in the product $P=\cos U \cos V$, in which U is increased and

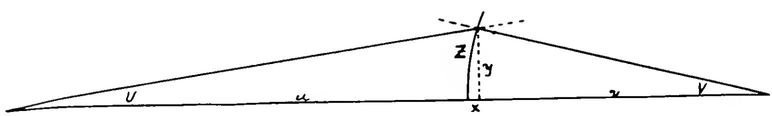


FIG. 3

V decreased by the same small angle. Differentiating this product and putting $dV=-dU$, the error in P is $dP=-\sin(U-V)dU$. In the least favorable case ($U=0$) the maximum error is less than 0.005 in the case of an $F/8$ lens, or 0.02 for an $F/3.5$ lens.

The general expression for relative illumination (4) may be expressed in the simpler form:

$$\frac{\rho}{\pi}=\frac{1}{(1-m^2)^2}\log\frac{\cos^2 U}{\cos^2 V}-\frac{\sin^2 U}{1-m^2} \qquad (5)$$

by putting $\rho=I/I_0$ and $m=v/u$ and substituting $\cos U$, $\cos V$, and $\sin U$ for their values in u, v, and R.

The form best adapted for computation is

$$\frac{\rho}{\pi}=\frac{1}{(1-m^2)^2}\log\frac{1+\left(\frac{a}{1+m}\right)^2}{1+\left(\frac{am}{1+m}\right)^2}-\frac{1}{1-m^2}\frac{1}{\left(\frac{1+m}{am}\right)^2+1} \tag{6}$$

$$=M^2\log\frac{1+S}{1+T}-M\frac{T}{1+T}$$

since the quantities given are m and aperture ratio of lens, $F/2R=1/2a$.

In the special case of object and image at equal distances ($u=v=2F$) from the zonal surface, (4), (5), and (6) are indeterminate but (3) integrates into

$$\frac{\rho}{\pi}=\frac{R^2(2u^2+R^2)}{(u^2+R^2)^2}=\frac{2w^2+1}{2(w^2+1)^2},\ w=\frac{R}{u}=\frac{2}{a} \tag{7}$$

The special case of an infinitely distant object is of frequent occurrence. In this case

$$\rho=-\pi\log\cos^2 V=\pi\log(1+a^2) \tag{8}$$

which, for V small, approximates closely to

$$\rho=\pi\sin^2 V=\pi a^2 \tag{9}$$

the form usually quoted. This is the relative illumination that would obtain if a flux density equal to that at the object came from the zonal surface itself.

In Table I are given computed ratios of flux density at image to that at the object for various apertures and various ratios of image

TABLE I

Aperture	$m=0$	$m=0.1$	$m=0.2$	$m=0.5$	$m=1$	$m=2$
1.........	0.704	0.580	0.521	0.333	0.179	0.0775
2.........	.1902	.1580	.1321	.0864	.0491	.0210
5.........	.0312	.0255	.0219	.01296	.00785	.00347
10.........	.00785	.00625	.00553	.00349	.00197	.000870
20.........	.00196	.00162	.00134	.000878	.000490	.000217
50.........	.000314	.000259	.000216	.000135	.000078	.0000346
100.........	0.000078	0.000065	0.000051	0.000044	0.000019	0.0000086

distance to object distance ($v/u=m$), neglecting losses by reflection and absorption.

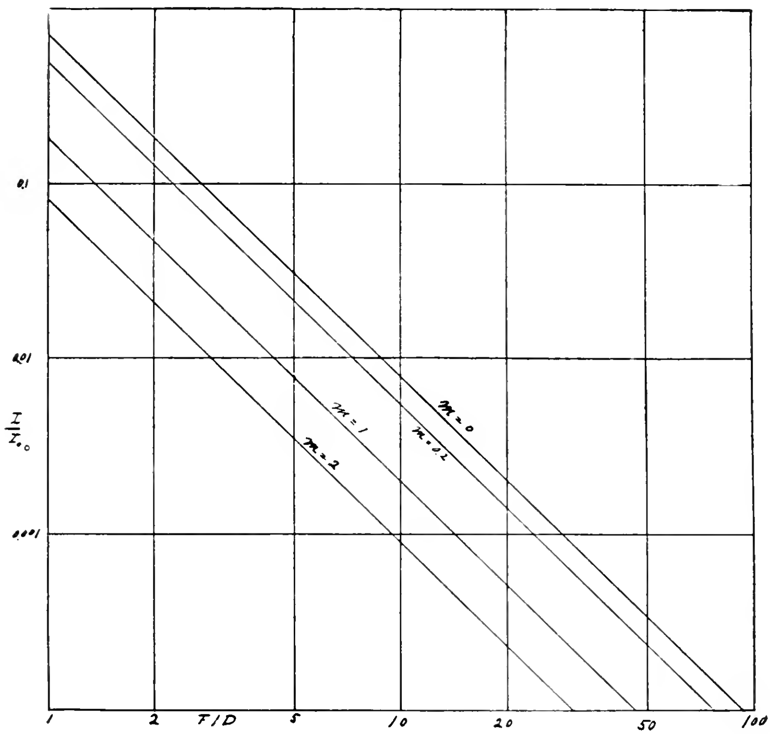


Fig. 4

The experimental data taken are given in the following tables and figures. They refer to the lenses listed below:

1. Cooke Process, 18″, Series V, No. 32051, tested at F/8 to F/90.
   E.F.L. 46.7 cm, $I_1/I_0=0.00475$, $m=0.08654$.
2. Cooke Process, 11″, Series V, No. 31796, tested at F/8 to F/32.
   E.F.L. 28.6 cm, $I_1/I_0=0.00475$, $m=0.0675$.
3. Cooke Process, $7\frac{1}{2}$″, Series V, No. 33991, tested at F/8 to F/32.
   E.F.L. 19.5 cm, $I_1/I_0=0.00475$, $m=0.0417$.
4. B. & L.-Zeiss Tessar, Series IC, No. 1320682, tested at F/5.4 to F/32.
   E.F.L. 15.05 cm, $I_2/I_0=0.00494$, $m=0.0309$.
5. Cooke, Series II, F/4.5, No. 40399, tested at F/4.72 to F/22.
   E.F.L. 20.93 cm, $I_1/I_0=0.00499$, $m=0.046$.

6. Fuess telescope objective (cemented doublet), F/5.3
   E.F.L. = 15.3 cm, Ap. 2.90 cm, $I_1/I_0 = 0.00494$, $m = 0.033$.
7. B. & L.-Zeiss Tessar, Series IC, No. 95933, 40 mm, F/4.5, tested at F/4.94.
8. Cooke Cinematograph, 2″, F/3.5, tested at F/3.67.
9. Zeiss-Krauss Cinematograph Tessar, F/3.5, 75 mm (77.0/19.5).
10. Zeiss-Krauss Cinematograph Tessar, F/4.5, 150 mm (152.1/32.3).

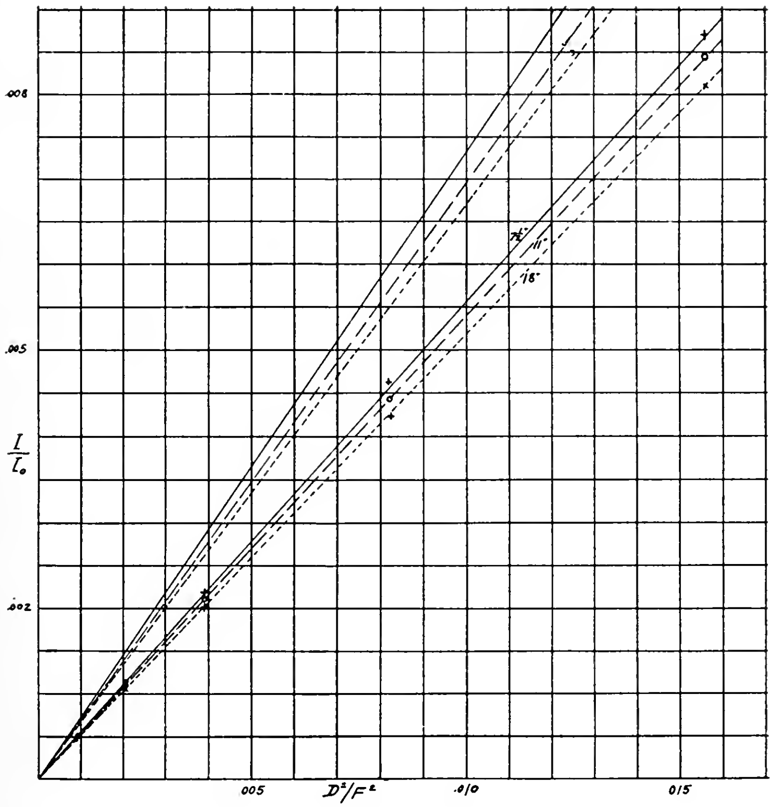


FIG. 5.—Tests of three process lenses

All apertures and focal lengths were determined on a precision lens bench; the marked stop points were correct to within the error in setting, except the lower marks on the high-speed lenses (F/4.5 and F/3.5). Uncertainty in setting the stop at a given mark (in measuring illumination) amounted to roughly 2 or 3 per cent, hence the results are uncertain to about 5 per cent. The

slope of the line relating I_2/I_0 to R^2/F^2, used in obtaining the specific transmission, is uncertain to certainly less than 3 per cent. The three process lenses show a sensibly linear relation between I_2/I_0 and D^2/F^2 ($D=2R$). The observed brightness divided by the computed brightness is taken as a measure of the percentage transmission. The constancy of this for different stop openings indicates the precision of setting and measurement.

TABLE II

Stop	18″ Cooke, Ser. V			11″ Cooke, Ser. V			7½″ Cooke, Ser. V		
	Obs.	Comp.	Trans.	Obs.	Comp.	Trans.	Obs.	Comp.	Trans.
F/8........	0.00820	0.01058	0.774	0.00841	0.01085	0.776	0.00870	0.01136	0.766
11.........	.00374	.00459	.816	.00441	.00570	.774	.00463	.00600	.770
16.........	.00206	.00264	.783	.00210	.00270	.779	.00202	.00283	.714
22.........	.00108	.00140	.773	.00116	.00143	.781	.00115	.00150	.766
32.........	.000524	.000661	.792	0.00051	0.00067	0.762	0.00050	0.00071	0.72
45.........	.000268	.000336	.796	....	....	....	....	....	....
64.........	.000156	.000166	.94	....	....	....	....	....	....
90.........	0.000100	0.000083	0.83	....	....	....	....	....	....

There are six free surfaces in these lenses. The sixth root of 0.77 is 0.956, a percentage transmission per surface corresponding to a refractive index of 1.53. Hence the observed brightness of image is slightly (about 4 per cent) higher than would be expected.

The Tessar and Cooke F/4.5 lenses gave results shown in Table III.

TABLE III

B. & L.-Z. Tessar IC				8″ Cooke, Series II			
Stop	Obs.	Comp.	Trans.	Stop	Obs.	Comp.	Trans.
F/4.74....	0.0249	0.0328	0.758	F/4.72....	0.0237	0.0321	0.739
5.66.....	.0184	.0238	.724	5.68.....	.0172	.0232	.744
8........	.00890	.0115	.766	8........	.00836	.0112	.749
11........	.00476	.00610	.765	11........	.00425	.00595	.715
16........	.00231	.00288	.804	16........	.00210	.00282	.664
22........	.00115	.00152	.765	22........	0.00117	0.00149	0.800
32........	0.00054	0.00072	0.750	....	....	....	....

The remaining small lenses (m sensibly zero) gave:

Fuess telescope objective, cemented doublet, F/5.32: $a=0.0940$, observed I_2/I_0 0.0257, computed 0.0278, ratio 0.921.

B. & L.-Zeiss Tessar, IC, F/4.5, 40 mm: F/4.94, $a=0.1012$, observed $I_2/I_0$0.0172, computed 0.0322, ratio 0.535.

2″ Cooke Cine, F/3.4, tested F/3.67: $a=0.1360$, observed $I_2/I_0$0.0393, computed 0.0585, ratio 0.673.

Zeiss-Krauss Cine Tessar, F/3.5, 75 mm, tested at F/3.69: $a=0.1355$, observed $I_2/I_0=0.0330$, computed 0.0579, ratio 0.571.

Zeiss-Krauss Cine Tessar, F/4.5, tested at F/4.71: $a=0.1063$, observed $I_2/I_0=0.0234$, computed 0.0355, ratio 0.660.

CONTRAST IN OPTICAL IMAGES

Large details of an image are of course of the same *relative* brightness as the corresponding details of the object. But details beyond the resolving power of the instrument may gain or lose enormously in contrast relative to their background, depending upon whether these details be points or lines darker or brighter than the background. Such observations as those on the minor satellites and planetary markings depend largely upon the choice of instruments.

For a portion of a telescopic image well above the limit of resolution, the relative brightness of image and object[1] is proportional to the square of the ratio of diameter of objective to its focal length (D^2/F^2). This applies to images formed on photographic plates or other focusing screens. The brightness of an image viewed by the eye is independent of aperture and focal length, $I/I_0=$constant. Details below the limit of resolution are spread out over a width (δ) proportional and approximately equal to $\lambda F/D$.

If the unresolved detail is the image of a point, its brightness is proportional to relative area of objective and diffraction disk or to D^2/δ^2, if the image of a fine line to D^2/δ. We have then for relative illumination of image and object, quantities proportional in these cases to a constant for a visual image; for an image on a plate or screen to:

$$D^2/F^2=A^2 \text{ for plain object}$$
$$D^2/\delta^2 =A^2D^2/\lambda^2 \text{ for point object}$$
$$D^2/\delta =AD^2/\lambda \text{ for line object}$$

From these relations the conditions for maximum contrast in the image are readily determined.

[1] See Nutting, *Applied Optics*, p. 79.

For *light* details[1] on a darker background, maximum contrast in a photographic image is obtained by increasing the brightness of detail over that of the background. If the object is a point, there is a clear gain in increasing diameter of objective D, keeping aperture ratio A constant. On the other hand, there is no gain by increasing relative aperture A by decreasing focal length F, keeping diameter D of objective constant.

For a bright-line object on a darker ground, relative contrast of image and object is proportional to DF/λ. This is enhanced by increasing either D or F or both. Increasing relative aperture by increasing D and decreasing F in the same proportion does not affect relative contrast. In a *visual* image, contrast of bright point details on a darker ground is proportional to A^2D^2, of line details to AD^2, hence there is a clear gain in increasing either aperture ratio or diameter of objective.

With *dark* details on a lighter background the conditions for maximum contrast are quite different. If the dark detail is a point, photographic contrast is proportional to $1/D^2$, and diameter of objective D should be small, aperture ratio A being of no consequence. If the detail is a dark line, then both D and F should be small to give best results.

In observing or photographing fine, dark details on Mars, for example, in so far as contrast is concerned, there is practically no advantage but a considerable disadvantage, aside from atmospheric disturbances, in using a large telescope. W. H. Pickering (4th Monthly Report on Mars) prefers in practice about an 8-inch objective as the best compromise.

Contrasts in dark details on a lighter ground in *visual* images are proportional to $1/A^2D^2$ and $1/AD^2$ for points and lines respectively; hence, in this case also, objectives of lesser aperture and focal length give better contrast.

While it may not be practicable to make use of these principles in the construction of a telescope for general purposes, they are of interest in explaining the widely varying images of the same object obtained with different instruments.

Eastman Research Laboratory
Rochester, N. Y.
January, 1914

[1] Wadsworth, *Astrophysical Journal* 6, 119–35, 1897.

ON THE INDIVIDUAL PARALLAXES OF THE BRIGHTER GALACTIC HELIUM STARS IN THE SOUTHERN HEMISPHERE, TOGETHER WITH CONSIDERATIONS ON THE PARALLAX OF STARS IN GENERAL[1]

By J. C. KAPTEYN[2]

I. INTRODUCTION AND SUMMARY

In this paper a first attempt is made to find the parallax of practically all the helium stars brighter than the sixth magnitude for that part of the sky which lies between galactic latitudes $\pm 30^\circ$ and galactic longitudes 216°–360°. I hope in a subsequent paper or papers to deal with the helium stars in the other parts of the sky. For the brighter stars of other spectral classes I have not here tried to derive individual parallaxes, but have discussed somewhat at length the prospects for the successful treatment of such an investigation. A few years ago the undertaking might well have seemed hopeless. Since the discovery of the phenomenon of "star-streaming," however, the outlook has become so much brighter that it seems necessary to look somewhat more closely into the matter. Finally, I have devoted a few pages to the consideration of what we may reasonably hope to achieve for the fainter stars.

At the fourth conference of the Solar Union in 1910, I made some preliminary statements on the results of a study of the helium stars.[3] Attention was drawn to the extraordinary parallelism shown by the proper motions of these, and at least a certain number of the early A stars, in some parts of the sky. Shortly afterward it appeared that for the region of the constellations of Scorpius and Centaurus the phenomenon had also been noticed independently by other astronomers. Opinion, however, was divided on the question whether or not the stars in this particular region of the sky form a separate group. While Eddington and I held that they

[1] *Mt. Wilson Contr.*, No. 82.

[2] Research Associate of the Carnegie Institution of Washington, Mount Wilson Solar Observatory.

[3] *Trans. Internat. Solar Union*, 3, 215–231, 1911.

do, this view was contested, even before my paper had appeared in print, by Campbell and B. Boss.

I hope that the present paper will sufficiently indicate the real state of affairs. Nevertheless, the question is, in my opinion, for the present of only secondary importance. The question of real importance is: Is the parallelism and equality of motion in this part of the sky of such a nature that we can derive individual parallaxes? Considerations as to whether or not this motion is almost wholly due to the progressive motion of the solar system, and as to whether the stars in other parts of the sky participate in it, may be interesting in themselves but do not touch the main conclusion either of my paper in 1910 or of the present one. This is the reason why I shall not reply expressly to the various objections raised.

As to the main point—the determination of individual parallaxes—though its possibility was pointed out in 1910, it could not then be carried out because sufficient radial-velocity data did not then exist. For the purpose of supplying such data, and more generally in order to provide a solid basis for a complete study of the B and early A stars, the Mount Wilson Solar Observatory resolved to place a large number of these stars on the observing program. Personally I tried to derive, and gradually to improve, whatever results could be obtained from the still incomplete data, in order to determine what further observations were most desirable. These observations are now approaching their termination and it will thus be possible very soon to bring the investigation of the helium stars, and of part of the A stars, to a provisional conclusion. In the meanwhile Campbell's catalogues of radial velocities of the B and A stars have appeared. For the northern sky the Lick measures, as well as those of the Yerkes, Allegheny, and other observatories, supplement the data collected at the Mount Wilson Solar Observatory in the most desirable way. For the southern sky Campbell's catalogues and the published contributions of the Cape Observatory constitute practically the whole of the available material.

The fact that the Mount Wilson observations are not fully completed, whereas for the southern sky we have definitive values which, for some time probably, will not be very greatly extended,

has been one of the reasons for beginning a somewhat more complete study of the helium stars with the Southern Hemisphere. Another and even more important reason for this choice lies in the far greater regularity shown by the astronomical proper motions in this part of the sky. It seems rational to begin the investigation with the study of the simpler case. It is this same consideration that made me restrict this study in the first instance to the stars within 30° of the galactic circle. By the exclusion of the higher latitudes we gain in homogeneity of material without losing many stars. It might even have been desirable to restrict our plan still more, for instance, by excluding the latitudes −20° to −30°, the longitudes beyond 330°, and the proper motions below a certain limit. The stars which appear to be exceptional would thus have nearly disappeared. I have preferred, however, not to go as far as this, but to cover a good part of the sky completely, in order to see what precision may now be obtained and also what difficulties remain to be faced.

The following short summary of the paper may make it more readily understood. It will at the same time supplement the information given in the paper itself.

All the helium stars between galactic latitudes ±30°, longitudes 216°–360°, in Boss's *Preliminary Catalogue*, which is complete to the sixth magnitude on Boss's scale (5.80 Harvard), but contains, besides, quite a number of fainter stars, are given at the end of the paper in three lists. The first two contain separately for galactic latitudes 0° to +30° and 0° to −30° all the stars having a secular motion of 1″.7 or greater. All the smaller proper motions are contained in the third list. The stars of the first two lists are plotted in Map 1, those of the third in Map 2. I have given the maps for the entire 360° of longitude, in order to show clearly the apparent tendency in these stars to clustering, a phenomenon that has been remarked by several astronomers. The most extensive of these "clusters" shown by Map 1 is between the longitudes approximately 200° and 340°. It is this group which forms the main subject of the present paper.

The total number of stars in Boss known to be of spectral class B is 752. Of these 655 (87 per cent) are between galactic

latitudes ±30°. The total number of B stars included in the present paper is 319, i.e., 42.4 per cent of all the Boss helium stars, and 48.7 per cent of those between galactic latitudes ±30°.

The first point is to determine, both by the proper motions and by the radial velocities, whether the group has a motion as a whole, and if so, the limits between which this is the case. This point is treated in detail in Sections 2–7. We meet with some difficulty in fixing the limit of the group on the side of the smaller longitudes. In order to arrive at a satisfactory conclusion about this point we

TABLE I

Group	l		b		100μ	n
A.	270° to 360°		0° to +30°		≦2″.4	96
B.	289	337	0	−30	3.0	29
C.	240	270	0	+25	1.7	28
D.	258	288	0	−30	1.7	20
E.	239	256	0	−20	1.7	25
F.	226	238	−20	+10	1.7	20
G.	217	225	−15	+10	≦1.7	11

TABLE II

Group	l		b		100μ	n
a.	195° to 216°		0° to −10°		≦1″.7	13
b.	195	216	−11	−25	1.7	6
c.	165	216	0	+30	1.7	9
d.	155	180	0	−14	1.7	11
e.	155	180	−15	−30	≦1.7	9

are compelled to include, as additional stars, those down to longitude 155°. The limit being fixed at 216°, these stars in lower longitudes are again discarded. They are of course contained in the maps, but not in the lists. The bulk of the stars retained were divided into the 7 groups indicated in Map 2, Plate II (at the end of this paper), and defined as in Table I (l=longitude; b=latitude; μ=total proper motion; n=number of stars).

The additional stars were subdivided as shown in Table II.

Some parts of the sky are not covered, but, as they contain very few stars, they may provisionally be disregarded. Only the

stars within the limits of Table I have been used in the derivation of the stream-elements. They have been marked in Lists 1 and 2 by their group-letters. The remaining stars are considered only when definitive elements have been obtained. The proper motions exceeding 1″.6 per century in the eight regions *E*, *F*, *G*, *a*, *b*, *c*, *d*, *e* have been plotted in Fig. 1. If in each of these areas we compute the average direction of the proper motion we get what has been

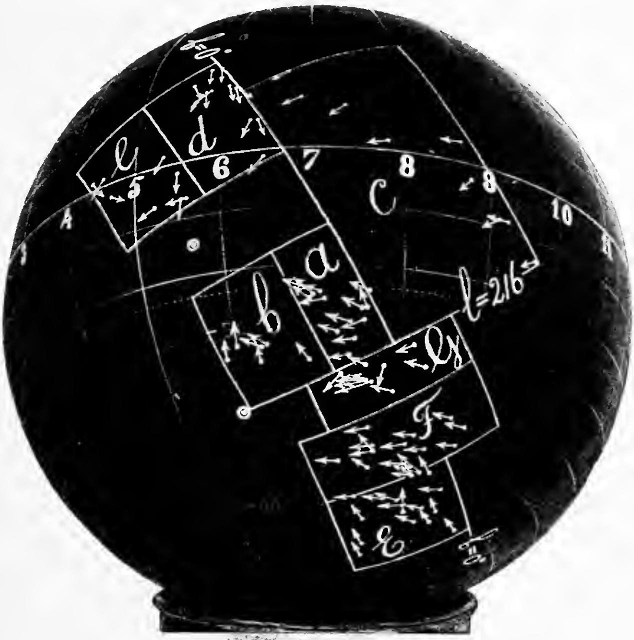


FIG. 1

represented in Fig. 2. Both figures make it plain that the directions of the proper motions converge, not toward a single point, but toward two rather widely separated points. A study of the figures further shows that the limit between the two streams thus indicated must lie somewhere near longitude 216°, and that there can be but little overlapping. Here, then, it was resolved to place the lower limit of the stars to be considered in the present study.

The community of motion in each of the parts *A*, *B*, ... *G* considered separately is very evident in our lists. It becomes

more striking if the proper motions are plotted. But is this motion the same for all the regions? In order to arrive at an unbiased judgment, an attempt is made to derive as complete elements as possible of the common motion in each part separately. The regions *E*, *F*, *G*, which are relatively small, are combined. In accordance with other investigations we have assumed from the outset that the observed radial velocities require a constant correction of -4.3 km. Campbell[1] finds -4.1. Fortunately our

FIG. 2

adopted value is exactly confirmed by the final solution in Section 8. In the cases shown in Table III a more or less reliable result is obtained. The conclusion is that for these regions there is community of motion.

For the regions *C* and *D* the data are not sufficient for the derivation of perfectly independent elements. They lie, however, between the regions *A* and *B* on the one side, and the regions $E+F+G$ on

[1] *Lick Bull.*, No. 196, p. 127, 1911.

the other (see Map 2); and both the position angles of the proper motions and the radial velocities agree with what is found from the elements furnished by surrounding regions. The conclusion is drawn, that, for the whole of the regions covered, the bulk of the stars move in a single stream whose elements are:

Vertex	18^h18^m, $+42°$	See (27)
Stream-velocity	$V=-18.3\pm0.9$ km	
Const. corr.	$K=-\ 4.3\pm0.5$ km	

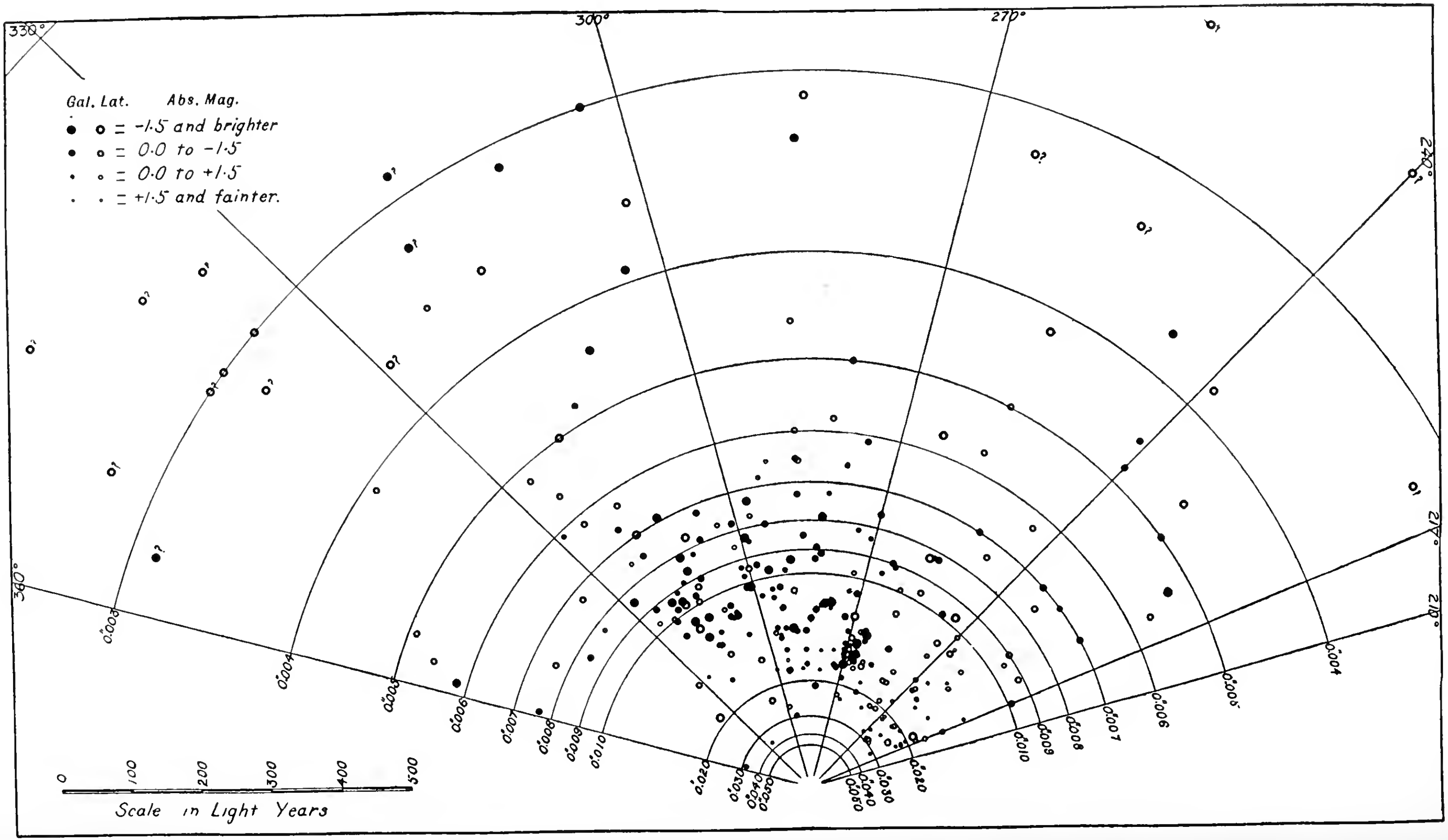
The residuals left by these elements in the mean position angles of the seven separate regions *A*, *B*, ... *G* are shown in Table XVIII, those in the radial velocities in Table XIX.

TABLE III

Region	Vertex		Stream-Velocity	P.E.	Reference Equations
A..............	18^h23^m	$+41.°5$	-15.3 km	±2.0	(1) and (2)
B..............	18 30	$+37$	....	....	(5)
E+F+G........	17 58	$+41.5$	-19.1	±0.75	(7) and (22)

The comparison of the motions of the individual stars, with the pure stream-motion according to these elements, leads to an estimate of the probable amount of the peculiar motion (Section 9). The probable amount r_u for the component in any direction is found to be only ±2.1 km per second. It is the smallness of this amount which suggests the possibility of applying to our case the method used for finding the parallax of the members of such groups as those of the Hyades and Ursa Major. The necessary formulae both for the parallaxes and for their probable errors are given in Section 12. The results obtained for the helium stars are summarized in Section 13. They enable us to draw a map in space of these stars (Map 3). Particulars may be found in Section 14.

A few stars have been excluded from this computation of the parallax on the ground that they do not belong to the group. These "exceptional" stars have been considered in Sections 10 and 11. The conclusion is reached that within the limits adopted for the present paper—and not counting the Vela group—not more than 4 per cent of the whole number of stars are extraneous to the group,



Map 3.—Arrangement of Helium Stars in Space

and these are mostly near the upper limit between 296° and 360° of galactic longitude and −9° and −30° of latitude. The Vela group, treated in Section 6, probably must be considered as a local group, much like the restricted Perseus group found by Eddington.[1] The data are as yet insufficient for the derivation of the elements of its motion.

The parallaxes for the stars of the main group once found, there is of course no theoretical difficulty in finding the luminosity-curve and the star-density at different distances from the sun. In practice it was found (Sections 15 and 16) that a somewhat satisfactory result could be obtained for the B0–B5 stars. That for the B5–B9 stars is less reliable. Still as the constants of the luminosity-curve for the B5–B9 stars were required for a particular purpose they have been deduced as well as the observations would permit. They are given in (48).

The main object of the paper, as far as the helium stars are concerned, would thus have been attained were it not that the proper motions which have been taken from Boss's *Preliminary Catalogue* were tacitly assumed to be systematically correct. For stars having in general such small proper motions as the B stars, the supposition is dangerous. A separate consideration of this point is therefore undertaken in Section 17, which leads to the result that a correction to Boss's proper motions in declination amounting to +0″003 is clearly indicated. The change in the stream-elements produced by such a correction has been determined. The altered parallaxes are given in the last column of the lists.

Section 18 is concerned with the question in how far the same method for finding the individual parallaxes—or a modification of it—may be applied with success to stars of the other spectral classes. In this section, as in all that precede, the parallax is derived from the motion of the stars. This practically restricts the method—at least for the present—to the brighter stars. In the last two sections, therefore, is considered the question as to what data we may hope to obtain for the distances of the faint stars, for which our knowledge of the motions, both astronomical and spectroscopic, is too slender to be of much value.

[1] *Monthly Notices*, **71**, 43, 1910.

2. REGION A: LONG. 270° TO 360°; LAT. 0° TO +30°; 100 $\mu \geqq 2''.4$

Very small proper motions are rare in this region, particularly below longitude 300°, as is shown by Table IV. In fact, there are only 13 stars in all for which 100 $\mu < 2''.4$. We therefore begin by confining our attention to the 96 stars for which 100 $\mu \geqq 2''.4$.

TABLE IV

100 μ	l 270°–299°	l 300°–330°	n
0''.0 to 0''.9	....	3	3
1.0 1.9	2	6	8
2.0 2.9	8	9	17
3.0 3.9	13	17	30
4.0 4.9	14	11	25
5.0 5.9	16	1	17
6.0 6.9	6	2	8
8.2	1	....	1
12.1	1	....	1
Total	61	49	110

TABLE V

	l	b	100 μ	a	δ	100 μ	p	n
A I	270° to 306°	0° to +30°	2''.5 to 3''.9	14h 23m	−45°.7	3''.30	219°.2	20
A II	270 306	0 +30	≧4.0	14 8	−46.7	5.50	223.6	39
A III	306 2	0 +30	≧2.4	16 23	−24.5	3.64	203.2	37
Mean (weight = number of stars)				15 4	−37.8	4.32	214.7	96

The parallelism of the astronomical proper motions of these stars is so great, and the radial velocities vary relatively so little (and on the whole regularly with the position in the sky), that there can be no serious question as to the community of motion for the group. Still, in order to dissipate whatever doubt might still exist, I have subdivided the group in three parts, for which the averages given in Table V were formed.[1]

Further, for the radial velocities see Table VI.

For the radial velocities the means were found in two different ways: first, by adopting as weight the number of stars observed

[1] In order to see how far the differences between the separate values are due to differences in position, compare with the values of p those computed by the help of the definitive elements in Tables XVII and XIX. Four stars, Boss 3458, 3699, 3716, 3929, were accidentally overlooked in forming the averages.

(the radial velocities marked (:) or depending on one observation receiving half-weight); second, by modifying the weights in such a way that the mean radial velocity became zero. From the second mean it follows at once that the vertex must lie exactly

$$90° \text{ from the point } 15^h 11^m, -35.^\circ 9 \qquad \text{(A)}$$

We may, however, obtain a result with somewhat greater weight by using the first mean value. Only in this case we have to assume a value for the stream-velocity. If we adopt −19 km per second, then from the last line but one in Table VI we conclude that, λ being

TABLE VI

	α	δ	$\rho-4.3$	Weight	Modified Weight
A I	$14^h 10^m$	$-45.^\circ 5$	$+4.^\circ 9$	8.5	6.0
A II	13 55	−46.0	+3.55	9	6.4
A III	16 46	−22.4	−5.8	9	9
Mean (actual weights)	14 57	−37.8	+0.8	26.5	....
Mean (modified weights)	15 11	−35.9	0.0	....	21.4

the distance to the vertex, $-19 \cos \lambda = +0.8$, whence $\lambda = 92.^\circ 4$. The uncertainty of the adopted stream-velocity is of hardly any importance, for we may vary it between 13.5 and 33 km without changing λ more than a degree. It seems preferable, therefore, to use the latter result on account of its greater weight. It defines a small circle

$$92.^\circ 4 \text{ from the point } 14^h 57^m, -37.^\circ 8 \qquad \text{(B)}$$

on which the vertex must be situated.

Finally, therefore, if we treat the region as a whole, the vertex is determined by the great circle defined by the last line of Table V and the small circle (B). The intersection of the two, which of course is nearly at an angle of 90°, gives

$$18^h 23^m, \quad +41.^\circ 5 \text{ (vertex A)} \qquad (1)$$

The determination by (A) and Table V, which is absolutely free from any supposition as to the stream-velocity, gives practically the same result.

I have also computed the vertex for the three subdivisions separately on the two assumptions $V=-19.0$ and $V=-25.0$ km. We find:

TABLE VII

VERTICES FOR SUBDIVISIONS

	$V=-19.0$		$V=-25.0$	
A I	18^h18^m	$+44°$	$18^h\ 6^m$	$+41°$
A II	18 6	+38	17 54	+36
A III	18 33	+45	18 48	+49

The influence of a wrong assumption for the stream-velocity is thus seen to be relatively small in all cases. Considering the uncertainty of the radial velocities, the differences in the positions of the three vertices, especially for $V=-19.0$, is also gratifyingly small. There thus seems hardly a doubt possible that the three form part of a single cosmical group, moving in the direction defined by the vertex (1).

This point being settled, we can derive the stream-velocity. As the region is so limited this determination will of course be rather weak. Using all the radial velocities and applying the constant correction -4.3 km, I find:

$$V=-15.3\pm 2.0 \text{ km} \tag{2}$$

The residuals furnish for the probable deviation of a radial velocity resting on more than one observation the value:

$$r_\rho = \pm 3.4 \text{ km} \tag{3}$$

If we assume the position of the vertex to be correct, V will be but little dependent on the assumed constant correction. Neglecting the correction altogether, we should find $V=-16.9$. But the position of the vertex itself is dependent on the correction. Supposing it to be -5.3 km instead of -4.3, the position becomes 18^h15^m, $+39°$ with $V=-17.1$. It thus appears that any admissible change in K does not affect very seriously either the position of the vertex or the velocity V.

3. REGION B: LONG. 289° TO 337°; LAT. 0° TO −30°; 100 $\mu \geqq 3''.0$

The parallelism of the motions is not so marked as in the preceding region as the summary in Table VIII shows at a glance. The two largest values included in the last column, though belonging to stars with considerable proper motions, Boss 4599 and 4913, have but little reliability because of the exceptionally great uncertainty in Boss's determination (compare with the probable errors given in Boss's catalogue). But even retaining these

TABLE VIII

p = position angle of proper motion	100 μ		
	$\leqq 1''.6$	$1''.7–2''.9$	$\geqq 3''.0$
<100°	2	1	1
100° to 120°	1	1	....
120 140	....	1	1
140 160	2	2	....
160 180	....	2	5
180 200	3	4	9
200 220	3	1	10
220 240	1	2	1
>240°	1	....	2
Totals	13	14	29

we find that for the stars with secular proper motions exceeding $3''$, 24 out of 29 values of p show deviations but little larger than can be accounted for by their probable errors r_p (see List 2).

For the smaller proper motions, where the accidental errors of the p's are of course more considerable, we find much the same state of affairs. There can be no doubt that, here again, we have to do with a very decided common motion. Still, in order to introduce as little uncertainty as possible, I will restrict myself provisionally to the proper motions exceeding $3''$ per century. In taking the mean, it is nearly immaterial whether we retain or reject the five most divergent values. I decided to reject them and found:

α	δ	100 μ	p	P.E.	n	
17^h45^m	$-46^\circ.2$	$5''.06$	$189^\circ.1$	$\pm 3^\circ.5$	24*	(4)

* In order not to produce exaggerated notions of the accuracy obtained I used the probable error found when we include the five rejected stars.

For the radial velocities the data are extremely poor. To enable the reader to judge the value of what we have, it seems necessary to write them out in full. See Table IX.

TABLE IX

Boss No.	α	δ	$100\,\mu$	$\rho-4.3$	Remarks	Weight
4094........	$16^h\ 1^m$	$-58°$	6″5	+ 1.7	1 obs.	0.5
4249........	16 38	−58	3.0	− 4.3	1 "	0.5
4156........	16 15	−49	3.3	− 3.3	1 "	0.5
4431........	17 24	−50	9.0	− 2.3	S.B. est.	0.5
4565........	17 59	−50	3.0	− 1.5	....	1
4657........	18 20	−46	5.3	− 5.1	....	1
4429........	17 24	−37	4.2	+12.7	S.B. est. rej.	....
4439........	17 27	−37	3.6	− 1.3	1 obs.	0.5
4737........	18 38	−36	5.3	− 2.9	....	1
4784........	18 49	−26	6.6	− 5.3	....	1
Mean...	17 47	−43.7	5.06	-3.01 ± 1.35	....	6.5

The result for Boss 4429 (υ Scorpii) was rejected. According to Campbell the star is a spectroscopic binary. The estimated value of the velocity of the center of mass may be greatly in error or the star may belong to the second stream, in which the velocity fits well. The position angle of the proper motion, however, does not favor this view. The probable error given for the mean was derived from the value (3) found in the preceding section. The mean velocity of −3 km is sufficiently small to allow the application of the method followed in the determination of the vertex for the region *A*.

Assuming $V=-19.0$, we find that for the mean position of Table IX $\lambda=80.°9 \pm 4.°1$. This value defines a small circle on which the vertex must lie; (4) likewise defines a great circle. The vertex must be at the intersection of the two. We thus get the position:

$$18^h30^m,\quad +37° \text{ (vertex } B) \tag{5}$$

which lies at a distance of not quite 5° from the vertex *A* (1). The divergence is fully accounted for by the probable errors.

Conclusion.—The stars of region *B* have a common motion which coincides with that of region *A*. The two regions together give a determination of the vertex, independent of radial velocities. It

is determined by the intersection of the directions defined by Table V and (4). I find

$$18^h33^m, \quad +45^\circ \text{ (vertex } A+B\text{, astronomical motion)} \qquad (6)$$

Owing to the smallness of the angle of intersection, which is about 30°, the position is not a strong one and may easily be in error by 6° or 7°. The agreement with (1) and (5), especially with the former, which is much the better of the two, is therefore all that can be desired. It may be taken as further proof—if such were required—of the identity of the stream-motions in the two regions.

4. REGIONS LONG. 217° TO 250°; LAT. −20° TO +10°; $100\,\mu \geqq 1''\!.7$
AND LONG. 155° TO 216°; LAT. −30° TO +30°; $100\,\mu \geqq 1''\!.7$

There is a sudden jump in the values of the p's, at longitude 216°. The phenomenon seems so remarkable and for the present investigation so important that I have prepared Fig. 1 (p. 47), which shows for the region the proper motions of more than 1″.6 per century. Together with Fig. 2 (p. 48), showing the average motions, it will make the matter much more evident than any verbal description.

There is plenty of evidence that we are near some vertex. Because of the rapid change in position angle near such points, which has a tendency to obscure the phenomenon in question, it becomes necessary to subdivide. The choice has to be made with some care. The immediate neighborhood of any point of convergence in which all systematic motion vanishes has to be avoided. The following subdivisions were finally adopted. They leave small parts of the sky uncovered. As they contain but few stars they may provisionally be left out of consideration. (See also Map 2.)

TABLE X

Region	l		b	
E	256° to	239°	−20° to	0°
F	238	226	−20	+10
G	225	217	−15	+10
a	216	195	0	−10
b	216	195	−11	−25
c	216	165	0	+30
d	180	155	0	−14
e	180	155	−15	−30

Taking into account only the stars of which the centennial motion exceeds 1″.6 we obtain the means in Table XI.

From the normal place for *F* the six stars, Boss 2095, 2166, 2171, 2191, 2360, 2382, were excluded. Together with a few other stars they seem to belong to a separate group which will presently be considered under the name of the Vela group (see Section 6). From each of the regions *G*, *a*, *d*, one star—Boss 2060, 1958, 1567—was excluded because of the exceptional direction of its motion. From region *b* Boss 1401 was omitted. Apparently it belongs in region *G*. Finally from *c* Boss 1935 was excluded. Its radial velocity would

TABLE XI

Region	Proper Motions					Radial Velocities			
	α	δ	100μ	p	n	α	δ	$\rho-4.3$	Weight
E.......	8^h38^m	−60°.2	3″.3	292°.5	25	8^h45^m	−59°.6	+16.5	10
F.......	8 27	−48.8	3.2	270.3	20	8 38	−48.5	+16.6	5
G.......	7 51	−37.1	3.0	254.2	11	7 36	−38.0	+19.8	1
a.......	7 16	−27.9	3.2	286.5	13	7 35	−27.0	+18.7	1
b.......	6 18	−33.2	4.2	333.5	6	6 1	−34.0	+20.7	1.5
c.......	8 0	− 0.3	3.4	249.9	9	7 36	+ 6.0	+14.5	3
d.......	5 54	+13.2	3.0	178.5	11	5 50	+14.3	+17.8	3
e.......	5 7	− 1.6	2.3	196.5	9	5 2	0.0	+15.0	3

assign it rather a place in the second stream. This star will be further discussed in a subsequent paper to be devoted to the second stream of helium stars. Boss 1994 in *a* ought probably to be placed in *G*. As, however, little would be altered by the change, I have not thought it worth while to make it.

The directions defined by the averages in Table XI have been plotted in Fig. 2 (p. 48). It shows at once that the proper motions of the regions *E*, *F*, *G* converge very nearly to a single point. The same is the case for the proper motions in *a*, *b*, *c*, *d*. The points of convergence, however, are not the same in the two cases, but lie pretty far apart. I find:

Regions	Convergent	Distance	
E, *F*, *G*	5^h58^m, −41°.5	1°, 1°, 2°	(7)
a, *b*, *c*, *d*	5 40 , −15	3.5, 0, 3, 3	(8)

In both cases the convergence toward a single point is extremely close, as appears from the above distance at which the several

directions pass these points. The direction for e, however, passes the point (8) at a distance of 12°. Five of the nine stars in the region have motions passing close to the convergent. If, excluding no star, we also take the direction for e into account, the convergent (8) is changed to:

Regions	Convergent	Distance	
a, b, c, d, e	$5^h 27^m, -12^\circ.5$	$4^\circ, 2^\circ.5, 0^\circ, 7^\circ, 8^\circ$	(9)

which lies still farther from the point (7).

Altogether I think that the evidence for the close convergence of the four directions a, b, c, d, or the five directions a, b, c, d, e, to a single point not far from (8) is absolutely convincing. It is scarcely less evident that the directions for E, F, G do *not* pass through (8). Their distances from this point are: E, 11° (25 stars); F, 21°.5 (20 stars); G, 25° (11 stars).

Most notable, perhaps, is the divergence for F, containing 20 stars. As has been said, six stars of the Vela group were here excluded. Had these been included, the distance would have been *increased* to 32°.5. Moreover, the convergence of the motions in E, F, G to the point (7) is almost perfect. Finally, this point is practically identical with the convergent of the regions A and B, which, as we shall see presently, has also to be adopted for the regions C and D. As the regions E, F, G form one continuous group in the sky with A, B, C, D (see Map 2), this coincidence of the convergents presents nothing surprising. All this, I think, leads inevitably to the conclusion that the stars of the regions on both sides of galactic longitude 217°, approximately, form two distinct groups.

I am fully aware that Boss's catalogue, from which the proper motions were taken, may be subject to systematic errors, and that these may have altered the direction of the proper motions in such a way that by correcting them the convergents (7) and (8) or (7) and (9) would be brought nearer together. May they not be brought into exact coincidence? I shall consider the question of systematic errors in Section 17, and the conclusion will be reached that there is really an indication of systematic error in the proper motions in declination. If the regions a, b, c, d, e do not

require the same correction, the removal of the error will indeed bring the convergent (8) nearer to (7) but by about 3° only.

The stars of the regions *a, b, c, d, e* have not yet been investigated for systematic error. Meanwhile, it is evident that to explain a difference of 80°, in the position angles of the proper motions in such close-lying regions as *G* and $a+b$ (see Fig. 1, p. 47) by difference of systematic error for the two regions is wholly out of the question. It may be urged that at best the normal for region *G* rests on too small a number of stars (11) to be wholly convincing. In order to see what value this objection has, I have drawn on the early A stars of this region. If, in order to make the material more comparable with the B stars, we exclude the two objects whose secular proper motions exceed 10″, we find eleven stars between the limits $\alpha = 7^h 0^m$ and $8^h 10^m$ and $\delta = -35°$ and $-48°$. The region nearly coincides with *G*, and gives the average

$$7^h 35^m, \ -40.^\circ 1; \ 100\,\mu = 3.''0; \ p = 260.^\circ 0 \ (11 \text{ A0–A3 stars}) \qquad (10)$$

The great circle defined by this direction has been drawn as a dotted line in Fig. 2 (p. 48). It confirms absolutely what was found from the B stars. If we exclude the star Boss 1826, which is almost at the antivertex itself, the direction would tend to throw this point even farther toward the south. A similar computation for region *F* gave

$$8^h 48^m, \ -47.^\circ 6; \ 100\,\mu = 3.''6; \ p = 267.^\circ 7 \ (13 \text{ A0–A3 stars}) \qquad (11)$$

which also agrees well with that in Table XI.

There is a consideration of quite another kind which still further confirms the practical correctness of the position (7) found for the antivertex of the stars *E, F, G*. At the January 1912 meeting of the Academy of Sciences in Amsterdam, I presented a paper in which it was shown that, as far as can be judged from existing data, the motions of all the somewhat richer star groups relative to the center of gravity of the whole stellar system are at least approximately parallel to the plane of the Milky Way. From this communication the following lines are taken:

Let

h = yearly motion of the solar system relative to the center of gravity of the stellar system.

β = galactic latitude of the apex of this motion.
v = yearly linear motion of a star group relative to the same center as h.
b = galactic latitude of its *true* antivertex.
V = yearly linear motion of this same group with respect to the sun.
B = galactic latitude of its *apparent* antivertex.

The motion v is the resultant of V and $-h$. Therefore, projecting on a normal to the Milky Way

$$V \sin B = v \sin b - h \sin \beta \tag{12}$$

We assume for the apex[1] $\alpha = 269\overset{\circ}{.}0$, $\delta = +32\overset{\circ}{.}0$, whence $\beta = +23^\circ$, and take $h = 19.5$ km per second (Campbell). Then, if the motion of the group is parallel to the Milky Way, we find from (12) as the condition for this parallelism

$$V \sin B = -7.6 \tag{13}$$

For the best values of the elements known to me we have:

TABLE XII

	V	$V \sin B$
Hyades	45.6 km	− 3.4 km
Ursa Major	18.4	−11.1
Scorp.-Centaur. (=A)	18.8	− 6.7
Perseus	18.0	− 4.1
2d type, 1st stream	32.6	− 8.4
A stars, 1st stream	27.7	− 6.1
B stars, 1st stream	22.0	− 6.7
2d type, 2d stream	18.4	− 8.5
A stars, 2d stream	24.5	−10.2
Mean	....	− 7.25

Considering the uncertainties, not only in the elements of the several groups, but also in the position of the Milky Way, the agreement seems very close. If now we assume that equation (13) also holds for the group $E+F+G$, we obtain a new datum for the determination of the stream-elements. Combining the mean values in Table XI found for ρ in the three separate groups E, F, G, we have

α	δ	b'	$\rho - 4.3$	
$8^h 38^m\!.5$	$-54\overset{\circ}{.}8$	$-8\overset{\circ}{.}7$	+16.74 km (weight 16)	(14)

[1] *Groningen Publication*, No. 21, 1908.

Let λ be the distance of this point from the antivertex, then

$$V \cos \lambda = 16.74 \tag{15}$$

This equation combined with (13) gives

$$\frac{\cos \lambda}{\sin B} = -2.20 \tag{16}$$

It is easily proved that the locus of the points on the sphere which satisfy this condition is a great circle at right angles to the arc connecting the point (14) with the pole of the Galaxy and intersecting this arc at a distance d from (14) determined by

$$\tan d = -\frac{7.6 + 16.74 \sin b'}{16.74 \cos b'} \tag{17}$$

from which

$$d = -18^{\circ}.5 \tag{18}$$

The great circle passes through the points (read from a globe) 6^h18^m, $-60°$; 6^h3^m, $-40°$; 5^h57^m, $-20°$. It is shown in Fig. 2 and we see that it cuts the average directions of the proper motion in E, F, G (Table XI) under favorable angles. The four circles intersect almost in a point. In fact, they all pass through a circle with a radius of barely 1° round the point 6^h0^m, $-41°$. The convergent (7) is thus strongly confirmed. The figure, finally, shows also the small circle (B) of Section 2, on which, according to the radial velocities, the antivertex for the region A must lie. It passes almost through the convergent (7) whereas the distance from (8) is about 22°.

5. STREAM-VELOCITY OF E+F+G

Assuming the correctness of the constant correction K, for which we used -4.3 km, the determination of the stream-velocity—owing to the nearness of the antivertex—must be relatively good, at least if we take into account the very limited number of stars with given radial velocity.

A small change in the position of the vertex will hardly affect the velocity. I will adopt the position

$$18^h20^m, \quad +42° \tag{19}$$

which is that best satisfying the directions defined by Table V (4), and Table XI. In fact I find that these directions pass the point (19) at the following distances:

A	at 0°.5	(96 stars)	
B	2.0	(24 ")	
E	2.0	(25 ")	(20)
F	2.4	(20 ")	
G	2.5	(11 ")	

The angles of intersection, taken from the globe are roughly

	F	*G*	
A	29°	53°	(21)
B	60	85	

The determination (19) is quite independent of the radial velocities and must be fairly good. With it the radial velocities of Table XI furnish in the usual way

$$V = -19.1 \pm 0.75 \text{ km} \qquad (22)$$

The probable error of a single determination of radial velocity, resting on more than a single measurement and still including peculiar motions, is found to be

$$r'_p = \pm 2.6 \quad \text{(16 full-weight stars)} \qquad (23)$$

The velocity (22) and the convergent (7) agree so closely with the corresponding values (2), (1), and (5) that we may conclude, with a certainty as great as the observations will allow, that the *stars in the regions E, F, G partake in the motion of A and B.*

6. THE VELA GROUP

We have already had occasion to refer to a group of stars in the constellation Vela, which stands out pretty clearly from the other stars. Rather less than half of the stars in the region

l		*b*		
230° to	235°	−10° to	0°	(*P*)
236	265	− 5	+5	

seem to belong to it.

In order to show what indication there is of such a group I computed the values of the divergences $p-p_0$ of the angle of position p from the angle of position p_0 of the great circle from star to antivertex, taking for the vertex a point near (19). This was done not only for the stars in region P, but also, as a comparison, for the adjacent region Q in higher and lower latitudes (remaining part of the region long. 230° to 265°; lat. −20° to +20°).

TABLE XIII

NUMBER OF STARS

$p-p_0$	P	Q	$p-p_0$	P	Q
−90° to −99°.....	1	....	−10° to −19°.....	6	4
−80 −89	....	....	0 − 9	3	3
−70 −79	4	....	+ 1 + 9	2	8
−60 −69	3	....	+10 +19	2	6
−50 −59	2	1	+20 +29	3	1
−40 −49	2	1	+30 +39	2	....
−30 −39	2	3	+40 +49	....	1
−20 −29	3	2			
			Total.........	35	30

TABLE XIV

Boss. No.	Sp.	Mag.	l	b	100 μ	p	r_p	p_c	O−C
2096....	B8	5.4	219°	− 3°	3″.7	189°	12°	244°	−55°
2095....	B1	4.3	229	−10	1.8	186	17	283	−97
2166....	B3	4.8	231	− 7	1.7	205	17	276	−71
2171....	B3	5.4	231	− 7	2.6	198	22	278	−80
2191....	B3	5.3	231	− 6	3.8	208	21	275	−67
2360....	B5c	5.5	233	− 1	3.4	195	13	264	−69
2382....	B0	4.9	234	− 1	2.3	211	22	264	−53
2577....	B8	5.2	243	+ 1	3.1	231	17	266	−35
2659....	B0	6.0	245	+ 3	1.7	216	28	263	−47
2669....	B0	6.5	245	+ 3	3.2	196	15	262	−66
2843....	B8	6.6	253	+ 3	1.7	211	25	259	−48
2860....	B3c	5.4	255	0	2.0	186	22	264	−78
2876....	B5	5.5	257	− 4	3.8	209	13	269	−60
3127....	B5	5.7	264	0	1.9	201	26	251	−50
3399....	Oap	5.6	272	− 2	2.6	203	15	241	−38

We see that whereas the region Q shows a single maximum at about $p-p_0=0$, P shows a double maximum; further, in P there are 26 negative and 9 positive values, whereas in Q the numbers are nearly equal. The stars given in Table XIV ought probably to be considered as members of the group.

The values of p_c (computed position angle) were computed with the definitive vertex found in Section 8. The extent in galactic longitude is relatively great; in latitude it is small. In this respect the group is much like the restricted Perseus group found by Eddington.[1] On the whole the reality of the group seems to me probable, though not beyond a doubt. The range in proper motion seems somewhat in excess of what might be expected in a physical group. Meanwhile, the probable errors are all rather large, so that the differences may very well be spurious in great part. In any case it would be very unsafe to include these stars in a discussion of the general group. The data are not sufficient for a reliable determination of the vertex and the group-velocity.

It might be different if good determinations of radial velocity were known for all or for a great part of the stars.[2] Such determinations might very possibly establish the reality of the group beyond a doubt.

7. REGION C: LONG. 240° TO 270°; LAT. 0° TO +25°; 100 $\mu \geqq 1''7$ REGION D: LONG. 258° TO 288°; LAT. −30° TO 0°; 100 $\mu \geqq 1''7$

Leaving out the six Vela stars within these limits, we get the averages given in Tables XV and XVI.

The quite abnormal radial velocity for Boss 3115 (List 1), depending on a single observation, has been rejected. The probable

TABLE XV

Region	α	δ	100 μ	p	P.E.	n	p Comp. by (19)	O−C
C.......	11^h52^m	−50°4	4″36	248°2	±2°2	28	243°9	+4°3
D.......	12 43	−68.6	3.46	243.0	3.2	20	246.8	−3.8

TABLE XVI

Region	α	δ	100 μ	$\rho-4.3$	P.E.	n	ρ Comp. by (22)	O−C
C.......	11^h45^m	−55°1	....	+10.5	±1.3	8	+11.65	−1.15
D.......	11 47	−68.4	....	+13.9	1.6	4.5	+12.53	+1.37

[1] *Monthly Notices*, **71**, 43, 1910.

[2] As far as I know there is only one observation available, a single measure of Boss 2860.

errors of the p's have been found by comparing in each region the individual values with the means for that region; those of the ρ's by assuming in accordance with (3) the probable error of one observation to be ± 3.4 km.

From the above data it seems impossible to derive reliable values of both the vertex and the stream-velocity. As, however, C and D lie between the regions A and B on the one side, and E, F, G on the other, and as further all the regions A, B, ... G form one continuous group in the sky, there is a strong presumption in favor of the view that both C and D belong to the same system as A, B, E, F, G. This presumption will give way to conviction if we find that both the position angle of the proper motions and the amount of the radial velocities agree with the values furnished by the stream-elements (19) and (22) for A, B, E, F, G. This proves really to be the case within the limits assigned by the probable errors; see Tables XV and XVI. For the general mean of the two regions together the agreement with these stream-elements is almost perfect.

We are thus driven to the belief that the stars in C and D join in the common motion of the regions A, B, E, F, G, so that finally we reach the conclusion that *the stars in the whole of the regions A, B, ... G have a common motion.* Provisionally this conclusion has of course to be restricted to the stars which have thus far contributed to our result, and even of these a few may have to be excluded.

8. DEFINITIVE ELEMENTS OF THE WHOLE GROUP A+B+C+D+E+F+G

Knowing this, we can now apply all of our data to the derivation of definitive elements for the greater group. If the constant correction K of the radial velocities were known with all desirable precision, it might be best to combine all the data furnished by proper motions and radial velocities for the determination of the direction and the amount of the stream-motion. Such, however, is not the case, and it is necessary to include the constant K among the unknowns of the problem. This being so, and the astronomical proper motions being by themselves sufficient for a satisfactory determination of the direction of motion, it seems

preferable not to make use of the radial velocities at all in that determination. I therefore simply drew on a globe the great circles defined by the data already given, but which, for clearness, are here repeated in Tables XVII and XVIII.

TABLE XVII

Region	α	δ	100μ	p_o	P.E.	n	p_c	O−C	$\frac{(O-C)}{\sin\lambda}$
A I.....	14^h23^m	−45°7	3″3	219°2	....	20	220°5	−1°3	1°3
A II....	14 8	−46.7	5.5	223.6	....	39	222.9	+0.7	0.7
A III....	16 23	−24.5	3.6	203.2	....	37	202.1	+1.1	1.1

TABLE XVIII

Region	α	δ	100μ	p_o	P.E.	n	p_c	O−C	$\frac{(O-C)}{\sin\lambda}$
A.......	$15^h\ 4^m$	−37°8	4″3	214°7	....	96	213°8	+0°9	0°9
B.......	17 45	−46.2	5.1	189.1	±3.5	24	186.2	+2.9	2.9
C.......	11 52	−50.4	4.4	248.2	±2.2	28	243.9	+4.3	3.5
D.......	12 43	−68.6	3.5	243.0	±3.2	20	246.6	−3.6	2.9
E.......	8 38	−60.2	3.3	292.5	....	25	294.8	−2.3	1.1
F.......	8 27	−48.8	3.2	270.3	....	20	274.6	−4.3	1.7
G.......	7 51	−37.1	3.0	254.2	....	11	247.5	+6.7	2.2
G′.......	7.35	−40.1	3.0	260.0	....	11	256.4	+3.6	1.2.

Because of the small number of stars in the important normal *G* I have added the normal *G′* (10) furnished by the early A stars of this region. All the directions pass through a circle with a radius somewhat greater than 3°, having its center at

$$18^h18^m,\quad +42^\circ \text{ (definitive vertex)} \tag{24}$$

This point was adopted as the definitive vertex of the whole group. The several directions pass it at distances given in the last column. The last column but one shows the divergences of the observed angles of position from the position angles of the circles passing through the vertex. The numbers of the last column show that, considering accidental errors alone, it is very unlikely that the position (24) is in error by more than 2°. The probable error derived in the usual way would certainly be found smaller than this.

The position of the vertex being known, we can now find the stream-velocity V from the whole of our data. Including as an unknown the constant correction K which must be applied to the observed velocities, the equations of condition take the form

$$V \cos \lambda - K = \rho$$

Each star of which the radial velocity has been determined gives an equation of this form. There are 74 such stars in all. Of these I rejected, as before, Boss 4429 and 3115. Eighteen stars, a few of which have been marked as very uncertain by the observers, the rest depending on a single observation, have been given half-weight. Three of the latter class were given weight 1/3 in order that all the observations with diminished weight might be combined to full-weight values. I thus obtained 61 equations of condition of equal weight. Solved by least squares they give

$$\left.\begin{aligned} V &= -18.21 \pm 0.87 \text{ km} \\ K &= -\ \ 4.38 \pm 0.50 \end{aligned}\right\} \quad (25)$$

The probable error of a full-weight observed radial velocity was found to be:

$$r'_\rho = \pm 2.95 \text{ km} \quad (26)$$

If the value of K were assumed to be known a priori, the probable error of V would be lowered to ± 0.68 km. This, however, seems to me unwarranted. It may even be said that the present determination, depending on the discussion of a collection of stars which form a single group, is the only quite unobjectionable one given up to the present. In conclusion I will adopt as the final elements of the whole group:

$$\left.\begin{aligned} &\text{Position of vertex (1900)} \quad 18^h 18^m, \quad +42^\circ \\ &V = -18.3 \pm 0.9, \quad K = -4.3 \pm 0.5 \end{aligned}\right\} \quad (27)$$

The representation of the single observations, both in position angle and in radial velocity, is shown in Lists 1, 2, 3. For the normals of position angle it has been shown in Table XVIII. For those in radial velocity it may be seen from the summary given in Table XIX.

The agreement between observation and computation leaves nothing to be desired. The last column, O−C′, will be explained in Section 17.

TABLE XIX

Region	α	δ	$\rho_0-4.3$	Weight	ρ_c	O−C	O−C′
A I......	$14^h 10^m$	$-45°.5$	+ 4.9	8.5	+ 4.3	+0.6	+0.8
A II.....	13 55	−46.0	+ 3.55	9	+ 4.9	−1.35	−1.4
A III.....	16 46	−22.4	− 5.8	9	− 6.9	+1.1	+1.2
A........	14 57	−37.8	+ 0.8	26.5	+ 0.6	+0.2	+0.2
B........	17 47	−43.7	− 3.0	6.5	− 1.3	−1.7	−1.3
C........	11 45	−55.1	+10.5	8	+11.2	−0.7	−0.6
D........	11 47	−68.4	+13.9	4.5	+12.1	+1.8	+2.1
E........	8 45	−59.6	+16.5	10	+16.1	+0.4	+0.5
F........	8 38	−48.5	+16.6	5	+16.6	0.0	−0.2
G........	7 36	−38	+19.8	1	+17.6	+2.2	+1.9

9. PROBABLE ERRORS AND PROBABLE AMOUNT OF PECULIAR MOTION

The value (26) of r'_ρ evidently includes both the error of observation and the peculiar motion. It is desirable to find the probable amount of each of the two separately. In Fig. 3, which is supposed

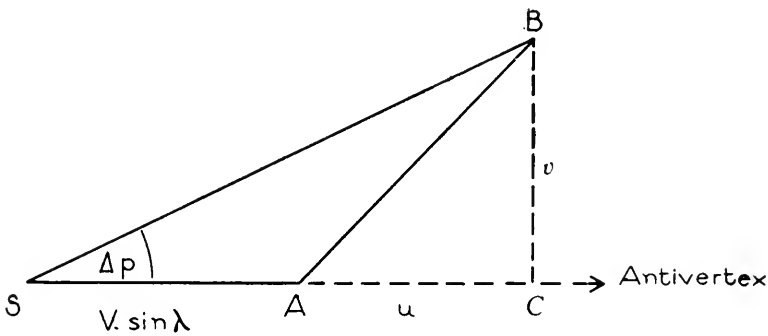


FIG. 3

to be in a plane at right angles to the line of sight, let *S* be any star; $V \sin \lambda$ the projection on the plane of the figure of the stream-velocity; *u* and *v* the components in this same plane of the star's linear peculiar motion. *SB* will be the total linear motion projected on this plane. It is this motion which is seen by the observer

under the angle μ, the total astronomical motion. Let finally Δp be the angle of SB with the great circle toward the antivertex.

It has been assumed—as a provisional hypothesis, to be replaced later by a real determination—that the peculiar motions are distributed according to Maxwell's law. Schwarzschild has already found that the assumption cannot be quite correct, but for the present purpose we cannot be led into serious error by adhering to it. I shall therefore suppose the law to hold. From this it follows that the quantities u and v will be distributed according to the error law. We may thus assume that

$$\text{Frequency of } u \text{ between } u \text{ and } u+du=\frac{h'}{\sqrt{\pi}}e^{-h'^2u^2}du \tag{28}$$

For v we have the same frequency law. From the figure we see that

$$(V\sin\lambda+u)\tan\Delta p=v \tag{29}$$

The stars near the vertex or antivertex cannot well serve for the present purpose. We shall therefore consider only stars for which $\sin\lambda$ is considerable For such stars, if they belong to the spectral class B, u will practically always be smaller than $V\sin\lambda$, so that, V being taken positive, the parenthesis in (29) will be positive. Consequently $\tan\Delta p$ and v will be of the same sign. If, therefore $\bar{v}$ and $\overline{\tan\Delta p}$ represent the averages of v and $\tan\Delta p$ respectively, all taken positively, and if we suppose that we have to do with a large number of stars all at the same distance λ from the vertex, we shall have

$$\bar{v}=V\sin\lambda\,\overline{\tan\Delta p}+\text{average value of }(u\tan\Delta p)$$

As the sign of u will be as often positive as negative and is independent of Δp, the last term will be vanishing. As further $\bar{u}$ is of course equal to $\bar{v}$ we find

$$\bar{u}=\bar{v}=V\sin\lambda\,\overline{\tan\Delta p} \tag{30}$$

This equation enables us to find the average values $\bar{u}$ and $\bar{v}$ from observed quantities, at least on the supposition that Δp is free from observation error. In order to satisfy this condition with at least some approximation, I took from the catalogue at the end of this paper the values of Δp of only those stars for which the

probable error r_p of the position angle, consequently of Δp, does not exceed 10°. I thus found

$$\left.\begin{array}{llll} & \sin\lambda & & \overline{\Delta p} \\ & 0.90 \text{ to } 1.00 & & 11°1 \text{ (101 stars)} \\ & 0.80 \quad 0.89 & & 9.8 \text{ (21 stars)} \\ & 0.70 \quad 0.79 & & 14.1 \text{ (9 stars)} \\ \hline \text{Means} & \sin\lambda=0.927 & & \overline{\Delta p}=11°1 \text{ (131 stars)} \end{array}\right\} \qquad (31)$$

This value must now be cleared of the effect of observation error. The probable errors, r_p, of the position angles have been inserted in Lists 1, 2, 3, at the end of this paper.[1] I find as the average value for the stars used, $r_p= \pm 6°3$, to which corresponds an average deviation of $\pm 7°5$. Therefore, freed of observation error,

$$\overline{\Delta p}=\sqrt{(11.1)^2-(7.5)^2}=8°2 \qquad (32)$$

Finally, by (30), if we assume $\overline{\tan \Delta p}=\tan \overline{\Delta p}$, which cannot be appreciably in error, $\bar{u}=\bar{v}=2.48$ km, to which corresponds the probable amount of u or v,

$$r_u=r_v=\pm 2.1 \text{ km per sec} \qquad (33)$$

If now r_ρ represents the true probable error of a radial velocity observation resting on more than one observation, *excluding* peculiar motion, we shall have (r_u representing the probable amount of the peculiar motion in any direction, therefore also in the line of sight)

$$r_\rho=\sqrt{r_\rho'^2-r_u^2} \qquad (34)$$

Therefore by (26) and (33)

$$r_\rho=\pm 2.1 \text{ km per sec} \qquad (35)$$

10. STARS BETWEEN GALACTIC LONG. 217° AND 360°, LAT. ±30°, 100 $\mu \geqq 1''7$, WHICH DIVERGE MARKEDLY FROM THE STREAM-MOTION

The stars thus far considered form but a part of the whole number between longitudes 217° and 360°. Small parts of the sky have been neglected altogether; everywhere we have kept above

[1] They were computed by the formula $r_p=57°3r/\mu$ in which r=probable error of the proper motion in either α or δ. These were assumed to be equal and I took for r the mean of the probable errors given in Boss's catalogue for the proper motions in α and δ. The derivation of the formula offers no difficulty.

certain limits of proper motion. Now that we have definitive elements, we shall take all of the stars into consideration. In the catalogue at the end of this paper the observed position angles p of the proper motions and the radial velocities ρ have all been compared with the values they would have had in case the stars followed the stream-motion exactly. We shall briefly examine the stars, whose motions seem not at all, or but indifferently, reconcilable with the stream-motion: first, those having centennial motions exceeding 1″.6.

A. Galactic Latitude North.—The position angles are extremely well represented. There are only four stars among 137 for which the deviation exceeds 40°. Meanwhile, we have excluded only stars for which the deviation exceeds 60°. There were two in this case:

Boss No.	p_o-p_c	r_p	$(p_o-p_c)\sin\lambda$	
2605	+62°	±34°	+45°	(36)
4225	−169	6	−154	

The first is pretty satisfactorily explained by the exceptionally large uncertainty in the observed proper motion. Thus, there is really but one case which cannot be admitted to the group on account of the direction of its motion.

In radial velocity the stars given in Table XX show divergences of over 10 km.

TABLE XX

Boss No.	Sp.	ρ_o	ρ_c	O−C
2428..........	B5	+32.0 (1 obs.)	+11.9	+20.1
3115..........	B9	−20.0 (1 obs.)	+12.6	−32.6
3176..........	B3	+28.0:	+14.3	+13.7
3187..........	B3	+25.0:	+14.8	+10.2
3699..........	B3	−16.6	+ 8.1	−24.7
3892..........	B8	+21.0 (1 obs.)	+ 4.9	+16.1
4190..........	B1p	−21.0 (1 obs.)	+ 4.9	−25.9
4287..........	B1l	−27.0	+ 3.0	−30.0

All the velocities have been taken from Campbell's catalogue of B stars. Six stars either have but one observation or are marked as uncertain. They will probably turn out to be spectroscopic binaries. No. 4287 also deviates largely in position angle, and must

certainly be excluded. No. 3699 has been retained in the expectation that, though not marked in any way in Campbell's catalogue, it may still turn out to be a binary. In all, therefore, the three stars Boss 2605, 4225, and 4287 have been excluded from the northern stars with centennial proper motions exceeding 1″.6. One of them may still turn out to be a member of the group.

B. Galactic Latitude South.—The following 13 stars show divergences in position angle of over 60° and were for that reason excluded: Boss 2060, 2109, 2165, 1558, 1469, 1566, 4392, 4796, 4599, 4913, 4739, 4783, 4954.[1] For the first six stars the great divergence is fairly explained by the vicinity of the antivertex. One of them, No. 2165, may prove to be a member of the second stream. Whether this be so or not will be settled as soon as the radial velocity is determined. There thus remain seven exceptional stars in a total of 115. These are all within the limits 296° to 360° of galactic longitude, −9° to −30° of latitude. We could hardly expect otherwise than that near the limits of the group there should be some overlapping of other groups.

In radial velocity we have diverging over 10 km the stars named in Table XXI.

TABLE XXI

Boss No.	Sp	ρ_o	ρ_c	O−C	l
4796......	B9	− 4.2	+5.9	−10.1	310°
4429......	B3	+17.0: S.B. est.	+1.1	+15.9	319
4498......	B8	−16.0	−0.7	−15.3	325
4637......	B8	−17.0 1 obs.:	+0.5	−17.5	325
4999......	B9	−17.0 S.B. est.	−2.6	−14.4	342

The first and the last also have abnormal deviations in p ($p_o - p_c$ = respectively, −122° and −57°). As furthermore they are within the limits for which we found the exceptional position angles, there seems to be little doubt that these two stars do not belong to our group. Of the rest, two are marked as uncertain. Considering the probability that the binary character of many stars, especially B stars, must have escaped detection, I have

[1] For Boss 1702 the great divergence is no reason for exclusion. It is only 3° distant from antivertex.

retained them and also No. 4498, which has no special uncertainty mark.

Summing up, we have excluded, of the stars with proper motions exceeding 1″.6, 17 in a total of 252. It is not improbable that seven of these really belong to the group. Their proximity to the antivertex makes a decision as yet impossible. Thus there may be not more than 4 per cent of the whole number of the stars which do not belong to the group.

II. STARS WITH SECULAR PROPER MOTIONS $\leqq$1″.6

In all there are within the area here under consideration 51 stars with this small proper motion. The direction of the motion is of course generally very uncertain. In order to decide whether they belong to the group, I followed two methods:

1. There are 21 stars—40 per cent of the whole number—for which the probable error of the proper motion is less than half the motion. For these I find the values of $p_o - p_c$ and $(p_o - p_c)\sin\lambda$ given in Table XXII.

TABLE XXII

p_o-p_c	$(p_o-p_c)\sin\lambda$	p_o-p_c	$(p_o-p_c)\sin\lambda$	p_o-p_c	$(p_o-p_c)\sin\lambda$
−39°	− 9°	−44°	−13°	−38°	−38°
+25	+ 4	−27	−11	+30	+27
−67	−27	+ 9	+ 5	−42	−38
−45	−14	+75	+43	− 3	− 3
+57	+20	− 5	− 3	+28	+25
−60	−25	+ 5	+ 5	−13	−12
− 1	0	− 2	− 2	−15	− 9

The mean absolute value of $(p_o - p_c)\sin\lambda$ is only 16°. This astonishingly low value leads, in my opinion, to a strong presumption of membership in the group. At the same time it furnishes a splendid testimony as to the reliability of Boss's proper motions.

2. If we exclude the values of the radial velocities (see List 3) depending on but one observation, and those marked as uncertain, there remain in all 9 values, which deviate from the computed values by the following amounts in kilometers: −3.3, −3.1, −1.3, +15.0, +8.1, −3.3, −15.5, +5.9, −7.9. Two, those of Boss 2467 and 5240, are widely divergent. The latter star is nearly at

the extreme limit of the group. The rest agree about as well as we should expect from the probable error (26) obtained before.

If we keep in mind the probability that even among the B stars for which three or more observations show no very marked divergences, there must still be numerous undetected spectroscopic binaries, we are led, I think, to the following conclusion: with the possible exception of a very few objects, the bright stars having centennial proper motions below 1″7 also belong to the group.

12. THE PARALLAXES AND THEIR PROBABLE ERRORS

As we are concerned with a group of stars, the members of which move so nearly parallel, we are naturally led to attempt to apply to them the method that has been used for finding the parallax of the members of such groups as the Hyades and Ursa Major.

The component of the linear motion of a star at right angles to the line of sight, in the plane containing the vertex is (see Fig. 3) $V \sin \lambda + u$ km per second. This is equivalent to $0.212 \, (V \sin \lambda + u)$ solar distances per year. The angle under which we see this motion, that is, the component of the proper motion along the great circle toward the antivertex, is $v = \mu \cos \Delta p = \mu \cos (p_o - p_c)$. The corresponding linear motion is v/π, π being the parallax. Equating the two we get for the parallax

$$\pi = \frac{v}{0.212 \, (V \sin \lambda + u)} \tag{37}$$

In our case, where $V = 18.3$ km (27), and the probable amount of u is ± 2.1 km (33), we may write without appreciable error, when $\sin \lambda$ is not too small,

$$\pi = \frac{v}{0.212 \, V \sin \lambda} \left(1 - \frac{u}{V \sin \lambda} \right)$$

As, in the case of numerous stars, the values of u were assumed to be distributed according to the same law as accidental errors (see Section 9), this is equivalent to saying that

$$\pi = \frac{v}{0.212 \, V \sin \lambda} \tag{38}$$

with a divergence whose probable amount is

$$r'_\pi = \frac{r_u}{V \sin \lambda} \pi \tag{39}$$

These formulae prove that we can thus gain considerable insight into the arrangement of the helium stars in space. With the values of V and r_u just quoted we find, for instance, for the stars 90° from the vertex, $r_\pi = 0.115\pi$ which, in the present state of science, must be considered a very high precision indeed.

Meanwhile, the value (39) does not represent the total probable error of the parallaxes. We must add the effect of the observation errors on the quantities v. This effect will be small in the case of large proper motions. It becomes very sensible in the case of many of the helium stars, which in general have small proper motions.

v being a component of the astronomical proper motion, its probable error must be about the same as that of any other component of this motion. I shall therefore take for it the mean, r_o, of the probable errors of the proper motion in right ascension and declination which are given in Boss's catalogue. We thus have P.E. of π due to observation error in $v = \dfrac{r_o}{0.212\,V \sin\lambda} = \dfrac{r_o}{\mu \cos(p_o - p_c)}\pi$

Therefore, finally (r_π being the total P.E. of π),

$$r_\pi = \pi\sqrt{\left(\frac{r_u}{V \sin\lambda}\right)^2 + \left(\frac{r_o}{\mu \cos(p_o - p_c)}\right)^2} \tag{40}$$

With such excellent data as those of the Boss catalogue, the increase in the probable error due to observation error will be found very small, especially in the Northern Hemisphere, as long as the total proper motion μ is not below a few hundredths of a second.

13. APPLICATION TO THE HELIUM STARS

In the case of the helium stars we find, introducing the values (27) and (33),

$$\pi = \frac{v}{3.88 \sin\lambda} = \frac{\mu \cos(p_o - p_c)}{3.88 \sin\lambda} \tag{41}$$

$$r_\pi = \pi\sqrt{\left(\frac{0.115}{\sin\lambda}\right)^2 + \left(\frac{r_o}{\mu \cos(p_o - p_c)}\right)^2} \tag{42}$$ [1]

[1] Owing to a mistake the computations for our catalogue have been carried through with the value $r_T = \pm 2.3$ instead of ± 2.1, so that instead of (42) we used

$$r_\pi = \pi\sqrt{\left(\frac{0.126}{\sin\lambda}\right)^2 + \left(\frac{r_o}{\mu \cos(p_o - p_c)}\right)^2}$$

As the consequence is only a slight increase in the probable errors of the values of π, I have not thought it worth while to repeat the computations.

With the aid of these formulae I have computed, accurate to 1 or 2 per cent of the amount, the parallaxes and their probable errors for all the stars with the exceptions of: (*a*) those which probably do not belong to the group (Vela stars and excluded stars); (*b*) those lying only a few degrees from the anti-vertex; (*c*) those having total centennial proper motions below 1″.0.

The computed parallaxes have been inserted in Lists 1, 2, 3, under the head π_1. The meaning of π_2 will be explained in Section 17. That of the other columns will be understood from the headings and the signification of the letters given at the end of this paper. For r_p see footnote on p. 71. The following summary will show at a glance the nature of the results obtained.

Of all the stars having centennial motions exceeding 1″.6 the percentages given in Table XXIII have parallaxes with probable errors within the limits given in the first column.

TABLE XXIII

Limits of P.E.	Galactic Latitude		Total 236 Stars
	North, 135 Stars	South, 101 Stars	
$r_\pi < \pi/6$	23 per cent	6 per cent	15 per cent
$\pi/6 < r_\pi < \pi/5$	33	7	21
$\pi/5 < r_\pi < \pi/4$	23	15	20
$\pi/4 < r_\pi < \pi/3$	12	24	17
$\pi/3 < r_\pi < \pi/2$	7	28	16
$r_\pi > \pi/2$	2	20	10
	100	100	99

For the stars north of the Milky Way the results appear to be very satisfactory. In 79 per cent of the cases the probable error is smaller than a fourth part of the parallax. For the stars south of the Galaxy they are not so good, owing mainly to the fact that so many of the stars are rather near the antivertex. For 38 per cent of these southern stars sin λ is below 0.60. In judging of the applicability of the method we have further to bear in mind that the entire group is south of the equator, in consequence whereof the accuracy of the proper motions—and of the parallaxes—is far below what would have been obtained had the group been in the Northern Hemisphere.

In consideration of the great importance which the knowledge of parallaxes has, it is natural to inquire as to what can be obtained in a similar way for the individual stars of the other spectral classes. It is true that among these we do not find any such great parallelism and equality of motion as among the B stars and before the discovery of the phenomenon of star-streaming it might well have seemed hopeless to apply to them the method here used for the helium stars.

Now, however, matters may turn out differently, at least in so far as we succeed in separating the members of the two streams. In each of them an approach to parallelism and equality of motion, though not so marked as for the helium stars, is very noticeable. Moreover, there are circumstances, such as the generally larger proper motion, the greater stream-velocity, at least of the first stream, etc., which compensate to a certain extent the influence of the greater inequality of motion. It is for these reasons that a separate section is devoted to a somewhat closer investigation of the matter (see Section 18).

14. MAP SHOWING THE ARRANGEMENT IN SPACE

With the aid of the parallaxes π_1 in the catalogue I have constructed a map showing the position in space of the stars treated in this paper (Map 3, p. 50). The positions shown are the projections on the plane of the Milky Way. The sun is supposed to be at the center. The polar co-ordinates therefore are: $r = \cos b / 10\pi$ and l, b and l representing the galactic latitude and longitude. The stars in the Northern Hemisphere are shown as black disks, those of the Southern as circles. The differences in absolute magnitude are indicated by differences in the diameters.[1] The scale is given on the map. The very uncertain positions are marked with an interrogation point.

In using this map we must, of course, not forget that as we get farther away from the sun we lose more and more the absolutely faint stars. This is a consequence of the fact that our stars are complete only to what in Boss's catalogue is the sixth apparent magnitude (=5.80 Harvard). If we except the stars, few in

[1] For this computation, see next section.

number, which in this catalogue are somewhat fainter than the limit, we find on the map at $\pi = 0''.0174$ only stars $+2.0$ or brighter; at $\pi = 0''.0069$ only stars 0.0 or brighter; at $\pi = 0''.0028$ only stars -2.0 or brighter, etc. For this and other evident reasons the map gives an idea of the relative star-density within each zone of constant distance but not—at least not without some computation—of the relative density in the consecutive zones.

I shall not indulge in speculations about the arrangement of the stars as shown by the map. It will be safer to wait until we shall have a similar representation for the whole of the bright helium stars near the Galaxy. I shall only draw attention to the pretty strong condensations at:

Gal. Long.		Gal. Lat.	Parallax	
220° to	240°	Mostly South	0''.0200 to	0''.0300
260	272	North and South	.0130	.0170
290	325	Mostly North	0.0090	0.0140

Of course we may see in the arrangement of these condensations the indication of a spiral structure. I shall not lay much stress on this, however, unless we find the same thing repeated in other parts of the sky.

15. ABSOLUTE MAGNITUDES AND THE LUMINOSITY-CURVE

The parallaxes being known, we can now find the absolute magnitudes. As in *Publication*, No. 11, of the Groningen Laboratory I will define the absolute magnitude of a star as the apparent magnitude it would show if placed at a distance corresponding to a parallax of $0''.1$. It is easily seen that if m = apparent magnitude, M = absolute magnitude, we have

$$M = m + 5 + 5 \log \pi \tag{43}$$

By means of this formula the values of M in the catalogue at the end of this paper have been computed. These I will now use for the derivation of the luminosity-curve. I will confine myself to the B0–B5 stars. There is evidently a rather large difference in luminosity between these and the B8–B9 stars, so that the two classes ought not to be combined. For the latter class by itself, however, the material seems too unsatisfactory. We shall have to wait for further data, or, perhaps, combine the B8–B9

stars with the A0–A3 stars. A provisional curve for the B5–B9 stars needed for a special purpose will be given farther on. See Table XXVI. As for the B0–B5 stars, let them be arranged according to both absolute magnitude and parallax. The process will be much simplified if we combine the stars of equal brightness within half a magnitude. We shall thus group the stars of absolute magnitude $+1.75$ to $+2.25$; those of magnitude $+1.25$ to $+1.75$, etc. Similarly we combine the stars within limits of parallax such that if a star were removed from the one limit to the other it would change its apparent brightness by just half a magnitude. We thus find Table XXIV.

The several columns are limited at such points that any star apparently fainter than 5.805 would necessarily find its place outside these limits. Some stars between apparent magnitudes 5.305 and 5.805 will also fall outside these limits, and are thus lost for our determination. This cannot be helped unless we take the intervals of magnitude, and correspondingly those of the parallaxes, smaller. The nature of the data does not seem to call for such refinement.

By the arrangement we are sure that within the limits of our table we have all the stars which exist, a condition essential for our purpose. Leaving out of consideration for the moment the numbers within parentheses, the construction of the table from the data in the catalogue will be understood if we add: first, that for stars precisely on or very near to the limit of two consecutive lines or columns, half a star was registered in the one and half in the other; second, the stars for which the centennial proper motion is $1''.6$ or smaller were kept separate from the rest by inserting them as a second number with the sign $+$. Take for instance the stars for $\log \pi = -2.361$ (mean of -2.411 and -2.311) and $M = -1.5$ (mean of -1.75 and -1.25). We find $2+1$, which means: two stars having $100\,\mu \geqq 1''.7$ and one having $100\,\mu \leqq 1''.6$. Third, the stars within $37°$ from the vertex or antivertex were excluded. For these the parallax, consequently the absolute magnitude, is more uncertain than for stars in other parts of the sky.

It is at once seen that this table furnishes a determination of the relative numbers of stars of different absolute magnitudes, that is, of the luminosity-curve. In fact, the numbers in each of

TABLE XXIV

Number of Stars

M	Log π: I −1.711 to −1.611	II −1.811 to −1.711	III −1.911 to −1.811	IV −2.011 to −1.911	V −2.111 to −2.011	VI −2.211 to −2.111	VII −2.311 to −2.211	VIII −2.411 to −2.311	IX −2.511 to −2.411	X −2.611 to −2.511	Total: Obs.	Comp.	O−C
+1.75 to +2.25	2 (0.6)						Domain of the stars apparently fainter than 5.80 (Harvard) and part of the stars 5.31 to 5.80				2	0.6	+1.4
+1.25 to +1.75	(0.7)	2 (2.4)									2	3.0	−1.0
+0.75 to +1.25	1 (0.7)	5.5 (2.6)	1.5 (4.0)								8	7.2	+0.8
+0.25 to +0.75	(0.7)	3 (2.5)	7 (4.0)	1.5 (6.4)							11.5	13.5	−2.0
−0.25 to +0.25	0.5 (0.6)	1 (2.3)	6 (3.6)	8 (5.7)	7 (8.7)						22.5	20.8	+1.7
−0.75 to −0.25	1.5 (0.5)	(1.9)	1 (2.9)	4.5 (4.8)	5.5 (7.2)	3.5 (5.8)					16	23.0	−7.0
−1.25 to −0.75	(0.4)	1 (1.4)	1 (2.2)	4 (3.7)	5 (5.5)	5+2 (4.4)	5+0.5 (3.5)				23.5	21.1	+2.4
−1.75 to −1.25	(0.3)	1 (1.0)	2 (1.6)	2 (2.8)	3 (3.8)	4 (3.1)	1+1 (2.5)	2+1 (2.1)			17	17.2	−0.2
−2.25 to −1.75	(0.2)	1.5 (0.6)	1 (1.0)	6 (1.7)	4 (2.5)	1 (2.0)	(1.6)	1+1 (1.3)	0+6 (4.5)		21.5	15.4	+6.1
−2.75 to −2.25	(0.1)	0.5 (0.4)	1 (0.6)	(1.0)	5 (1.5)	1 (1.2)	0+2 (0.9)	(0.8)	0+2 (2.7)	0+1 (1.5)	12.5	10.7	+1.8
−3.25 to −2.75	(0.1)	(0.2)	(0.3)	(0.5)	(0.8)	(0.6)	(0.5)	(0.4)	(1.5)	0+2 (0.8)	2	5.7	−3.7
−3.75 to −3.25	(0.1)	(0.1)	(0.2)	0.5 (0.3)	1 (0.4)	1 (0.3)	(0.3)	(0.2)	0+1 (0.7)	(0.4)	3.5	3.0	+0.5
−4.25 to −5.75	....	(0.1)	(0.1)	0.5 (0.1)	(0.2)	(0.1)	(0.1)	(0.1)	0+1 (0.3)	(0.2)	1.5	1.3	+0.2
Brighter.	....	....	....	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.2)	(0.1)	0	0.8	−0.8
Totals	5	15.5	20.5	27	30.5	17.5	9.5	5	10	3	143.5	143.3	

the vertical columns show the curve. The extent of the curve shown by the several columns—that is, by the stars in the several shells at different distances from the sun—is different, but as the part of the curve common to two consecutive shells is generally considerable, nothing is easier than to reduce the numbers in all the shells to what they would have been had the stars all been in one and the same shell. It is evident that we must not combine the numbers without such a reduction (as has been done by some astronomers) because the volume and the star-density in the consecutive shells is or may be very different. Executing the combination in the way which insures the greatest weight to the results, I find the curve[1] shown in Table XXV.

TABLE XXV

LUMINOSITY-CURVE B0–B5 STARS

M	Number	n	Formula (44)	O−C
+1.5	61	3	61.7	− 0^5
+1.0	74	8	65.8	+ 8
+0.5	58	11^5	64.4	− 6^5
0.0	66	22^5	57.9	+ 8
−0.5	34	16	48.0	−14
−1.0	42^5	23^5	36.5	+ 6
−1.5	27	17	25.5	+ 1^5
−2.0	25^5	21^5	16.4	+ 9
−2.5	12^5	12^5	9.7	+ 3
−3.0	2	2	5.3	− 3
−3.5	3^5	3^5	2.7	+ 1
−4.0	1^5	1^5	1.2	0

I omitted the number for $M = +2.0$, which is too uncertain. The column n shows the total numbers of stars that have contributed to each separate result. They are meant to give a rough idea of the relative reliability of the numbers in the preceding column, which must be considered as representing the observed luminosity-curve.

I have tried to represent these numbers by the analytical expression derived for the luminosity-curve of all the spectral classes together,[2] which Schwarzschild[3] found to be consistent with the

[1] The two stars for $M = +2.0$, $\log \pi = -1.661$, were first equally divided over the two absolute magnitudes $+2.0$ and $+1.5$.

[2] *Groningen Publication*, No. 11, 1902.

[3] *Astronomische Nachrichten*, **190**, 361, 1912.

other empirically derived frequency-curves, determining, of course, the constants so as to represent the observations. I found

$$\log \text{Number} = 1.763 + 0.1285\, M - 0.0726\, M^2 \qquad (44)$$

by which the last column but one of Table XXV was computed. The agreement seems as good as could have been expected. We may write this formula in the form (taking the total number of stars as our unit)

$$\text{Frequency} = \frac{h}{\sqrt{\pi}} e^{-h^2(M-K)^2} \qquad (45)$$

in which

$$h = 0.409, \qquad K = 0.885 \qquad (46)$$

The luminosity-curve of the B0–B5 stars is thus found to be a Gaussian error-curve. The absolute magnitudes deviate from the value K as do observed quantities in the case of a probable error of 1.17 magnitudes.

This luminosity-curve is to be considered as an observed curve only as far, approximately, as the maximum at $M = +0.9$. The intensely interesting question arises: What will be the curve for fainter absolute magnitudes? Will it still follow formula (44) or (45)?

In order to settle this question satisfactorily we shall necessarily have to wait for data for the stars having apparent magnitudes fainter than 6.0, especially of those of great proper motion. For several years such data have been collected at Groningen. Unfortunately the spectral classification of these stars is still wanting.

In the absence of such data nothing really definitive can be maintained. Still we are not altogether without an indication. According to Miss Maury's classification of the spectra of the Pleiades, we have in this group not a single star of the class B0–B4. Of the B5–B9 stars she finds the numbers in the third column of the summary given in Table XXVI.

The faintest star classified as B is 6.61 (Hertzsprung). The stars fainter than this are of the type A, those fainter than 9.5 mostly F (Tickhoff). As the stars are practically all at the same distance, the apparent magnitudes must differ from the absolute values only by a constant. The numbers of stars given for the

different apparent magnitudes thus show: (*a*) That, beginning at the brighter extremity the luminosity-curve of the B5–B9 stars first rises to a maximum and then falls off again to zero. Notwithstanding the small numbers of stars this point must be considered as well established. (*b*) That the curve is roughly symmetrical with respect to the maximum. Meanwhile, it cannot be taken as at all certain a priori that the luminosity-curve in such a local group, in which all the stars probably started on their career at the same time, is identical with that of the stars in the rest of the sky. Whether there is such identity or not ought to be settled by observation. A group like the Pleiades, if we knew its parallax, might also throw some light on the question.

TABLE XXVI

LUMINOSITY-CURVE B5–B9 STARS

m	M	Number	Lum.-Curve
≦2.0	−1.7	....	0.2
2.5	−1.2	....	0.1
3.0	−0.7	....	0.3
3.5	−0.2	1	0.5
4.0	+0.3	2	0.9
4.5	+0.8	3	1.3
5.0	+1.3	0	1.6
5.5	+1.8	3	1.8
6.0	+2.3	2	1.8
6.5	+2.8	2	1.6
7.0	+3.3	....	1.2
7.5	+3.8	....	0.8
8.0	+4.3	....	0.5
8.5	+4.8	....	0.2
≧9.0	+5.3	....	0.2
Totals	....	13	13.0

In a communication to the Academy of Sciences of Amsterdam (February meeting, 1912) I made an estimate of this parallax, founded on the hypothesis that the motion of the group in space referred to the center of gravity of the whole stellar system is parallel to the plane of the Milky Way. We have already had occasion to refer to the probability of such parallelism in the last part of Section 4. This hypothesis led to the parallax

$$\pi(\text{Pleiades}) = 0\overset{''}{.}018 \qquad (47)$$

With the aid of this value the apparent magnitudes in Table XXVI were converted into absolute magnitudes, which are shown in the second column. On the other hand, I derived, as well as the material contained in the present paper permits—which, as was said just now, is not very well—the luminosity-curve of the B5–B9 stars. I find for it the equation (45) in which

$$h=0.508, \quad K=2.00 \tag{48}$$

If, with these values, we compute the distribution of the total of 13 stars over the different absolute magnitudes, we find the last column of Table XXVI. Considering the circumstances, the agreement between the numbers of this column and the observed numbers is tolerable. It seems by no means impossible that among the stars fainter than 7.0, classified as A stars, there are two or three which ought really to have been classified as B8 or B9. If this were so, the agreement would leave nothing to be desired.

How all this bears on our question is evident. We have, however, to bear in mind that the luminosity-curve defined by the constants (48) is again to be considered as given by observation only up to a certain point, in this case about $M=+2.0$. Still a maximum near this magnitude is clearly indicated. In the Pleiades the maximum must lie somewhere near $M=+1.3$ or $+1.4$. The difference is easily accounted for by the uncertainties in the two determinations and in the parallax. It would even disappear altogether, and at the same time the range of magnitude from the brighter stars up to the maximum would become nearly identical, if it should turn out that two or three stars fainter than 6.6 in the Pleiades are really B8 or B9. All this leads to the belief that the absolute brightness of the Pleiades must be fairly normal, that is, about equal to that of other stars of the same spectrum in other parts of the sky.

This being granted, we may apply what we find in the Pleiades to the stellar system at large. Therefore, the foregoing conclusions (*a*) and (*b*) would hold for the whole sky. They are directly obtained only for the stars B5–B9, but it seems very unlikely that the stars B0–B5 should follow a very different law.

16. THE STAR-DENSITY

The number of Bo–B5 stars per unit of volume, at different distances from the sun, is easily obtained from Table XXIV. If the stars of the first three columns are taken together and if we call N_0 the total number of stars in these three shells, N_4 that in shell IV, etc., we find Table XXVII.

TABLE XXVII

$\dfrac{\text{Stars } M \leqq +0.75 \text{ in I+II+III}}{\textit{Idem} \text{ in IV}}$	$=\dfrac{29}{27}$	Therefore $N_4 = 0.931\ N_0$
$\dfrac{\text{Stars } M \leqq +0.25 \text{ in IV}}{\textit{Idem} \text{ in V}}$	$=\dfrac{25.5}{30.5}$	" $N_5 = 1.196\ N_4$
$\dfrac{\text{Stars } M \leqq -0.25 \text{ in V}}{\textit{Idem} \text{ in VI}}$	$=\dfrac{23.5}{17.5}$	" $N_6 = 0.745\ N_5$
$\dfrac{\text{Stars } M \leqq -0.75 \text{ in VI}}{\textit{Idem} \text{ in VII}}$	$=\dfrac{14}{9.5}$	" $N_7 = 0.679\ N_6$
$\dfrac{\text{Stars } M \leqq -1.25 \text{ in VII}}{\textit{Idem} \text{ in VIII}}$	$=\dfrac{4}{5}$	" $N_8 = 1.25\ \ N_7$
$\dfrac{\text{Stars } M \leqq -1.75 \text{ in VIII}}{\textit{Idem} \text{ in IX}}$	$=\dfrac{2}{10}$	" $N_9 = 5.00\ \ N_8$
$\dfrac{\text{Stars } M \leqq -2.25 \text{ in IX}}{\textit{Idem} \text{ in X}}$	$=\dfrac{4}{3}$	" $N_{10} = 0.75\ \ N_9$

From these numbers, if we consider that we have approximately volume I+II+III = volume IV, and further that the volume of each following shell is almost exactly double that of the preceding one, we find Table XXVIII.

TABLE XXVIII

Limits of Distance		Densities	
0 to	8	$\Delta_0 = 1.00$	
8	10	$\Delta_4 = 0.93$	
10	13	$\Delta_5 = 0.56$	
13	16	$\Delta_6 = 0.21$	
16	20	$\Delta_7 = 0.07$	
20	26	$\Delta_8 = 0.04$	0.06
26	32	$\Delta_9 = 0.11$	
32	41	$\Delta_{10} = 0.04$	

Of course these densities, particularly at distances beyond $20 (\pi = 0\overset{''}{.}005)$, are very uncertain. If, neglecting shell X, we combine VIII and IX, and compare this directly with I, II, and III, we get $\Delta_{8+9} = 0.06$. The absolute number of stars per unit of

volume of absolute magnitude $M=0$ is obtained by dividing the numbers in Table XXVIII by about 150. The numbers for the other absolute magnitudes are then obtained by the aid of the luminosity-curve (45) and (46).

The numbers of Table XXVIII show that we have to do with a group which thins out very rapidly as we go outward, that is, with a group pretty sharply limited in distance as well as in longitude and latitude. This conclusion is independent of the faintest stars. I mean that it ought not to be altered if later we should be able to take the stars fainter than 6.0 into consideration.[1]

Toward the greater longitudes (see Map 1) the collection of stars with which the present paper deals is limited at about 330°, though a number of outlying objects beyond this limit are included. Toward the north and the south the group cannot extend much beyond ±30° of galactic latitude. We have not discussed the parts of the sky beyond these latitudes, but it is easy to convince oneself that the helium stars thin out very rapidly there. Toward the south the thinning out begins in much lower latitudes. Toward the north such is the case below longitude 260°. In the smaller longitudes there is a pretty sharp limit at 217° (see Section 4).

In the direction opposite the sun it was found just now that the group has thinned out enormously at the distance corresponding to a parallax of 0.″004. Toward the sun (see Map 3) it can hardly extend much beyond $\pi=0.''040$. In most longitudes it does not reach this limit. Outside these limits there must be, on all sides, an extensive space with hardly any helium stars. Those within the limits have a common motion relative to the sun which may be due wholly to the sun's own motion, from which the individual motions diverge very little—not much more, perhaps, than the individual stars in the Ursa group.[2] I think that we ought to call such a collection of stars a *physical group*. If exception is taken to the term, we must invent another with which to indicate the peculiar nature of the collection.

[1] It is true that we have here neglected the scattering of light in space. Within the distances with which we have to deal here the influence of such a scattering must certainly be small.

[2] Cf. Ludendorff, *Astronomische Nachrichten*, **183**, 113, 1909; **195**, 369, 1913.

17. SYSTEMATIC ERRORS IN BOSS'S CATALOGUE

Thus far we have not considered the possibility of any systematic error in the proper motions of the stars in Boss's catalogue. As in such a work the error must certainly be relatively small, there can scarcely be introduced any appreciable uncertainty by neglecting it altogether, if all the stars with which we have to deal have moderately large proper motions. In the present case, however, the proper motions are generally small. It thus becomes extremely desirable to consider whether means cannot be devised to correct our results for any remaining systematic error.

If we accept the result reached in Section 10, that practically all the helium stars in our regions, whatever their proper motions, form a single system, the derivation of such a correction is by no means hopeless. For it must be evident that the influence on the position angles, p, of a systematic error in the proper motions in right ascension ($\delta\mu_\alpha$) or in declination ($\delta\mu_\delta$) will be the more sensible the smaller the proper motion. Therefore, if such an error exists, we must find that for any point of the sky, though the direction of the motion of all the stars be really the same, this direction, as derived from Boss's data, will be different for stars of small and great proper motion. We shall have to accept as the systematic catalogue-correction one that will make the difference disappear.

Mathematically speaking, the problem is indeterminate. We obtain only one equation between the two unknown quantities $\delta\mu_\alpha$ and $\delta\mu_\delta$, but in many cases the circumstances of the case will in great part remove the indeterminateness. For stars with small proper motion, let μ_1 and p_1 represent the average proper motion and position angle. Let μ_2 and p_2 represent the same quantities for stars of larger motion in the same part of the sky. Neglecting quantities of the order of the squares of $\delta\mu_\alpha/\mu$ and $\delta\mu_\delta/\mu$, we find, in order that the two sets of stars may furnish the same value for the position angle, that the corrections $\delta\mu_\alpha$ and $\delta\mu_\delta$ must satisfy the equation

$$p_2-p_1=\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)\cos p\cdot\delta\mu_\alpha-\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)\sin p\cdot\delta\mu_\delta \qquad (49)$$

in which, to the order of our approximation, we may take for p any value sufficiently near p_1 and p_2. For the regions A, B, ... this equation gives the conditions in Table XXIX.

TABLE XXIX

A, -0.82	$\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)\delta\mu_\alpha$	$+0.57$	$\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)\delta\mu_\delta$	$=$	$\frac{p_2-p_1}{57.3}$	(96	stars)
B, -0.99	"	$+0.15$	"	$=$	"	(24	")
C, -0.37	"	$+0.93$	"	$=$	"	(28	")
D, -0.45	"	$+0.89$	"	$=$	"	(20	")
E, $+0.38$	"	$+0.92$	"	$=$	"	(25	")
F, 0.00	"	$+1.00$	"	$=$	"	(20	")
G, -0.27	"	$+0.96$	"	$=$	"	(11	")

The stars having centennial motions below 1″.0 are few in number and their position angles are of course exceedingly uncertain. Their consideration cannot help us appreciably, and we shall neglect them in what follows.

In the region A there are only two, and in B only seven stars, for which the total proper motion lies between 0″.010 and 0″.016, whose position angles have not at the same time probable errors exceeding 30°. The average for these nine stars was compared with that for the stars of greater proper motion. We have, of course, reduced to the same point of the sky. In order to make this reduction small we combined the normals with the respective weights 1 and 3. We thus found the two averages:

α	δ	μ	p	P.E.	
17^h5^m	$-44°.1$	0″.049	195°.5	$\pm 3°$	(50)
17 5	-44.1	0.0135	192.0	± 6	

Substituting these values into the equation obtained by combining the first two of Table XXIX, also with the weights 1 and 3, we find

$$51\delta\mu_\alpha + 14\delta\mu_\delta = +0''.061 \tag{51}$$

The difference in the values of p in (50) is far within the probable errors of the determination. Moreover, even if real, it will only necessitate a correction to the proper motion either of right ascension or of declination, by which the position angle of the normals for A and B in Table XVIII (and these have been used in the determination of the vertex) would be changed hardly more than one degree.

In region D there is not a single star with proper motion below 0."017 and r_p below 30°. The remaining regions C, E, F, G are by far the most important for fixing the true position of the vertex. With the exception of a relatively small part, of which the declination exceeds −60°, the whole region is practically contained between the limits

$$\left.\begin{array}{l} 7^h0^m \text{ and } 12^h30^m \text{ in } \alpha \\ -30^\circ \text{ and } -60^\circ \text{ in } \delta \end{array}\right\} \quad (52)$$

We shall assume that the corrections $\delta\mu_\alpha$ and $\delta\mu_\delta$ are constant for this region. In order to avoid the trouble of reducing all the results to exactly the same point in the sky, I operated with the values of $p_o - p_c$, the values of p_c being computed for the vertex (27). The results are in Table XXX.

TABLE XXX

Limits of μ	$\bar{\mu}$	$p_o - p_c$	Probable Error	Smoothed	n	Weight
0."010 to 0."019	0."015	−11°	±7°.5	−2°.2	13	2
0.020 0.025	0.023	− 1.5	4.0	−0.8	11	7
0.026 0.029	0.027	+10	5.0	0.0	7	5
0.030 0.039	0.034	+ 2	4.5	+1.1	11	6
0.040 0.049	0.046	− 2	2.5	+3.1	13	14
0.050 0.079	0.063	+12	4.5	+6.0	8	5
≧0.080	....	....	....	....	0	....

The weights have been roughly computed from the interagreement of the observations. There is evidently a gradual change in $p_o - p_c$ with $\bar{\mu}$. As the numbers run very irregularly I have smoothed them by assuming that they must satisfy the condition $p_o - p_c = a + b\mu$. By least squares we find $a = -4.7$ and $b = 169$. Combining now the first two smoothed values and similarly the last two, we find as the basis of further calculations

$$\left.\begin{array}{cc} \mu & p_o - p_c \\ 0''.019 & -1^\circ.5 \\ 0.050 & +4.6 \end{array}\right\} \quad (53)$$

These values must now be substituted into the conditions of Table XXIX for the regions C, E, F, G, combined into one. Giving half-weight to the last, which rests on only 11 stars, I find

$$-1.3\delta\mu_\alpha + 31.0\delta\mu_\delta = +0.106 \quad (54)$$

The influence of a systematic correction in the proper motion in right ascension thus turns out to be quite negligible. No reasonable value assumed for it will change our conclusion as to the value of $\delta\mu_\delta$ or the position of the vertex. We may thus safely neglect it altogether and so find

$$\delta\mu_\delta = +0\overset{''}{.}003 \tag{55}$$

It is worthy of remark that, according to (51), the same correction will also reduce practically to zero the small difference between the values for p found from the stars with great and small proper motions in the regions A and B.

Summarizing, we conclude: the systematic correction needed by the proper motions in right ascension for the regions A and B is so small that it cannot have any serious influence on the direction of the vertex furnished by the stars in these regions. For the remaining regions the influence of such a correction, even if somewhat more considerable, must be practically negligible (see co-efficients in Table XXIX). A systematic correction of the proper motions in declination will affect very little the corrections found for A and B. For the remaining regions there is evidence of such a correction to the amount of $+0\overset{''}{.}003$. This correction will at the same time improve the agreement of the small and great proper motions in regions A and B.

If, therefore, we adopt for all of our regions the systematic corrections

$$\delta\mu_\alpha = 0\overset{''}{.}000 \qquad \delta\mu_\delta = +0\overset{''}{.}003 \tag{56}$$

we shall obtain the corrected values of p given in Table XXXI which must replace the normals of Table XVIII.

TABLE XXXI

Region	α	δ	$100\,\mu$	Corrected p	n	$(O-C)\sin\lambda$
A..........	$15^h\ 4^m$	$-37\overset{\circ}{.}8$	$4\overset{''}{.}3$	$217\overset{\circ}{.}0$	96	0
B..........	17 45	-46.2	5.1	189.6	24	1.5
C..........	11 52	-50.4	4.4	251.8	28	3
D..........	12 43	-68.6	3.5	247.4	20	2.5
E..........	8 38	-60.2	3.3	297.3	25	2.3
F..........	8 27	-48.8	3.2	275.7	20	3
G..........	7 43	-38.5	3.0	262.6	22	0

These give for the corrected position of the vertex

$$18^h 24^m, \quad +39^\circ \tag{57}$$

The distances at which the several directions pass this point, read from the globe, are in the last column of Table XXXI. They are quite as satisfactory as those for the normals in Table XVIII. Adopting this new position we have also to repeat the computation of the stream-velocity from the radial motions. I now find

$$V = -18.0 \text{ km}, \quad K = -4.8 \text{ km} \tag{58}$$

The residuals left by this solution are shown in the last column O−C′ of Table XIX. They are also quite as good as those of solution (27). The correction $\delta\mu_\delta$ here found agrees at least in sign with the correction $-0.''0023 \cos \alpha$ given by Boss himself.[1] For the region (52) this correction amounts on the average to $+0.''0017$; for all of the regions, to $+0.''0014$. Altogether I think that the present solution (57) and (58) is to be preferred, and I regret now that I did not base the further discussion on it. The changes introduced are, however, generally slight. Those in μ and p have not been given in the lists. They are easily computed by

$$\left.\begin{aligned} \delta\mu &= +0.''003 \cos p \\ \delta p &= -\frac{0.17 \sin p}{\mu} \end{aligned}\right\} \tag{59}$$

Only the new parallaxes (π_2) are given. These will be found in the last column of the Lists 1, 2, 3. The values of M were also computed, but as they seldom differ more than 0.1 or 0.2 mag. I have omitted them.

18. POSSIBILITY OF APPLYING THE METHOD TO OTHER SPECTRAL CLASSES

In Section 13 we were led to the question whether results for parallaxes could not be obtained for the stars of other spectral classes in the same way as for the B stars. In order to obtain an insight into the matter I began by deriving provisional values for the stream-elements of the other classes, confining myself to the

[1] *Preliminary Gen. Cat.*, Introduction, p. xxviii.

stars belonging to the first stream. The second stream is decidedly less promising.

In what follows, therefore, the supposition is involved that we succeed in separating the two streams, that is, in determining to which of the two streams each star belongs. Now that the data for radial velocities are becoming more abundant, at least for the brighter stars, the doubtful cases will be confined more and more to narrow limits. The elements adopted are given in Table XXXII.

TABLE XXXII

	Vertex		V	P.E.	K	P.E.
A stars............	$18^h 22^m$	+18.8	−27.2	±1.3	0.0	±0.8
F, G, K stars......	18 3	+14.6	−30.0	....	0.0	....

For the A stars they were taken from a still unpublished investigation by myself. The stream-velocity depends on the radial velocity of 70 stars, mostly obtained at Mount Wilson. For the F, G, K stars the vertex was taken from Eddington's paper in the *Monthly Notices* of November 1910. The stream-velocity used is somewhat smaller than would follow from Eddington's and Campbell's determinations.

With the aid of these elements I derived the values of r_π for the several classes. For the A stars I used the radial velocities which served for the determination of V, rejecting all the values depending on only one or two observations. For the F, G, K stars I used the velocities in *Lick Observatory Bulletin*, No. 229. I confined myself to stars having a secular proper motion exceeding $3''$, which does not deviate in position angle from the direction of the stream-motion (projected on the sphere) by more than $\pm 15°$.

TABLE XXXIII

	Average Radial Peculiar Motion	r_u=0 845×prec. col.	n
A stars.............	7.1 km	± 6.0 km	44 stars
F "	10.6	± 9.0	31 "
G "	10.7	± 9.0	21 "
K "	12.8	±10.8	55 "

I further excluded all the stars for which, according to the Lick *Bulletin*, the observed velocity is uncertain (1 or 2 obs., S.B. est., values marked (:) or without decimal). I thus found, freeing the radial velocities from the sun's motion, Table XXXIII.

Two G stars and two K stars which show quite exceptional divergences (respectively $+60.2$, -41.2, $+72.5$, $+67.5$) were excluded. It would seem that the exclusion of four stars in a total of 151 is sufficiently justified by the probability of undiscovered orbital motion. Substituting these values into (39) we find

$$\left.\begin{array}{lll} \text{A stars} & r'_\pi \sin\lambda = \pm 0.221\pi & \\ \text{F} \quad " & \quad " \quad = \pm 0.300\pi & \\ \text{G} \quad " & \quad " \quad = \pm 0.300\pi & \pm 0.330\pi \\ \text{K} \quad " & \quad " \quad = \pm 0.360\pi & \end{array}\right\} \qquad (60)$$

There is a second means of deriving the value r_u and consequently of r'_π. Eddington[1] finds for the first stream stars of spectrum A, F, G, K, M, $hV = 1.516$, where $h = 0.4769/r_u$, consequently

$$r_u = 0.314V \qquad (61)$$

Assuming that this result is valid for the aggregate of the F, G, K stars, we have by (39)

$$\text{F, G, K stars} \qquad r'_\pi \sin\lambda = \pm 0.314\pi \qquad (62)$$

which agrees well with the value for $F+G+K$ in (60). Taking $V = 30.0$ from Table XXXII, we find for the mean of the two determinations

$$r_u = \pm 9.6 \text{ km}, \qquad r'_\pi \sin\lambda = \pm 0.320\pi \qquad (63)$$

On account of the factor $\sin\lambda$, the probable error r'_π becomes rapidly larger near the vertices. In order to obtain somewhat more precise notions, let us confine our attention for the moment to that part of the sky—almost 77 per cent of the whole—which is more than 40° from these vertices. The mean value of $\sin\lambda$ will be 0.891 and we shall have on an average

$$\left.\begin{array}{ll} \text{A stars} & r'_\pi = \pm 0.248\pi \\ \text{F, G, K,} \quad " & r'_\pi = \pm 0.360\pi \end{array}\right\} \qquad (64)$$

To these values ought to be added the effect of the observation errors in v, which are very different for different stars and different

[1] *Op. cit.*, p. 35.

regions of the sky. We may, however, for the present neglect them, because their effect, which for the B stars in this paper is very material, will be much less for the stars of the later types for which both the parallaxes and the proper motions are as a rule much larger. In the Northern Hemisphere, at least, and for stars down to magnitude 6.0 this effect must be almost vanishing. In a catalogue like that of Boss's about half of the stars (not of the B type) will be included in the present discussion. For this half we shall have the probable errors (64).

I fear that in the judgment of many the precision implied by these probable errors will not be appreciated at its true value. Errors of 25 and 36 per cent of the whole may seem excessive. In a certain sense they are, but if we consider for a moment the present standpoint of the science, we may be led to mitigate this judgment. Direct parallax determination cannot deal at all with the bulk of the stars. It has to limit itself to those nearest our system. For the moment this is practically a restriction to the stars with large proper motion. Thus direct determinations can at most teach us something about the structure of only an infinitesimal part of the universe. But even for this part direct determinations do not give better results than we here find possible for the bulk of the stars for which we have the necessary data for proper motion. Take for instance the most extensive and valuable series of such determinations at present in existence. I find that at the Yale Observatory 178 stars, having a proper motion over half a second yearly, have been observed for parallax. Of these the number of parallaxes with a probable error

$$\left.\begin{array}{l}\text{below } 0.248\pi \text{ is } 36\text{, or } 20 \text{ per cent of the whole}\\ \text{below } 0.360\pi \text{ is } 55\text{, or } 31 \text{ per cent of the whole}\end{array}\right\} \quad (65)$$

That is, we find that 20 and 31 per cent of the Yale stars of large proper motion are better determined than the bulk of our stars; 80 and 69 per cent are not so well determined. If then the method yields results for the bulk of the stars not inferior to those furnished by direct parallax determinations for only a small fraction of them, there is reason to view the results (64) with less dissatisfaction.

But there is more. We can obtain results very materially better than those furnished by (38), though, as far as I can see, we cannot

say exactly how much better. The following reasoning will make this clear. The divergences of our parallaxes from their true values are due solely to the divergence of the individual linear motions of the stars from the stream-motion. In the case of stars having exactly the stream-motion the parallax found by (38) must be faultless.[1] Even in the case of divergences, no error will be introduced as long as the component of the linear *peculiar* motion in the stream-direction is zero ($u=0$). Therefore, the greatest errors must be looked for in the cases for which the divergence of the velocities in the stream-direction is greatest, that is, for considerable positive or negative values of u. In general, of course, we have no means of judging whether the linear motion u is considerable or not. We have only angular motion to guide us. Still we can distinguish a class of stars in which considerable positive values of u must be abnormally frequent, and another class in which there must be exclusively negative values of u which are considerable. In the first the parallaxes found by formula (38) must be predominatingly too great, in the second they must all be too small.

The first class is that of the stars for which the value of the angular motion v is great (great proper motion stars). The second is that of the stars for which the v-component of the angular proper motion is zero or negative. For the second class the proposition is at once evident. Our formula gives zero or negative parallaxes, which are of necessity too small. As to the first, it must be evident that, *ceteris paribus*, the stars nearest our sun will have the greatest angular motion. But it is also no less evident that, *ceteris paribus*, the stars having the greatest linear velocity will show the greatest angular motion. Therefore, among the great proper motion stars we must have, not only a predominance of near stars, but also—and this is the point at present—stars of extreme linear motion. In conclusion, therefore, our formula must give systematic deviations for the extreme values of v; (a) for the extreme positive values of v it must give the parallaxes preponderatingly too great, therefore too great on the average; (b) for the zero or negative values of v the parallax found is always too small.

[1] For the moment we neglect the observation errors in the proper motion.

It must be evident—and this leads to the point we are aiming at—that, if we succeed in getting rid of these systematic divergences without increasing the errors for the intermediate values of v, we shall diminish the average probable error.

It is easy to see how this can be done. We have only to find the average value of the parallax for all the stars of any given magnitude which have the same given value of v. Then if for each of these individual stars we adopt this mean value as its parallax, systematic error is evidently absolutely avoided.

The execution of this plan demands the knowledge: (*a*) of the star-density D at different distances from the sun; (*b*) of the luminosity-curve. Near the sun, up to $\pi=0\rlap{.}''02$ or $0\rlap{.}''015$, I will assume the density D to be constant; for greater distances I will use the formula derived by Schwarzschild.[1] For the luminosity-curve I followed *Astronomical Journal*, No. 566, according to which

$$\left.\begin{array}{l}\text{Frequency abs. mag. } M \text{ to } M+\delta M=\dfrac{h}{\sqrt{\pi}}e^{-h^2(M-K)^2}\delta M \\ h=0.248, \qquad K=9.5\end{array}\right\} \quad (66)$$

The derivation of the necessary formula will be found in the appendix forming the last section of this paper. The computation, which is pretty laborious, was carried through for only the few cases inserted in Table XXXIV which will be sufficient for the present purpose.

TABLE XXXIV

VALUES OF $\bar{\pi}$ (FOR $V \sin\lambda=27.0$ km)

Mag.	v					
	$0\rlap{.}''00$	$0\rlap{.}''25$	$0\rlap{.}''50$	$1\rlap{.}''00$	$2\rlap{.}''00$	$4\rlap{.}''00$
0.5..........	$0\rlap{.}''013$	$0\rlap{.}''051$	$0\rlap{.}''090$	$0\rlap{.}''165$	$0\rlap{.}''306$	$0\rlap{.}''576$
2.5..........	0.006	0.044	0.080	0.150	0.282	0.537
4.5..........	0.003	0.039	0.074	0.139	0.264	0.506
6.5..........	0.002	0.036	0.069	0.130	0.249	0.477

The computation was carried through for the value $V=30$ km (Table XXXII) and $\sin\lambda=0.90$. For other values of $\sin\lambda$, I multiplied the values of the table by $0.90/\sin\lambda$. This is not wholly accurate, but as $\sin\lambda$ is below 0.70 for only five of the

[1] *Astronomische Nachrichten*, 185, 81, 1910.

stars, no serious error can have been introduced. As the table shows, the difficulty that to $v = 0''.00$ correspond negative values of π has been removed. In order to see in how far the representation for the very great values of v has been improved we must make a comparison with the directly measured parallaxes.

TABLE XXXV

No. Gron. 24	Mag.	μ	v	π Obs.	r_π	π Computed		
						By (38)	By (*d*)	By (67)
304	0.3	0''.33	0''.33	+0''.056	±0''.036	0''.073	0''.060	0''.090
321	6.6	0.33	0.31	+0.077	0.035	0.069	0.044	0.052
280	0.1	0.35	0.33	+0.094	0.015	0.117	0.061	0.117
346	1.3	0.37	0.37	+0.138	0.014	0.059	0.065	0.074
148	6.1	0.42	0.41	−0.011	0.037	0.065	0.053	0.052
66	0.2	0.44	0.44	+0.066	0.020	0.078	0.085	0.094
139	5.9	0.44	0.43	+0.092	0.040	0.071	0.059	0.056
223	4.1	0.45	0.45	+0.066	0.021	0.093	0.081	0.073
310	7.0	0.48	0.09	+0.077	0.028	0.021	0.015	0.027
121	3.1	0.50	0.48	+0.061	0.036	0.079	0.070	0.074
164	2.2	0.51	0.51	+0.129	0.043	0.080	0.076	0.081
120	6.1	0.53	0.51	+0.085	0.028	0.093	0.073	0.063
252	6.3	0.53	0.13	+0.048	0.037	0.025	0.024	0.032
174	3.6	0.55	0.53	+0.058	0.015	0.084	0.074	0.074
224	6.8	0.56	0.49	+0.069	0.037	0.128	0.097	0.070
286	6.8	0.62	0.49	+0.021	0.018	0.124	0.094	0.069
162	6.3	0.63	0.60	+0.038	0.027	0.094	0.074	0.062
134	5.6	0.68	0.67	+0.071	0.020	0.112	0.090	0.072
130	5.5	0.75	0.72	+0.038	0.030	0.120	0.097	0.076
351	3.8	0.75	0.72	+0.041	0.044	0.117	0.099	0.085
247	6.7	0.83	0.67	+0.137	0.038	0.185	0.140	0.085
318	3.6	0.83	0.61	+0.097	0.037	0.117	0.115	0.087
339	6.9	0.84	0.81	+0.017	0.014	0.146	0.098	0.074
129	3.3	1.09	1.07	+0.092	0.009	0.172	0.143	0.108
14	5.8	1.34	0.98	+0.162	0.045	0.155	0.118	0.084
210	6.7	1.38	1.38	+0.133	0.031	0.301	0.217	0.109
6	4.3	2.07	1.64	+0.148	0.027	0.265	0.204	0.124
349	5.7	2.11	2.11	+0.157	0.008	0.353	0.243	0.128
7	2.9	2.24	2.22	+0.143	0.007	0.359	0.283	0.163
194	0.2	2.28	1.23	+0.075	0.006	0.236	0.221	0.165
36	5.9	2.31	1.11	+0.143	0.027	0.208	0.157	0.096
199	0.3	3.66	2.34	+0.759	0.010	(0.368)	(0.320)	(0.204)
17	5.3	3.75	3.75	+0.112	0.007	0.590	0.422	0.171
324	5.6	5.25	5.09	+0.311	0.004	1.112	0.571	0.230

Only the stars of considerable proper motion are worth considering. The measured values of the others are mostly illusory. Further, only those stars for which we have radial velocities can be assigned to the proper stream with some certainty. Of stars of known radial velocity having proper motions exceeding 0''.30 yearly,

I found 59. Of these, 34 fit better in the first stream; 18 in the second, while for 7 it is quite uncertain to which stream they belong. The 34 assigned to the first stream will be found in Table XXXV. Some few diverge widely from the stream and in reality may not belong to it. As, however, we know a priori that among these stars of extreme proper motion we must look for extreme divergences, I have been careful to exclude no star as long as it clearly fits better in stream I than in stream II. The stars have been arranged in the order of the amount μ of the total proper motion.

The fifth column of Table XXXV shows the observed parallaxes, the sixth their probable errors. The seventh and eighth show the values computed respectively by the formulae (38) and (*d*) of the appendix (Table XXXIV). Taking the means of the first 11 stars, the 11 following, and the last 12, we get, giving equal weight to each star, Table XXXVI.

TABLE XXXVI

μ	$\overline{\mu}$	π Obs.	π Computed		
			By (38)	By (*d*)	By (67)
0".30 to 0".52	0".420	+0".077	+0".073	+0".061	+0".072
0.52 0.83	0.660	+0.064	+0.109	+0.089	+0.070
≧0.84	2.360	+0.188	+0.355	+0.250	+0.138
Id. excl. 199	2.242	+0.136	+0.354	+0.243	+0.132

We clearly see that, according to expectation, formula (38) yields too high values for the very high values of μ. This becomes even more evident if we exclude No. 199 (α Centauri) which is very extreme and probably does not belong to the first stream. The agreement becomes considerably better by the new formula (*d*), but it cannot be denied that even this formula, for values of the proper motion exceeding 0".84, gives values still too high. The cause may lie in part in the adoption of the value $V=30$ km. It is only in the first third of our table that A stars are included; for the rest all the stars are of the second type and for these the stream velocity, according to Eddington,[1] adopting Campbell's value 19.5 km for

[1] *Monthly Notices*, November 1910.

the sun's velocity, is 32.6 km. With this value the computed numbers for the stars with proper motions exceeding $0\overset{''}{.}52$ must be diminished by 9 per cent.

Probably, however, the main cause of the divergence lies in the fact found by Schwarzschild[1] that the large deviations of u from the stream-motion are more frequent than they should be according to Maxwell's law, which has also been assumed in this paper. This must change our results in the desired direction. The same thing is proved by the radial velocities. Among 31 stars of the second type having proper motions exceeding $0\overset{''}{.}30$, I find eight peculiar radial motions exceeding 29 km, and six exceeding 38.5 km. Assuming Maxwell's law and $r_u = 9.6$ (63), these numbers would be respectively 1.3 and 0.2.

Schwarzschild's results are not directly applicable to our case and it will be necessary in a thorough discussion of the matter to include a reliable determination of the law of the velocities u. My assistant Zernike is at present working on the subject, using the radial velocities recently published by the Lick Observatory. This determination ought to be supplemented by a discussion of the astronomical proper motions of the stars of a single stream according to the method worked out by Schwarzschild. After this, the agreement with observation ought to leave little to be desired.

Not improbably we can still reduce the accidental divergences a little by the derivation of a formula giving π as a function of μ instead of v. In this way we return to the point of view taken in *Groningen Publication*, No. 8, with the following differences, however: (*a*) We have here considered a single stream and not all of the stars together. The errors of the individual parallaxes must thus be greatly reduced. In fact it is only this subdivision into two streams which makes the derivation of individual parallaxes promising. (*b*) We have a better insight into the probable errors of these individual parallaxes, at least of their upper limit. (*c*) The new formula will be theoretical, while that in *Groningen Publication*, No. 8, was derived empirically. Schwarzschild has already derived such a theoretical formula,[2] but this also applies to all of the stars in the two streams together.

[1] *Astronomische Nachrichten*, **190**, 361, 1912. [2] *Ibid.*, p. 373.

Meanwhile, the theoretical formula not yet being available, I have derived the following empirical equations for the representation of the observed parallaxes:

$$\log \pi = -0.866 - 0.036m + 0.505 \log \frac{v}{\sin \lambda} \tag{67}$$

$$\log \pi = -0.861 - 0.036m + 0.476 \log \mu \tag{68}$$

The values computed by (67) have been given in Tables XXXV and XXXVI. The coefficient for m was adopted in accordance with Schwarzschild's formula; the two others were derived from the data in Table XXXV, α Centauri being excluded It seems reasonable to expect that the definitive formula will represent these observed parallaxes nearly as well.

It is an unfortunate circumstance that the only direct way of ascertaining the degree of reliability of the parallaxes furnished by any formula, viz., that by direct comparison with the observed parallaxes, is so little satisfactory. That this is so, is not only a consequence of the relatively great uncertainty of the observed parallaxes, but also of the fact that the only really reliable observed values belong to stars of very large proper motion. Now these stars, as has already been remarked, must to a great extent be exceptional. There must be among them a high percentage of exceptionally large linear motions, therefore a high percentage of large divergences between theoretical and observed parallaxes. Necessarily we get by comparison with observations an exaggerated idea of the errors of the formula. We must get a better insight into these errors, by the earlier considerations of this section.

From another point of view, on the contrary, we have to consider the fact that exceptionally large errors must be most frequent for the large proper motion stars, as an advantage. For just these stars—those least amenable to a theoretical formula—have been and will in the future be preferred by parallax observers, so that for them too we shall get reliable data. It will only be necessary that in these direct determinations we give the greater care to the smaller parallaxes, in order that these too may become trustworthy within a moderate fraction of the total amount.

To sum up: formula (38) furnishes parallaxes for the individual stars, the probable errors of which are shown in (64). We can,

however, greatly reduce the larger deviations left by this formula if, instead, we use formula (*d*) of the appendix. Meanwhile it appears from the comparison with observations that even this formula leaves systematic errors for stars having a proper motion considerably over half a second yearly. The cause of this must lie in part in the fact that the value adopted for the stream-velocity of the second-type stars is too small, but for the greater part probably in a deviation of the distribution of the star-velocities from Maxwell's law. We must expect that formula (*d*) or another in which v is replaced by μ will systematically agree with observations as soon as we shall have a reliable determination of the velocity law for the members of a single stream. Such a formula will again reduce the errors of the theoretical value, but we must expect that even such an improved formula will leave rather large errors for the stars of exceptionally large proper motions. If, therefore, we leave the stars with proper motions of, say, 0.″50 or more for direct parallax determination, the probable error for the rest—the great bulk of the stars—will once more be lessened.

We may thus hope to find values for the parallaxes which will, perhaps, justify a first attempt to locate the stars in space. The attempt will, no doubt, be a very crude one. Still, it may lead to the disentanglement of some of the most salient points of the arrangement of the stars in space.

Meanwhile, before such an attempt can be made, we shall have to assign each of our stars to the stream to which it belongs. The astronomical proper motion being known, the possibility of this operation will depend in a great measure on the knowledge of the radial velocities. It thus becomes evident how urgent is our need for radial velocities of stars with well known proper motions, such, for instance, as those in the Boss catalogue.

It is rather probable that in the course of this preparatory work we shall find further means of reducing the probable errors of our theoretical parallaxes. The study of the helium and A stars led to the recognition of a certain number of local groups. The motion of these groups is appreciably different from that of the main stream though still in the main following either the one or the other. If it be true that the second type is only a later phase

in the evolution of the stars, then the expectation seems justified that among the second-type stars too we shall recognize partial streams. The knowledge of the radial velocities will be a great help in this discovery. Not only shall we find more decisive parallelism and equality of velocity in such partial groups, but the equality of motion of what is left of the main stream will be increased by the omission of such groups. Every increase in this equality, however, will mean decrease in the probable errors of our parallaxes. Even now the separate treatment of different parts of the sky may do much. It certainly does so in the case of the helium stars.

Finally, I wish to say that the method of the present article, embodied in equation (*d*) of the appendix, was not applied to the helium stars, in the first and most important place, because, owing to the fact that r_π is so much smaller for these stars, any marked increase in accuracy is not to be expected; in the second place, because the new formula is dependent on the law of the star-density and the luminosity-curve. Both these elements have been derived for the stars B0–B5, but not, or at least not satisfactorily, for the B8–B9 stars. Even for the former it might be well to wait for more extensive data.

19. PARALLAX DERIVED FROM APPARENT MAGNITUDE

In all that precedes, the finding of the parallax depends on an accurate knowledge of the astronomical and, for part of the material at least, of the radial motion. For the brighter stars, we have excellent data for the former element in Boss's already often-quoted catalogue. For the latter our data are rapidly increasing. For the fainter stars, on the contrary, what we know is not only extremely fragmentary, but it is evident that our further progress must be slow. As a consequence of this state of affairs, such investigations as were considered in what precedes, can extend our knowledge to but a small fraction of the whole universe. In our endeavor to conclude from this to the structure of the whole, we shall naturally try to get some help at least from other elements which may be obtained quickly for all or for a considerable part of all the stars we see in our telescopes.

Thus, for instance, the number of stars for each class of magnitude furnishes one of the most precious data. The magnitudes themselves have also been frequently used, though often in a way not unobjectionable. In what follows I shall attempt to turn them to account in a different way, starting from the supposition that we know the luminosity-curve.

For the sake of clearness and convenience, I shall here consider only the B0–B5 stars. The application to other spectral classes or to all of the stars together will present no difficulty as soon as for these too we know the luminosity-curve. First consider:

Problem I: Of a group of early B stars, all at practically the same distance from the sun, we have given the average apparent magnitude $\overline{m}$ *of all the members brighter than* m_0. *What is the parallax* π *of the group?*

As the frequency-curve has the form (45) and as

$$M = m + 5 + 5 \log \pi,$$

we get at once

$$\overline{m} = \frac{\int_{-\infty}^{m_0} m e^{-h^2(m+5-K+5\log\pi)^2} dm}{\int_{-\infty}^{m_0} e^{-h^2(m+5-K+5\log\pi)^2} dm} \tag{69}$$

which is readily transformed to

$$\overline{m} = K - 5 - 5\log\pi - \frac{1}{2h} \frac{e^{-P^2}}{\int_{-\infty}^{P} e^{-z^2} dz} \tag{70}$$

in which

$$P = h(m_0 - K + 5 + 5\log\pi) \tag{71}$$

Many tables have been given of the integral in the formula. In case π is known the value of $\overline{m}$ is readily computed. For the B0–B5 stars we have by (46) $h = 0.409$, $K = 0.885$. If further we take

$$m_0 = 5.80 \tag{72}$$

which is the limit on the Harvard scale to which Boss's catalogue is complete, we get the values of $\overline{m}$ in the second column of Table XXXVII.

The other columns will be explained presently. For the solution of our problem we have to enter this table with the argument

m and take out the corresponding value of π. Meanwhile, the applicability of the formulae is greatly restricted by the condition that all the stars must be practically at the same distance. Therefore,

Problem II: Of a group of early B stars, ranging over a wide interval of distance, given the average apparent magnitude of all the stars brighter than m_0; required the average parallax of the group.

TABLE XXXVII

π	$\overline{m}$		$\overline{me^{-1/8m}}$		Dev. from (76)
0.″001	5.304		2.724		+0.005
		135		30	
.002	5.169		2.694		0
		114		26	
.003	5.055		2.668		− 1
		281		76	
.006	4.774		2.592		− 1
		247		76	
.009	4.527		2.516		− 1
		226		74	
.012	4.301		2.442		+ 1
		213		75	
.015	4.088		2.367		+ 2
		200		77	
.018	3.888		2.290		+ 1
		190		76	
.021	3.698		2.214		+ 1
		182		77	
.024	3.516		2.137		0
		169		77	
.027	3.347		2.060		− 1
		161		76	
.030	3.186		1.984		− 1
		484		248	
.040	2.702		1.736		+ 3
		413		239	
0.050	2.289		1.497		+ 17

Suppose we have a_1 stars of parallax π_1, a_2 of parallax π_2, etc. The preceding table furnishes the values of $\overline{m}_1$, $\overline{m}_2$, ... corresponding to these parallaxes. The total averages $\overline{\pi}$ and $\overline{m}$ will be

$$\left.\begin{aligned}\overline{\pi}&=\frac{a_1\pi_1+a_2\pi_2+\ \ldots\ldots}{a_1+a_2+\ \ldots\ldots}\\[1em]\overline{m}&=\frac{a_1\overline{m}_1+a_2\overline{m}_2+\ \ldots\ldots}{a_1+a_2+\ \ldots\ldots}\end{aligned}\right\}\qquad(73)$$

We know nothing in general of the numbers $a_1, a_2, \ldots$. We have given only the average $\overline{m}$. If, entering Table XXXVII with the argument $\overline{m}$, we take out the corresponding value π_0 of the parallax, we shall certainly find π_0 different from $\overline{\pi}$. It is evident that this must generally be so in the evaluation of any quantity unless the tabulated value and the argument are linear functions of each other. In the present case $\overline{m}$ is not such a function, therefore π_0 must be different from $\overline{\pi}$.

For a moderate range of distance the difference $\pi_0 - \overline{\pi}$ will be small; still, as we usually know little or nothing about the range, it will generally be unsafe to take π_0 for $\overline{\pi}$. The difficulty may be avoided in the following way. In the preceding problem where we had to deal with stars of one and the same π, we expressed $\overline{m}$ as a function of π. It is this function which turns out to be non-linear. As, however, the individual magnitudes of all our stars are generally known, we may as easily express the average value of any function $\phi(m)$ of m as a function of π. We shall choose the form $\phi(m)$ in such a way that $\overline{\phi(m)}$ becomes a linear function of π.

Writing down the value of $\overline{\phi(m)}$ from analogy with equation (69), we thus have to determine $\phi(m)$ in such a way as to satisfy the equation:

$$\frac{\int_{-\infty}^{m_0} \phi(m) e^{-h^2(m+5+5\log\pi-K)^2} dm}{\int_{-\infty}^{m_0} e^{-h^2(m+5+5\log\pi-K)^2} dm} = A + B\pi \qquad (74)$$

where A and B are constants.

I have not succeeded in finding the general solution of this integral equation; but as there is no real objection to taking for $\phi(m)$ any suitable form with any number of constants, it cannot be difficult to find an expression which will make the first member of (74) practically linear over a wide range of parallaxes.

For the present purpose I succeeded amply by the determination of a single constant, assuming for $\phi(m)$ the form

$$me^{-pm} \qquad (75)$$

For $p=1/8$ we find the values of the average amount of this function inserted in the third column of Table XXXVII.[1] The last column shows the divergences from the linear form

$$2.7445-25.3\,\pi \tag{76}$$

Between $\pi=0''.001$ and $\pi=0''.040$, which is a range wider than that of the parallaxes of the stars treated in this paper, these differences are practically vanishing. If, in the application to faint stars, even this range is deemed insufficient, we shall no doubt be able to extend it by the introduction of more constants.

In the special case of the bright B0–B5 stars we finally get the solution of our problem either by the use of Table XXXVII or by the formula obtained by adopting (76):

$$\bar{\pi}=0''.1844-0''.0395\,\overline{me^{-1/8m}} \tag{77}$$

With a little table giving the values of the last term for values of m, the whole computation becomes extremely simple. I have applied this method to the B0–B5 stars in the several groups of this paper, entering the table first with the argument $\bar{m}$ and second with the argument $\overline{me^{1/8m}}$. The results are shown by Table XXXVIII.

The values in the fifth and sixth columns are practically identical in the present case. I have added the average parallax as found from the proper motion. They represent the best of what we know of these parallaxes. The agreement with the values derived from the magnitudes, which is perhaps best shown by the last column, may seem too rough to give them any claim to consideration.

In my opinion, however, their value is considerable. They show that some idea of average distance can be obtained from

[1] If we put $k=p/2h^2$

$$\gamma=5-k+5\log\pi \qquad \theta=h(m_0+\gamma+k)$$
$$\epsilon=h(m_0+\gamma) \qquad g=1/2p(k+2\gamma)$$

we get

$$\overline{me^{-pm}}=-e^{g}\frac{(\gamma+k)\int_{-\infty}^{\theta}e^{-z^2}dz+\frac{1}{2h}e^{-\theta^2}}{\int_{-\infty}^{\epsilon}e^{-z^2}dz}$$

by which the average values of me^{-pm} for Table XXXVII have been computed.

the magnitudes alone, a precious knowledge in all those cases where we have as yet no data for the proper motions, or where that motion is so small that it becomes useless as a measure of distance.

TABLE XXXVIII

Region	Average Sp.	$\overline{m}$	$\overline{me^{-1/8m}}$	π from $\overline{m}$	π from $\overline{me^{-1/8m}}$	π from μ	n	log $\frac{\text{col. 7}}{\text{col. 6}}$
				$100\,\mu \geqq 1''.7$				
A I........	B3.0	4.26	2.468	0″.0126	0″.0110	0″.0081	19	−0.13
A II........	B3.4	4.11	2.368	.0147	.0150	.0132	25	−0.06
A III.......	B2.8	4.20	2.443	.0134	.0120	.0089	28	−0.13
B..........	B2.9	3.97	2.317	.0168	.0169	.0112	14	−0.18
C+*D*.......	B3.3	3.98	2.337	.0166	.0162	.0129	23	−0.10
E+*F*+*G*....	B3.3	4.59	2.547	.0082	.0078	.0173	24	+0.35
				$1''.0 \leqq 100\,\mu \leqq 1''.6$				
A+*B*+*C*+*D*	B3.5	4.93	2.637	.0043	.0042	.0033	11	−0.10
E+*F*+*G*....	B3.0	4.88	2.642	0.0049	0.0040	0.0075	17	+0.27

The following examples, though still resting on inadequate data, will be sufficient as an illustration.

First example: Perseus cluster.—In a recent paper by Adams and Van Maanen[1] it has been shown that the stars belonging to the h and χ Persei cluster have a high radial velocity (−43 km), which enables us to distinguish surely between members and non-members of the cluster. Within the limits

$$\left.\begin{array}{ll} \alpha\ 1900 & 2^h8^m0^s \text{ to } 2^h17^m0^s \\ \delta\ 1900 & +56^\circ20' \text{ to } +57^\circ0' \end{array}\right\} \quad (78)$$

I find four B0–B5 stars, all members of the group, brighter than 7.20 (Harvard scale). Arranged in order of magnitude they are (spectra from *Astronomical Journal*, No. 648):

Harvard Mag.	Sp.	α 1900	δ 1900	
6.40	B3	$2^h12^m12^s$	$+56^\circ42'$	
6.42	B2	9 52	+56 35	(79)
6.66	B4	12 3	+56 40	
7.05	B2	9 47	+56 34	

[1] *Astronomical Journal*, No. 648, p. 186, 1913.

The mean of the magnitudes is

$$\bar{m}=6.63 \tag{80}$$

Substituting

$$m_0=7.20, \qquad h=0.409, \qquad K=0.885, \tag{81}$$

the formulae (70) and (71) lead to

$$\left.\begin{array}{ll} \pi & \bar{m} \\ 0''0005 & 6.71 \\ 0.0010 & 6.54 \\ 0.0020 & 6.37^5 \end{array}\right\} \tag{82}$$

Interpolation shows that to the observed value (80) of $\bar{m}$ corresponds the parallax

$$\pi=0''0007 \tag{83}$$

which we conclude must be the parallax of the cluster. The uncertainty is of course still great.

We get an idea of the uncertainty by the following consideration. Given that on a certain area there are four stars between apparent magnitudes 6.30 and 7.20 and given the parallax, we can easily compute the number of stars brighter than 6.30 and the number of stars between, let us say, 7.20 and 9.0 to be expected within the same area. For with any given parallax we can at once transform our apparent magnitudes into absolute values, and the relative frequency of these is given by the luminosity-curve (45) and (46). I find the data given in Table XXXIX.

TABLE XXXIX

NUMBER OF STARS

π	Apparent Magnitude		
	$-\infty$ to 6.30	6.30 to 7.20	7.20 to 9.0
0''004	4.3	4.0	9.5
.002	2.3	4.0	18.8
.001	1.3	4.0	38.8
0.0005	0.7	4.0	79.3

We conclude at once that the parallax cannot be as high as 0''004 because in that case we ought to find within the area (78) at least four members of the cluster apparently brighter than 6.30

whereas there is certainly not a single one. Even the parallax 0″.002 seems scarcely admissible for a similar reason. Therefore, even the extremely meager data available in the present case give us the right to conclude with some probability that the parallax of h and χ Persei must be below 0″.002. To go farther would be dangerous. With such a small number of stars, the limits chosen for the area and for the apparent magnitudes are necessarily somewhat arbitrary. Moreover, relatively great accidental deviations from a normal distribution are to be feared.

Table XXXIX shows further how the observations of fainter stars will soon enable us to make a more accurate estimate. Even in the absence of radial velocities we shall be able to improve the result simply by the classification of the spectra for the fainter stars, for the number of such stars not belonging to the group on an area as small as that now under consideration must be very small and is capable of rough evaluation.

Considerations like these suggest the use of some such quantity as

$$\frac{\text{Number of stars apparently brighter than } m_1}{\textit{Idem} \text{ between } m_1 \text{ and } m_0}$$

instead of $\bar{m}$, for the derivation of π, I have briefly tried this modification of the method and think there are cases in which it may be preferable. The full development of the whole matter must, however, be delayed for a future publication.

Second example: Small Magellanic Cloud.—As far as I know there does not exist a complete catalogue of carefully determined magnitudes of all the stars in the small Magellanic Cloud. But there is an extensive list of variables in *Harvard Annals*, **60**, pp. 90–96. I assume that if we take the maximum light of these stars, we shall have a set of magnitudes that will be a fair specimen of all the magnitudes of the group. Fifty-four stars, well distributed over the cloud, mag. 14.0 or brighter—that is, on account of abbreviation to one decimal, brighter than 14.05—give

$$\bar{m} = 13.44 \tag{84}$$

From this datum I derived the parallax on two suppositions: (1) that we have to do with a set of stars similar in absolute magni-

tude to all the stars in the rest of the sky. As it may be urged that by going down only to magnitude 14.05 we include only the absolutely bright stars which therefore will belong exclusively or preferentially to the spectral types B or A, I supposed: (2) that we have to do with B0–B5 stars. The truth will probably lie between the two. We have for

I. "All the stars" $m_0=14.05$, $h=0.248$, $K=9.5$

II. B0–B5 stars $m_0=14.05$, $h=0.409$, $K=0.885$ Cf.(46)

The values of h and K for "all the stars" were taken from *Astronomical Journal*, No. 566, the formula there given being written in the form (45). By the aid of these values I find from (70) and (71) Table XL.

TABLE XL

π	$\overline{m}$	
	All Stars	B0–B5
0″00005	13.43	13.38
.00010	13.36	13.16
.00015	13.32	12.97
.00020	13.27	12.78
.0004	13.15	12.15
.0008	13.01	11.15
.0016	12.82	9.83
.0032	12.56	8.36
.0064	12.20	6.85
.0128	11.72	5.35
0.0256	11.09	3.84

Interpolation, or rather extrapolation, for the value (84) thus gives

Parallax of Small Magellanic Cloud

$$\left.\begin{array}{ll}(1) & 0''00004 \\ (2) & 0.00004\end{array}\right\} \quad (85)$$

The result thus turns out to be independent of the spectral class of the stars. I have chosen the small Magellanic Cloud because another determination of its parallax, by a totally different method, has recently been made by Hertzsprung.[1] The value found—if we write out more decimals than was done by the author—is

$$\pi=0''000084 \quad (86)$$

[1] *Astronomische Nachrichten*, **196**, 201, 1913.

The two results are at least of the same order of magnitude. As the value (84) of $\overline{m}$ which served for the derivation of (85) is professedly unreliable, Hertzsprung's result is probably the better of the two. But, as I possess a good photograph of the small cloud, I hope soon to secure the measures necessary to improve the present determination. Meanwhile, the determinations (83), (85), and (86), provisionally at least, must not be taken too seriously, for it has been tacitly assumed in all of them that there is no scattering or absorption of light in space. Owing to the enormous distance of the Magellanic Cloud the existence of any appreciable absorption will totally change the value derived for its parallax. Let a be the absorption in magnitudes per unit of distance ($\pi = 0.''1$), I then find that Hertzsprung's result will become

a	π	Abs. in Mags.	
0.000	0.″000084	0.00	
.005	.000223	2.24	
.010	.000318	3.14	(87)
.015	.000398	3.77	
0.020	0.000470	4.26	

Therefore, as long as the question of absorption in space is not settled, we can hardly maintain that we know anything of the parallax of this object.

Moreover, it must be acknowledged that even leaving alone the question of the existence of absorption of light in space, the matter is still in a somewhat unsatisfactory state. The deviations found a moment ago, especially those for the *E*, *F*, *G* group, both for the stars with greater and those with smaller proper motion, are somewhat exceptional and of the same sign. It would seem that in this region the B stars are absolutely somewhat fainter than the bulk of the early B stars. Unless such systematic deviations of the absolute brightness prove to be quite exceptional, very accurate data for distance cannot be expected from the method. Meanwhile it will be better to wait for further data and a more reliable luminosity-curve before entering on a more thorough discussion.

20. PARALLAX DERIVED FROM THE SPECTRUM OR THE COLOR INDEX

I think that at the present moment we can hardly refuse to admit that, apparent magnitude and spectral lines being the same, the color index of the stars farthest away is greatest. Corresponding to the change in color index, there must of course be a change in the spectrum, and the spectrum must yield an even more sensitive measure of the phenomenon than the color index. In a paper now ready for press[1] I have brought together whatever evidence, published and unpublished, I could collect.

Let us assume, as a first approximation, that this increase of color index is proportional to the distance, so that we may put

$$\text{Color index} = g + Cr \qquad (88)$$

The constant g, different for the several spectral classes, can easily be determined from the stars near our system. In fact, save for small corrections, it has already been determined by several astronomers. If now we suppose that by some means or other we have also found the value of C, it is evident that equation (88) will yield the value of the parallax as a function of the color index, which is directly measurable. The presence of the constant g restricts the application to stars of known spectrum. For these, however, it is not impossible that we shall obtain individual parallaxes. If so, then a very great field will be opened by the method. The question will have to be settled whether C is an absolute constant or whether it changes—and if so, in what manner—with the position in the sky, with the spectrum, and possibly with other circumstances.

The determination does not necessarily require a knowledge of the cause or causes to which the change of color index is due, however desirable such information in itself may be. If the form (88) proves insufficient, we may take another with more constants and thus proceed quite empirically. The method must become the more valuable, and proportionally the more precise, the farther away the objects are. If the phenomenon holds for the small Magellanic Cloud—which it does not necessarily—then the total

[1] *Mt. Wilson Contr.*, No. 83; *Astrophysical Journal*, **40**, 1914.

effect cannot well be less, I think, than a full magnitude. It may be much greater.

On the whole, the small Magellanic Cloud may furnish an extremely valuable contribution to the whole question, in particular to the discrimination between possible causes and eventually to the determination of the constant C. The possession of three different methods for the derivation of its distance—that of Hertzsprung and the two indicated in this and the preceding section—is exceedingly valuable because of the mutual control and the increase in confidence with which the results of each will be regarded.

APPENDIX (TO SECTION 18). COMPUTATION OF THE AVERAGE VALUE $\bar{\pi}$ OF THE PARALLAX FOR STARS OF ANY GIVEN APPARENT MAGNITUDE AND A GIVEN VALUE OF v

Consider an infinitely small cone having its vertex in the sun, which cuts the area ω from a sphere drawn round the sun with radius unity. Take as unit of distance that corresponding to $\pi=0''.1$. The infinitely small volume dw cut from this cone by the two spherical surfaces round the sun, having the radii r and $r+dr$ will be

$$dw=\omega r^2 dr$$

Let

$\Delta_{m,r}$ = number of stars of *apparent* magnitude m per unit of volume at the distance r from the sun. This quantity must not be confounded with

D_r = number of stars of any arbitrarily chosen *absolute* magnitude, M per unit of volume at distance r.

Then

$$\omega r^2 \Delta_{m,r} dr$$

will be the number of stars of apparent magnitude m in dw. According to formula (28) of the text, the fraction

$$\frac{h'}{\sqrt{\pi}} e^{h'^2 u^2} du$$

will have linear velocities between $V \sin\lambda+u$ and $V \sin\lambda+u+du$. For the value of h' we have by the aid of formula (63) of the text

$$h'=\frac{0.4769}{r_u}=\frac{0.4769}{9.6}=0.050 \qquad (a)$$

The linear motion $V \sin \lambda + u$ is expressed in kilometers per second. Expressed in solar distances per year it becomes $0.212(V \sin \lambda + u)$. If v be the angle (in seconds) under which we see this motion, then

$$\frac{v}{\pi} = 0.212(V \sin \lambda + u)$$

from which, because

$$\pi = \frac{0.''1}{r} \tag{b}$$

$$u = 47.2rv - V \sin \lambda \tag{c}$$

(u and V in kilometers per second; v in seconds per year; r in units defined by (b)). Therefore

$$\left.\begin{array}{l}\text{Number of stars in } dw \\ \text{having angular motion} \\ v \text{ between } v \text{ and } v+dv\end{array}\right\} = 47.2\frac{h'\omega}{\sqrt{\pi}} e^{-h'^2(47.2rv - V \sin \lambda)^2} r^3 \Delta_{m,r} dr dv$$

and on account of (b)

$$\bar{\pi}_v = \frac{1}{10} \frac{\int_0^\infty e^{-h'^2(47.2rv - V \sin \lambda)^2} r^2 \Delta_{m,r} dr}{\int_0^\infty e^{-h'^2(47.2rv - V \sin \lambda)^2} r^3 \Delta_{m,r} dr} \tag{d}$$

It remains only to find $\Delta_{m,r}$. Let

$$\text{Frequency abs. mag. } M \text{ to } M+dM = \phi(M)dM \tag{e}$$

$\eta = \phi(M)$ represents what is usually called the luminosity-curve. As $M = m - 5 \log r$, we readily see that

$$\Delta_{m,r} = D_r \phi(m - 5 \log r) \tag{f}$$

For D_r the best available analytical form, in my opinion, is that given by Schwarzschild:[1]

$$\log D_r = a_0 + a_1(5 \log r) - a_2(5 \log r)^2 \tag{g}$$

where

$$a_0 = +0.488, \qquad a_1 = +0.097, \qquad a_2 = +0.0088 \tag{h}$$

For the luminosity-curve I used the form following at once from that given in *Astronomical Journal*, No. 566.

$$\phi(M) = \frac{h}{\sqrt{\pi}} e^{-h^2(M-K)^2} \tag{i}$$

[1] *Astronomische Nachrichten*, **190**, 361, 1912.

where

$$h = 0.248, \qquad K = 9.5 \tag{j}$$

We thus get

$$\log \Delta_{m,r} = a_0 + a_1(5 \log r) - a_2(5 \log r)^2 - h^2 \text{ mod. } (m - K - 5 \log r)^2 \tag{k}$$

in which the first term, being a constant, may be disregarded. Meanwhile Schwarzschild's form (g) has the defect that it gives D zero for $r = 0$. It cannot, therefore, be accepted for very small values of r. In ordinary cases this defect can scarcely be of any importance; but for the application to the large proper motion stars with which we are more particularly concerned, for all of which r must be small, its use would certainly be objectionable. For these objects I took D_r = constant, which is equivalent to neglecting a_1 and a_2 in (k). Table XXXIV of the text, therefore, was computed for the arguments $v = 0''25$ and higher by substituting in (d):

$$\log \Delta_{m,r} = -h^2 \text{ mod. } (m - K - 5 \log r)^2 \tag{l}$$

and then integrating numerically. For $v = 0''00$ (l) was replaced by (k). For this particular value of v the expressions in (d) become rigorously integrable. We get:

$$\log \bar{\pi}(v = 0''00) = -1.816 - 0.150m \tag{m}$$

by the aid of which the first column of Table XXXIV was calculated.

NOTATION USED IN THE LISTS AND IN THE PAPER

The letters used exclusively in Section 4 have there been explained.

a = absorption of light in magnitudes per unit of distance ($\pi = 0''1$)
b = galactic latitude
h = $0.4769/r$ = modulus of precision in the frequency-curve (45)
h' = modulus of precision in the frequency-curve (28)
K = constant correction of observed radial velocities; also median in frequency-curve (45)
l = galactic longitude
λ = distance on the sphere: star—vertex
μ = total proper motion
μ_a = proper motion in right ascension (arc of great circle)

μ_δ = proper motion in declination

$\left.\begin{matrix}\delta\mu_a \\ \delta\mu_\delta\end{matrix}\right\}$ = systematic corrections to Boss's μ_a and μ_δ

m = apparent magnitude

m_0 = limiting apparent magnitude

M = absolute magnitude (= app. mag. star would show at $r=1$)

n = number of stars

p = position angle of proper motion. Also: spectrum peculiar

Δp = $p_o - p_c$

π = parallax

π_1 = parallax computed by formula (38) and elements (27)

π_2 = parallax computed by formula (38) and elements (57), (58), (59)

r = probable error; also distance: $r = 0\rlap{.}''1/\pi$

r_p = $57\rlap{.}^\circ3\, r/\mu$ = probable observation error of position angle

r_π = total probable error of π computed by (38) including observation error in μ

r'_π = total probable error of π computed by (38) excluding observation error in μ

r_ρ = probable error of a radial velocity resting on more than one observation, excluding peculiar motion

r'_ρ = total probable error of a radial velocity resting on more than one observation including both observation error and peculiar motion

$\left.\begin{matrix}r_u \\ r_v\end{matrix}\right\}$ = probable amount of the quantities u and v

ρ = radial velocity

$\left.\begin{matrix}u \\ v\end{matrix}\right\}$ = components of the peculiar linear motion at right angles to the line of sight, the one toward the antivertex, the other at right angles thereto (see Fig. 3, Section 9)

υ = component of the angular proper motion μ along the great circle toward the antivertex

V = stream-velocity in kilometers per second

C = (in column of spectra) spectrum composite

c = (in column of spectra) spectrum lines sharp

LIST 1

GALACTIC LATITUDE +; $100\mu \geq 1''.7$

Boss No.	Sp.	Harv. Mag.	1900 α	1900 $\delta-$	Gal. Long.	Gal. Lat.	100μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit 0''.0001	$\frac{r_\pi}{\pi}$	M	π_2 Unit 0''.0001
2600	B3	4.96	9h6	14°	217°	29°	3''.1	238°	+19.0	129°	+ 11°	7°	+ 3.2	....	99	0.20	−0.1	97
2190	B	6.10	8.2	32	218	2	2.1	239	....	155	0	17	....	G	129	42	+1.6	127
2257	B	5.82	8.4	35	223	3	6.5	278	....	154	+ 33	16	....	G	322	44	+3.3	337
2605	B2	4.74	9.6	23	224	23	1.7	295	....	134	(+ 62)	34	....	Excl.	....	....	....	....
2428	B5	5.54	9.0	42	232	3	4.6	267	+32.0*	150	+ 10	10	+11.9	F	232	31	+2.3	236
2483	B3	5.15	9.2	43	234	5	3.4	268	....	148	+ 9	11	....	F	165	31	+1.2	173
2472	B	5.92	9.1	46	236	2	2.9	258	....	149	− 6	18	....	F	137	39	+1.6	147
2790	B9	5.65	10.4	30	238	24	4.5	257	+25.0†	129	+ 18	6	+ 9.2	F	142	20	+1.4	146
2640	B8	5.26	9.8	45	240	7	3.4	249	....	142	− 7	13	....	C	141	30	+1.0	143
2577	B8	5.16	9.5	51	243	1	3.1	231	....	....	....	16	....	Vela	....	....	....	....
2659	B	6.00	9.8	51	245	3	1.7	216	....	....	....	28	....	Vela	....	....	....	....
2669	B	6.47	9.9	51	245	3	3.2	196	....	....	....	15	....	Vela	....	....	....	....
2674	B5	3.70	9.9	54	247	1	2.3	255	+14.4	143	− 12	11	− 4.5	C	99	28	−1.3	94
2755	B5p	4.65	10.3	56	251	1	1.8	270	+ 9.0	139	+ 6	17	− 9.3	C	70	36	−1.1	70
2843	B8	6.62	10.6	55	253	3	1.7	211	....	....	....	25	....	Vela	....	....	....	....
2860	B3p	5.44	10.6	59	255	0	2.0	186	0.0*	....	....	22	....	Vela	....	....	....	....
3073	B8	4.68	11.6	34	255	27	3.0	272	....	119	+ 35	10	....	C	74	26	−0.9	76
2905	B3	6.64	10.8	58	256	1	2.2	262	....	136	0	20	....	C	82	40	+1.2	82
2992	B5	4.26	11.3	54	257	6	3.9	243	....	131	− 9	6	....	C	134	20	−0.1	130
3115	B9	4.40	11.8	33	258	28	5.6	264	−20.0*	117	+ 28	4	−32.6	C	142	16	+0.2	144
3048	B8	4.82	11.5	54	259	7	6.5	267	+12.0*	128	+ 17	6	− 3.6	C	206	19	+1.3	206
3091	B8	5.44	11.7	45	259	16	6.8	270	....	123	+ 29	6	....	C	181	20	+1.7	184
3057	B3	5.84	11.5	60	262	2	2.8	293	....	131	+ 37	21	....	C	76	49	+0.2	77
3168	B	5.62	12.1	41	263	22	2.9	234	....	118	− 3	17	....	C	85	34	+0.3	80
3162	B5	4.81	12.1	50	264	12	3.0	260	+16.4	122	+ 19	12	+ 2.4	C	88	27	−0.5	86
3165	B3	2.88	12.1	50	264	12	4.3	246	....	122	+ 5	4	....	C	130	17	−1.6	130
3127	B5	5.70	11.9	62	264	0	1.9	201	....	....	....	25	....	Vela	....	....	....	....
3227	B9	5.42	12.3	35	264	27	4.4	235	....	112	+ 2	10	....	C	122	22	+0.8	119
3176	B3	4.20	12.1	52	265	10	4.9	243	+28.0	123	0	6	+13.7	C	150	19	+0.1	150
3232	B9	5.77	12.3	35	265	27	4.3	252	....	112	+ 19	9	....	C	114	21	+1.2	113
3187	B3	3.08	12.2	58	266	4	4.8	246	+25.0	125	− 1	4	+10.2	C	151	18	−1.0	148
3239	B3	5.04	12.3	51	266	12	7.8	242	....	120	+ 2	6	....	C	231	18	+1.8	233
3249	B9	5.60	12.4	38	266	24	3.1	238	....	113	+ 4	8	....	C	87	20	+0.3	82
3236	B5	5.14	12.3	63	267	0	5.4	222	....	127	− 26	6	....	C	158	20	+1.1	148
3237	B1	1.58	12.3	62	267	0	4.7	228	+ 7.0	127	− 20	4	− 8.3	C	142	18	−2.7	133
3238	B1	2.09	12.3	62	267	0	4.9	251	....	127	+ 3	4	....	C	158	18	−1.9	151
3245	B3	4.16	12.4	50	267	13	4.7	233	....	118s	− 5	6	....	C	138	18	−0.1	135
3300	B8	4.79	12.6	39	269	24	6.3	227	+13.0	112	− 6	5	+ 1.8	C	172	17	+1.0	170
3314	B8	6.25	12.6	56	269	7	4.8	236	....	121	− 4	7	....	C	144	0.20	+2.0	138

3305	B8	5.02	$12^{h}6$	59°	270°	4°	2′.5	241°	+19.5	122°	− 1°	15°	+ 5.5	C	76	0.30	−0.6	73
3312	B9	6.23	12.6	55	270	8	5.3	231	....	120	− 9	7	....	C	154	20	+2.2	150
3328	B1	1.50	12.7	59	270	4	5.6	240	+13.0	122	0	3	− 1.0	A II‡	170	16	−2.2	165
3324	B3	4.86	12.7	56	270	7	5.5	224	+18.2	120	− 15	7	+ 4.8	A II	157	18	+0.8	153
3345	B8	6.14	12.8	39	271	24	5.3	224	....	110	− 7	7	....	A II	142	18	+1.8	138
3357	B3	4.84	12.8	59	271	4	5.1	219	....	121	− 20	9	....	A II	144	21	+0.6	139
3358	B3	4.26	12.8	57	271	6	3.6	248	+13.2	120	+ 9	9	− 0.2	A I	107	21	−1.1	105
3359	B3	5.46	12.8	57	271	6	2.7	231	+ 7.0*	120	− 8	16	− 6.4		83	32	+0.1	75
3369	B8	5.29	12.8	51	271	12	5.2	225	....	117	− 10	9	....	A II	148	21	+1.1	143
3366	Oe5	5.58	12.8	56	271	7	4.5	235	....	119	− 3	9	....	A II	133	21	+1.2	128
3390	B3	4.96	13.0	48	273	15	5.9	221	....	113	− 12	6	....	A II	162	18	+1.0	159
3393	B3	4.40	13.0	49	273	13	4.2	235	....	114	+ 1	6	....	A II	119	18	−0.2	115
3419	B8	4.76	13.1	59	273	4	6.1	236	....	118	− 1	7	....	A II	179	19	+1.0	175
3457	B5	4.62	13.3	60	274	2	3.9	241	....	118	+ 6	11	....	A I	113	24	−0.1	112
3456	B3	6.51	13.3	60	274	2	2.9	217	....	118	− 18	15	....	A I	82	30	+1.1	77
3451	B3	5.70	13.3	52	275	10	5.7	213	....	114	− 19	6	....	A II	153	18	+1.6	145
3458	B	6.10	13.3	52	275	10	2.9	274	....	114	+ 42	20	....	....	60	50	0.0	62
3521	B1	2.56	13.6	53	278	9	4.1	229	+ 6.0	112	0	5	− 5.2	A II	114	17	−2.2	109
3528	B9	5.40	13.6	54	278	8	6.8	218	....	113	− 12	6	....	A II	188	18	+1.8	182
3535	B	6.30	13.6	56	278	6	2.4	210	....	115	− 20	16	....	A I	65	32	+0.3	56
3576	B8	5.42	13.8	52	280	9	6.7	228	....	110	+ 1	5	....	A II	184	16	+1.7	180
3592	B8	5.84	13.8	52	280	9	5.8	236	....	110	+ 9	8	....	A II	156	20	+1.8	152
3615	B1	0.86	13.9	60	280	1	4.1	219	+12.0*	114	− 9	3	+ 0.3	A II	116	15	−3.8	111
3575	B3	5.87	13.8	46	281	15	4.4	208	....	107	− 18	10	....	A II	113	23	+1.1	109
3565	B2	3.32	13.7	42	282	19	2.8	230	+12.3	105	+ 4	9	+ 3.3	A I	74	21	−2.3	70
3593	B2p	3.06	13.8	47	282	14	8.2	230	....	107	+ 5	2	....	A II	220	14	−0.2	217
3643	B3	5.20	14.1	57	282	4	3.6	247	....	112	+ 21	11	....	A I	91	26	0.0	92
3564	B2	3.53	13.7	41	283	20	4.2	234	+ 6.0†	104	+ 8	6	− 2.7	A II	112	17	−1.2	107
3602	B3	4.05	13.9	42	283	19	3.4	233	+ 5.3	103	+ 9	8	− 3.1	A I	90	19	−1.2	85
3670	B5	4.41	14.2	56	283	4	3.2	229	+ 3.5	111	+ 4	12	− 7.4	A I	89	26	−0.8	83
3603	B3	4.17	13.9	44	284	17	4.6	226	+ 4.0*	104	+ 2	5	− 4.7	A II	122	16	−0.4	119
3577, 3578	B5	4.72	13.8	32	286	29	6.0	230	+14.1	97	+ 7	4	+ 7.6	A II	154	15	+0.7	150
3586	B5	4.76	13.8	31	286	30	2.8	221	+ 8.0†	96	− 2	9	+ 1.8	A I	73	21	−0.9	68
3621	B3	4.54	14.0	41	286	19	3.8	208	+12.4	102	− 15	6	+ 4.3	A I	97	18	−0.5	85
3732	B5	4.14	14.5	49	287	10	5.1	223	....	103	+ 3	7	....	A II	133	18	−0.2	132
3699	B3	4.65	14.3	45	287	14	3.0	197	−16.6	102	− 24	8	−24.7	....	71	21	−1.1	67
3716	B2	4.60	14.4	50	287	9	5.4	256	− 0.6	105	+ 35	5	− 9.6	....	117	16	−0.1	116
3713	B9	5.49	14.4	45	288	14	6.8	219	....	101	− 1	5	....	A II	179	16	+1.8	176
3688	B5	4.55	14.3	39	289	20	4.9	215	+ 9.2	97	− 5	5	+ 2.7	A II	128	16	+0.1	124
3745	B2	2.89	14.6	47	289	11	3.5	217	+ 8.0†	102	− 1	6	− 0.1	A I	92	19	−2.3	88
3724	B3C	2.65	14.5	42	290	16	4.9	224	0.0	98	+ 6	4	− 6.9	A II	128	0.15	−1.8	124

* 1 observation.

† Spect. Binary, estimated velocity of center of mass.

‡ Also used in C.

LIST 1.—*Continued*

Boss No.	Sp.	Harv. Mag.	1900 α	1900 $\delta -$	Gal. Long.	Gal. Lat. +	100μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit 0".0001	r_π π	M	π_2 Unit 0".0001
3808	B8	5.78	14h8	48°	291°	10°	3".7	223°	....	99°	+ 7°	12°	....	A I	96	0.25	+0.7	93
3747	B3	4.09	14.6	37	293	20	4.4	204	....	93	− 13	6	....	A II	111	17	−0.7	105
3783	B5	4.49	14.7	43	293	14	5.0	225	+ 7.2	97	+ 9	6	+ 0.7	A II	128	17	0.0	126
3838	B5	4.72	15.0	47	293	10	4.0	215	....	99	0	6	....	A II	104	16	−0.2	100
3888	B8	4.36	15.2	48	294	8	5.9	212	....	98	− 1	5	....	A II	154	16	+0.3	148
3815	B2p	2.81	14.9	43	294	13	6.9	223	0.0†	96	+ 7	3	− 6.2	A II	177	14	−1.0	173
3862 } 3863 }	B9, A	4.14	15.1	48	294	8	12.1	237	....	98	+ 23	2	....	A II	289	14	+1.4	288
3852	B3	4.39	15.0	45	294	11	3.1	230	....	97	+ 16	11	....	A I	78	24	−1.1	73
3707	B8	5.00	14.4	29	295	29	4.2	217	....	89	− 2	9	....	A II	108	16	+0.2	105
3818	B3	3.35	14.9	42	295	14	3.6	208	+10.0†	96	− 8	6	+ 3.8	A I	94	17	−1.8	87
3706	B8	5.11	14.8	37	295	19	4.3	242	....	84	+ 26	9	....	A II	102	22	+0.2	100
3965	B9	5.48	15.5	52	295	2	5.4	218	....	100	+ 8	9	....	A II	140	21	+1.2	135
4011	B9	5.96	15.7	53	295	0	5.4	223	....	101	+ 14	8	....	A II	138	20	+1.7	133
3865	B3	4.92	15.1	44	296	11	5.1	230	....	96	+ 16	8	....	A II	128	21	+0.5	123
3905	B3	3.74	15.3	44	297	10	2.8	215	+14.0†	94	+ 3	9	+ 8.4	A I	72	21	−2.0	68
3892	B8	5.78	15.2	40	298	14	4.1	218	+21.0*	92	+ 6	9	+16.1	A II	106	21	+0.9	102
3954	B3	4.84	15.5	45	298	8	3.9	189	+ 5.0†	94	− 21	12	− 0.6	A I	93	27	−0.4	86
3896	B2	3.43	15.2	40	299	14	3.3	192	....	92	− 20	8	....	A I	80	21	−2.0	73
3832	B3	5.45	15.0	32	300	22	6.4	217	....	88	0	7	....	A II	165	18	+1.5	163
3776	B9	5.80	14.7	26	301	29	2.6	216	....	85	− 1	11	....	A I	67	23	−0.1	63
3950	B3	2.95	15.5	41	301	11	3.9	204	....	91	− 6	5	....	A I	100	15	−2.0	94
3910	B3	4.69	15.3	36	302	17	3.8	215	....	88	+ 4	6	....	A I	98	17	−0.4	94
4128	B4	5.36	16.1	47	302	2	6.6	207	....	94	+ 3	6	....	A II	170	17	+1.5	165
3929	B5	5.52	15.4	36	302	16	4.1	183	....	88	− 29	9	....	....	93	22	+0.4	87
4190	B1p	5.46	16.4	46	304	1	1.9	238	−21.0*	92	+ 37	23	−25.9	....	39	53	−1.6	37
3991	B5	4.82	15.6	34	306	16	4.9	221	+12.0*	85	+ 12	7	+ 9.3	A II§	124	18	+0.3	121
4064	B3	3.61	15.9	38	307	11	4.4	207	....	86	+ 1	5	....	A III	113	16	−1.1	108
4076	B5	4.97	15.9	38	307	10	4.4	217	+ 5.0*	86	+ 12	8	+ 2.0	A III	111	20	+0.2	108
4006	B8	5.61	15.7	34	307	15	2.7	214	....	84	+ 7	16	....	A III	70	31	−0.2	63
4018	B9	4.11	15.7	33	308	16	2.9	198	....	82	− 9	8	....	A III	76	19	−1.5	69
4224	B	6.14	16.5	43	308	2	4.8	220	....	88	+ 20	7	....	A III	116	20	+1.5	113
4091	B3	4.33	16.0	36	308	11	4.1	211	....	85	+ 6	5	....	A III	106	16	−0.5	100
3973	B3	3.80	15.5	29	309	20	4.1	216	....	81	+ 7	5	....	A III	107	16	−1.0	104
4287	B1p	4.88	16.8	42	311	1	2.0	163	−27.0	......	(− 34)	18	(−30.0)	Excl.	......	......	......	......
4248	B3	5.68	16.6	41	311	2	4.0	228	....	86	+ 20	9	....	A III	82	24	+0.2	87
4052	B3	4.02	15.8	29	312	18	3.3	203	+ 3.2	79	− 4	5	+ 2.4	A III	87	16	−1.3	80
4019	B3	4.77	15.7	25	314	21	4.2	213	....	75	+ 6	5	....	A III	111	16	0.0	107
4200	B3	4.33	16.4	34	314	9	2.6	200	....	80	− 1	10	....	A III	68	22	−1.5	61
4277	B3p	3.09	16.7	38	314	3	3.1	193	....	83	− 5	6	....	A III	81	17	−2.4	74
4281	B2	3.64	16.8	38	314	3	3.4	214	+ 1.7	83	+ 16	7	− 0.4	A III	86	0.18	−1.7	82

4034	B3	4.66	15h8	25°	315°	21°	3".4	200°	....	75°	− 7°	5°	....	A III	90	0.16	−0.6	85
4062	B2p	3.00	15.9	26	315	19	3.9	203	....	75	− 3	4	....	A III	104	15	−1.9	99
3944	B3	5.59	15.5	16	310	30	2.8	207	Bin.	70	− 5	6	....	A III	77	17	0.0	70
4058	B8	5.41	15.9	24	316	20	5.2	237	....	75	+ 30	10	....	A III	120	24	+0.8	121
4115	B3	4.70	16.1	28	317	16	4.7	216	....	76	+ 11	4	....	A III	122	15	+0.1	120
4066	Bp	2.54	15.9	22	318	22	4.1	198	....	72	− 9	3	....	A III	111	14	−2.2	106
4033	B3	5.06	15.8	20	319	25	3.7	202	Bin.	71	− 6	4	....	A III	100	15	+0.1	95
4158	B1	3.08	16.2	25	319	17	3.4	200	....	73	− 3	4	....	A III	91	15	−2.1	86
4218	B	2.91	16.5	28	319	12	3.9	197	+ 1.5	74	− 3	3	+ 2.2	A III	105	14	−2.0	99
4198	B3	4.87	16.4	25	320	15	2.8	188	....	72	− 14	8	....	A III	73	20	−0.8	67
4321	B9	5.06	16.9	32	320	5	6.3	190	....	76	− 6	5	....	A III	165	16	+1.1	161
4178	B5	5.22	16.3	23	321	17	2.4	208	....	71	+ 5	6	....	A III	65	18	−0.7	59
4086	B1	2.90	16.0	20	321	23	3.1	201	− 9.5‡	70	+ 6	3	− 7.5	A III	85	14	−2.4	85
4087	B1	5.06	16.0	20	321	23	3.4	223										
4093	B2	4.13	16.0	20	321	22	3.4	192	−11.0	70	− 14	5	− 9.0	A III	90	16	−1.1	85
4117	B3	4.29	16.1	19	323	22	3.4	200	....	69	− 5	4	....	A III	94	16	−0.8	88
4059	B3	4.68	15.9	14	324	28	3.4	206	....	66	− 2	5	....	A III	96	17	−0.4	90
4183	B3	4.85	16.3	18	325	20	3.1	188	Bin.	66	− 15	6	....	A III	85	18	−0.5	79
4399	B3	3.37	17.3	25	328	6	3.1	182	− 0.9	68	− 10	4	+ 1.7	A III	83	16	−2.0	77
4495	B3	6.16	17.7	27	329	0	2.5	217	....	68	+ 30	12	....	A III	61	29	+0.1	57
4225	B	2.70	16.5	10	334	23	2.1	35	....	.....	(−169)	6	....	Excl.	.....	.....	.....	.....
4465	B8	4.65	17.5	8	344	11	2.5	189	....	52	− 3	7	....	A III	82	21	−0.8	75
4573	B8	5.79	18.0	8	348	5	2.6	126	− 5.3	50	− 58	16	+ 2.2	A III	47	55	−0.9	46
4302	B8	4.29	16.8	10‖	357	29	7.3	231	....	38	+ 24	2	....	A III	278	20	+1.4	283
4545	B3	4.81	17.9	4‖	359	12	1.9	205	−11.0	38	+ 18	10	− 0.9	A III	75	0.28	−0.8	67

* 1 observation.

† Spect. Binary, estimated velocity of center of mass.

‡ Velocity center of mass from orbit.

§ Also used in A III.

‖ Declination north.

LIST 2

GALACTIC LATITUDE −; $100\mu \geqq 1''.7$

Boss No.	Sp.	Harv. Mag.	1900 α	1900 δ−	Gal. Long.	Gal. Lat.	100μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit $0''.0001$	$\frac{r_\pi}{\pi}$	M	π_2 Unit $0''.0001$
1910	B3	5.11	7h2	37°	217°	10°	1″.7	220°	+18.0	167°	− 27°	21°	− 4.1	*G*	177	...	+1.4	165
2096	B8	5.41	7.8	36	219	3	3.7	189	...	...	...	12	...	Vela	...	...	...	...
1702	B8	3.18	6.6	43	219	19	2.0	177	+26.0*	177	−103	9	+ 3.4	*G*	3° from antivertex			
2017	B5	4.91	7.6	38	220	7	2.3	255	+24.1	164	+ 3	15	+ 2.2	*G*	211	0.52	+1.5	222
2018	A†	5.74	7.6	38	220	7	2.5	265	...	164	+ 13	20	...	*G*	221	56	+2.5	243
2019	B5	5.78	7.6	38	220	7	2.4	263	...	164	+ 11	15	...	*G*	221	52	+2.5	243
2026	B8	5.48	7.6	38	220	6	2.4	280	...	164	+ 28	15	...	*G*	193	54	+1.9	222
2038	B5	6.44	7.7	38	220	6	2.8	255	...	164	+ 3	13	...	*G*	257	50	+3.4	257
2048	B8	5.86	7.7	38	220	6	2.6	270	...	164	+ 18	15	...	*G*	230	52	+2.6	238
2060	B3	5.11	7.7	38	220	5	1.7	126	...	164	(−126)	21	...	Excl.	...	...	...	...
2141	Od	2.27	8.0	40	224	4	3.5	282	...	160	+ 25	5	...	*G*	243	38	−0.8	266
2109	B5	5.42	7.9	43	225	6	1.9	32	...	162	(+127)	34	...	Excl.	...	...	...	...
2062	B2	5.26	7.8	46	227	9	2.5	253	...	163	− 23	21	...	*F*	204	60	+1.8	196
2165	B3	5.16	8.1	44	228	5	2.2	153	...	159	(−112)	27	...	Excl.	...	...	...	...
2095	B1	4.32	7.8	48	230	10	1.8	186	...	...	...	17	...	Vela	...	...	...	...
1558	B1	4.84	6.1	55	230	27	2.4	250	...	166	(−118)	12	...	Excl.	...	...	...	...
1925	B	5.50	7.3	52	231	16	3.6	264	...	165	− 46	21	...	*F*	248	73	+2.4	238
2119	B1p	4.50	7.9	49	231	9	3.0	258	...	162	− 24	14	...	*F*	225	49	+1.2	210
2166	B3	4.79	8.1	47	231	7	1.7	205	...	...	...	16	...	Vela	...	...	...	...
2171	B3	5.40	8.1	48	231	7	2.6	198	...	...	...	21	...	Vela	...	...	...	...
2191	B3	5.28	8.2	47	231	6	3.8	208	...	...	...	20	...	Vela	...	...	...	...
2127	B3	6.43	7.9	50	232	10	1.7	253	...	162	− 32	27	...	*F*	125	68	+1.9	113
2128	B3	6.65	7.9	50	232	10	2.1	305	...	162	+ 20	25	...	*F*	167	62	+2.8	177
2226	B2	4.90	8.3	48	232	5	1.7	263	+19.0	157	− 10	25	− 2.1	*F*	113	54	+0.1	114
2270	B	6.11	8.4	46	232	3	4.6	251	...	157	− 18	8	...	*F*	292	35	+3.4	277
2360	B5c	5.54	8.7	46	233	1	3.4	195	...	...	...	13	...	Vela	...	...	...	...
2111	B3	3.60	7.9	53	234	12	3.8	297	+19.6	160	+ 3	5	− 1.9	*F*	288	38	+0.9	298
2382	B	4.89	8.8	46	234	1	2.3	211	...	...	...	21	...	Vela	...	...	...	...
2250	B2	5.23	8.4	51	235	7	4.7	258	...	157	− 22	13	...	*F*	292	40	+2.5	277
2337	B3	5.48	8.6	48	235	3	4.8	257	...	155	− 13	12	...	*F*	288	38	+2.8	275
2041	B	6.44	7.7	58	238	16	2.5	258	...	158	− 54	17	...	*E*	104	45	+1.4	90
2319	B5	5.60	8.6	53	238	6	3.7	301	...	155	+ 21	13	...	*E*	209	39	+2.1	208
2322	B5	5.38	8.6	53	238	6	4.4	263	...	155	− 17	9	...	*E*	258	35	+2.6	243
2325	B3	3.68	8.6	53	238	6	2.0	307	+18.0*	155	+ 27	14	− 2.9	*E* ‡	111	40	−1.1	116
2340	B9	5.68	8.6	53	238	6	3.8	295	...	154	+ 15	12	...	*E*	216	36	+2.3	220
2341	B5	5.04	8.7	53	238	6	2.6	274	+19.4	153	− 6	17	− 1.2	*E*	149	42	+0.9	145
2433	B9	5.42	9.0	52	240	3	2.2	306	...	151	+ 32	19	...	*E*	102	47	+0.4	105
2225	B5	6.07	8.3	58	241	11	3.6	267	...	154	− 29	11	...	*E*	181	36	+2.4	179
2298	B3	5.40	8.5	58	241	10	4.9	278	...	153	− 12	9	...	*E*	274	0.33	+2.5	259

2136.	B3	4.96	8h0	63°	243°	16°	2°.0	319°	. . . .	154°	+ 5°	21°	. . . .	*E*	117	0.46	+0.3	123
2331.	B2	4.42	8.6	59	243	10	2.5	256	. . . .	152^5	− 35	13	. . . .	*E*	110	40	−0.4	108
1469.	B5	5.15	5.8	67	244	30	2.7	295	+18.0	155	(− 80)	12	− 2.9	Excl.	. . . . .	. . . . .	. . . . .	. . . . .
2526.	B3	2.63	9.3	55	244	3	2.1	256	+21.9§	147^5	− 19	11	+ 2.2	*E*	96	31	−2.5	89
2406.	B8	3.98	8.9	60	245	9	6.2	336	+24.5	150	+ 48	7	+ 4.4	*E*	211	31	+0.6	218
1566.	B9	5.21	6.2	69	246	28	6.7	298	+25.0‖	153	(− 65)	5	+ 4.4	Excl.	. . . . .	. . . . .	. . . . .	. . . . .
2473.	B3	3.56	9.1	59	246	7	4.1	263	+23.3§	149	− 20	7	+ 3.3	*E*	193	27	0.0	188
2581.	B5	4.20	9.5	59	247	5	1.8	308	+21.7	146	+ 30	15	+ 2.2	*E*	74	38	−1.5	76
2477.	B3	4.18	9.2	62	248	9	4.7	276	+16.0	145	− 4	7	− 3.3	*E*	213	25	+0.8	215
1795.	B8	5.52	6.9	71	249	25	2.9	344	. . . .	151	− 4	14	. . . .	*E*	156	36	+1.5	158
2179.	B5	4.46	8.1	68	249	18	2.8	321	. . . .	150	+ 3	12	. . . .	*E*	144	33	+0.3	144
2604.	B9	4.67	9.6	61	250	6	3.9	299	+26.0*	146	+ 19	9	+ 6.5	*E*	170	29	+0.9	169
2228.	B9	5.44	8.3	71	252	18	4.6	319	. . . .	147	+ 2	8	. . . .	*E*	220	27	+2.1	224
2811.	B5	3.58	10.5	61	255	2	2.6	286	+24.0	138	+ 17	11	+ 6.1	*E*	96	28	−1.4	100
2859.	B3	5.20	10.6	64	257	4	1.7	294	. . . .	137^5	+ 24	20	. . . .	*E*	57	45	−1.0	61
2862.	B	3.03	10.6	64	257	4	2.4	293	+16.0	137^5	+ 23	8	− 1.8	*E*	83	25	−2.4	87
2876.	B5	5.54	10.7	64	257	4	3.8	209	. . . .	. . . .	. . . .	13	. . . .	Vela	. . . . .	. . . . .	. . . . .	. . . . .
2733.	B8	3.56	10.2	70	258	11	2.8	266	. . . .	140	− 16	9	. . . .	*D*	109	26	−1.2	103
2969.	B9	5.52	11.2	64	260	3	2.8	257	. . . .	133^5	− 6	15	. . . .	*D*	99	33	+0.5	95
2606.	B	5.24	9.6	80	262	20	4.4	279	. . . .	137	− 23	6	. . . .	*D*	155	22	+1.1	148
3054.	B9	3.34	11.5	62	262	0	4.8	242	+11.0‖	132	− 15	5	− 5.5	*D*	164	19	−0.7	157
2103.	B5	4.52	11.7	63	264	1	2.0	225	. . . .	130	− 31	21	. . . .	*D*	57	47	−1.7	50
3889.	B3	4.62	10.7	80	265	19	4.8	265	+22.0	134^5	− 19	5	+ 4.9	*D*	165	21	+0.7	162
3111.	B8	5.10	11.8	65	265	3	3.9	247	. . . .	130	− 9	10	. . . .	*D*	131	24	+0.7	125
3200.	B3	4.26	12.2	63	266	1	5.0	247	+19.9	127	− 2	8	+ 4.6	*D*	161	21	+0.3	157
3134.	B9	5.05	11.9	78	268	16	4.4	247	. . . .	132	− 19	6	. . . .	*D*	146	20	+0.8	140
3199.	B5	4.38	12.2	79	268	16	4.9	284	. . . .	131	+ 20	3	. . . .	*D*	158	18	+0.4	160
3289.	B3	2.94	12.5	69	269	6	4.2	240	+13.5	130	− 10	7	− 2.6	*D*	137	21	−1.4	125
3269.	B5	4.04	12.4	72	269	9	4.6	253	. . . .	129	0	7	. . . .	*D*	152	20	+0.1	146
3320.	B3	3.26	12.7	68	270	5	4.1	225	. . . .	126	− 22	7	. . . .	*D*	121	21	−1.3	115
3429.	B8	4.95	13.1	67	273	4	3.2	234	. . . .	123	− 7	11	. . . .	*D*	98	25	0.0	93
3646.	B3	5.88	14.2	66	279	5	1.7	239	. . . .	118	+ 11	30	. . . .	*D*	50	54	−0.6	44
3925.	B5p	5.65	15.3	73	281	14	2.3	182	. . . .	120	− 36	17	. . . .	*D*	56	39	−0.7	47
3956.	B8	5.76	15.5	73	281	14	2.4	222	. . . .	119	+ 7	16	. . . .	*D*	71	29	0.0	64
3886.	Oe5	5.50	15.2	60	287	3	2.5	233	. . . .	108	+ 18	19	. . . .	*D*	65	38	−0.5	60
3874.	Oe5	5.24	15.2	61	287	3	2.2	216	. . . .	109	0	17	. . . .	*D*	60	33	−0.9	55
3872.	B1	5.95	15.2	60	287	3	2.2	256	. . . .	109	+ 40	17	. . . .	*D*	46	41	−0.8	44
3901.	B5C	4.54	15.2	59	289	2	4.3	193	. . . .	107	− 22	9	. . . .	*B*	108	21	−0.3	100
4268.	B9	6.30	16.7	65	292	13	2.3	195	. . . .	108	− 3	17	. . . .	. . . .	62	34	+0.2	54
4013.	B5	5.84	15.7	55	294	1	3.7	163	. . . .	102	− 46	13	. . . .	*B*	68	35	0.0	61
4094.	B9	5.79	16.0	58	294	5	6.5	207	+ 6.0‖	104	+ 1	7	− 2.7	*B*	173	0.18	+2.0	166
4392.	B3	5.88	17.2	63	296	15	2.4	118	. . . .	107	(− 74)	23	. . . .	Excl.	. . . . .	. . . . .	. . . . .	. . . . .

* Spect. Binary, estimated velocity of center of mass.

† Inserted because of community of motion with Nos. 2017, 2019, 2026, 2038, 2048.

‡ Also used in *F*.

§ Velocity center of mass from orbit.

‖ 1 observation.

LIST 2.—*Continued*

Boss No.	Sp.	Harv. Mag.	1900 α	1900 δ−	Gal. Long	Gal. Lat.−	100 μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit 0″.0001	r_π π	M	π_2 Unit 0″.0001
4253	B	5.76	16h6	58°	297°	9°	5″.2	209°	......	102°	+10°	10°	......	B	134	0.21	+1.4	128
4249	B3	5.94	16.6	58	297	9	3.0	180	0.0*	102	−19	16	−8.1	B	76	32	+0.3	69
4426	B8	3.70	17.4	61	298	15	10.2	213	......	104	+23	3	......	B	250	14	+0.7	243
4324	B3	5.88	16.9	58	299	10	3.2	206	......	102	+10	13	......	B	82	28	+0.4	74
4762	B2	4.42	18.7	62	300	24	1.9	235	......	104	+59	14	......	......	27	48	−3.4	24
4666	B8	4.81	18.4	62	300	22	3.4	192	......	104	+13	9	......	B	88	21	−0.4	80
4156	B5	5.49	16.2	49	302	0	3.3	200	+1.0*	95	−2	15	−4.9	B	85	31	+0.1	79
4296	B8	6.57	16.8	50	304	5	3.1	216	......	94	+19	15	......	B	77	30	+1.0	73
4611	B5	5.54	18.2	56	305	18	4.7	236	......	98	+53	9	......	B	73	29	−0.2	71
4431	B3	2.97	17.4	50	308	9	9.0	201	+2.0†	93	+11	2	−3.3	B	227	14	−0.3	223
4398	B3	5.50	17.3	47	310	7	4.5	171	......	91	−21	11	......	B	108	25	+0.7	102
4796	B9	5.03	18.8	53	310	23	2.3	52	−4.2	95	(−122)	19	−10.1	Excl.	......	......	......	......
4565	B1ac	3.90	18.0	50	311	14	3.0	203	+2.8	92	+19	8	−2.1	B	75	20	−1.7	68
4404	B8	5.10	17.3	44	313	5	4.0	217	......	87	+26	12	......	B	93	26	−0.1	90
4677	B8	5.05	18.4	46	315	16	4.0	201	......	88	+22	9	......	B	95	22	−0.1	89
4657	B3	3.76	18.3	46	315	16	5.3	191	−0.8	88	+11	5	−4.5	B	134	16	−0.6	129
4680	B5	5.33	18.4	46	316	17	1.9	196	......	88	+17	17	......	......	46	35	−1.4	39
4429	B3	2.80	17.4	37	319	2	4.2	183	+17.0†	80	−7	5	+15.9	B	111	16	−2.0	105
4439	B2	1.71	17.4	37	319	3	3.6	186	+3.0*	80	−3	5	+1.9	B	95	15	−3.4	89
4474	B2	2.51	17.6	39	319	5	2.8	201	......	82	+13	6	......	......	70	18	−3.3	64
4760	B9	5.64	18.7	44	319	19	2.5	154	......	85	−22	15	......	......	60	33	−0.5	53
4929	B8	4.24	19.2	45	320	25	1.9	183	−4.0	87⁵	+14	13	−6.8	......	46	27	−2.5	39
4599	B3	5.52	18.1	41	320	11	5.7	256	......	84	(+73)	17	......	Excl.	......	......	......	......
4509	B8	5.97	17.8	35	323	5	1.7	225	......	77	+39	24	......	......	35	57	−1.3	33
4690	B9	6.55	18.4	39	323	14	2.8	176	......	81	−2	12	......	B	73	30	+0.8	64
4691	B9	5.95	18.4	39	323	14	3.8	203	......	81	+25	11	......	B	96	23	+0.8	91
4498	B8	4.83	17.7	32	325	3	2.4	173	−16.0	74	−14	12	−15.3	......	62	26	−1.3	55
4936	B8	4.11	19.3	41	325	24	12.9	168	......	84	0	2	......	B	336	14	+1.7	334
4637	B8	5.39	18.3	37	325	11	2.4	152	−17.0*	78	−28	15	−17.5	B	55	33	−0.9	51
5060	B9	5.39	19.7	40	327	29	3.8	130	......	84	−34	11	......	B	83	27	0.0	80
4792	B5	5.41	18.8	38	327	18	2.9	127	+6.0*	80	−47	12	+4.9	......	50	36	−1.1	48
4734	B3	4.82	18.6	36	327	15	5.3	180	+1.4	78	+3	9	+0.9	B	139	20	+0.5	136
4913	B5	5.61	19.2	36	330	22	7.4	257	......	79	(+88)	16	......	Excl.	......	......	......	......
5108	B3	4.39	19.9	36	332	29	4.7	160	......	81	−2	10	......	B	122	21	−0.2	118
5040	B6	5.56	19.7	32	335	26	2.9	192	......	76	+28	11	......	......	66	26	−0.3	63
4739	B8	3.30	18.6	27	335	12	4.8	94	......	68	(−83)	3	......	Excl.	......	......	......	......
4784	B3	2.14	18.8	26	337	13	6.6	173	−1.0	68	−1	1	+1.6	B	183	35	−1.6	182
4718	B9	5.75	18.5	24	337	9	3.0	195	......	65	+17	7	......	B	82	18	+0.2	76
4999	B9	4.66	19.5	25	342	22	7.6	109	−17.0†	68	(−57)	1	−14.4	Excl.	......	......	......	......
4783	B5	5.04	18.8	16	347	9	2.4	250	−5.7	58	(+77)	16	−0.3	Excl.	......	......	......	......
4816	B5	5.36	18.9	13	349	9	2.5	182	−12.7	56	+10	15	−6.8	......	75	32	−0.2	72
4954	B8	5.68	19.3	15	351	15	2.3	95	−13.0	59	(−72)	13	−7.9	Excl.	......	......	......	......
4883	B3	5.37	19.1	8	356	9	1.7	156	−11.5	51	−13	9	−4.3	......	53	0.23	−1.0	49

* 1 observation.

† Spect. Binary, estimated velocity of center of mass.

LIST 3

Helium Stars for Which $100\mu < 1''.7$

Boss No.	Sp.	Harv. Mag.	1900 α	1900 $\delta-$	Gal Long.	Gal. Lat.	100μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit $0''.0001$	$\frac{r_\pi}{\pi}$	M	π_2 Unit $0''.0001$
1907	B3	4.68	7h2	37°	217°	−10°	1''.5	208°	......	167°	− 39°	22°	......	......	148	0.74	+0.5	114
1845	B3	4.85	7.1	40	218	−13	1.3	283	+19.0	170	+ 25	20	− 3.3	......	182	83	+1.1	217
2188	B3	5.12	8.2	36	221	0	0.9	125	......	156	−122	46	......	......	......	......	......	......
2187	B3	4.77	8.2	36	221	0	1.3	180	......	156	− 67	23	......	......	57	66	−1.4	49
2089	B3	4.53	7.8	39	222	− 5	1.6	210	......	162	− 45	16	......	......	92	58	−0.7	86
2342	B2	3.70	8.7	33	223	+ 7	1.3	299	+17.0	150	+ 57	18	− 3.1	......	52	48	−2.7	60
2217	B3	5.17	8.3	36	223	+ 1	1.4	188	......	156	− 60	29	......	......	70	71	−0.6	54
1719	B	6.47	6.6	48	225	−21	1.4	321	......	174	− 19	31	......	......	6° from antivertex			
2249	B	5.30	8.4	42	227	− 1	1.5	259	......	156	− 1	24	......	......	94	53	+0.2	94
1884	B8	4.88	7.2	48	227	−15	1.6	38	......	168	(+103)	33	......	Excl.	......	......	......	......
2070	B	4.25	7.8	46	228	− 9	1.4	231	+20.5	163	− 44	23	− 1.3	......	89	71	−0.9	77
2267	B	5.06	8.4	44	230	− 2	1.2	194	......	156	− 71	38	......	......	50	1.03	−1.4	40
2196	B	6.02	8.2	46	230	− 5	1.1	248	......	158	− 19	35	......	......	70	75	+0.2	70
2094	B3	4.83	7.8	49	230	−10	1.5	293	......	162	+ 8	36	......	......	125	75	+0.3	136
2332	B5	5.23	8.6	45	232	− 1	1.1	221	......	153	− 44	52	......	......	46	1.27	−1.5	36
2266	B5	5.52	8.4	48	233	− 4	1.3	247	0.0	156	− 27	13	−21.0	......	75	57	−0.1	71
2467	B5	4.96	9.1	44	235	+ 3	1.4	270	+35.0	149	+ 9	27	+15.0	......	70	54	−0.8	73
2408	B5	4.77	8.9	52	239	− 3	1.1	195	......	151	− 80	38	......	......	38	1.06	−2.3	27
2629	B	5.68	9.7	44	240	+ 8	1.1	270	......	144	+ 14	34	......	......	48	63	−0.9	48
2363	B3	4.63	8.7	56	240	− 7	0.5	259	......	153	− 26	94	......	......	......	......	......	......
2575	B3	5.35	9.5	49	241	+ 3	1.5	340	......	145	+ 75	28	......	......	45	77	−1.4	56
2414	B3	5.08	8.9	59	244	− 8	1.2	280	......	150	− 5	20	......	......	62	43	−0.9	66
2702	B3	5.10	10.1	51	246	+ 4	1.0	241	......	141	− 21	37	......	......	37	74	−2.1	33
2457	B3	4.86	9.1	70	254	−15	0.8	310	+26.0*	144	+ 9	62	+ 6.9	......	......	......	......	......
2880	B5	5.10	10.7	64	257	− 4	1.4	210	......	138	− 40	37	......	......	42	85	−1.8	30
2878	B8	5.43	10.7	64	257	− 4	0.1	270	......	138	0	360	......	......	......	......	......	......
2867	B3	5.09	10.7	63	257	− 4	1.4	291	......	138	+ 21	30	......	......	50	60	−1.4	53
3151	B3	4.98	12.0	63	265	− 1	0.6	288	......	128	+ 36	68	......	......	......	......	......	......
3802	B2	5.42	14.8	62	285	− 3	1.1	207	......	112	− 13	41	......	......	31	72	−2.2	22
3668	B3	4.10	14.2	46	286	+13	1.2	228	......	104	+ 5	24	......	......	32	45	−3.4	27
4405	B1	3.51	17.3	56	302	−12	1.3	189	......	100	− 2	17	......	......	34	32	−3.9	26
4180	B5	4.71	16.3	47	303	+ 1	1.0	217	+ 7.0*	94	+ 15	73	+ 1.4	......	20	1.27	−3.2	21
4496	B5	5.90	17.7	54	305	−14	1.3	347	......	96	(+160)	39	......	Excl.	......	......	......	......
4288	B	5.34	16.8	42	311	+ 1	1.2	194	−18.0*	86	− 3	32	−21.0	......	31	56	−2.2	24
4274	Oe	5.37	16.8	41	311	+ 1	1.0	101	......	86	(− 96)	35	......	Excl.	......	......	......	......
4642	B5	5.42	18.3	44	317	−15	1.6	142	......	86	− 38	24	......	......	34	0.55	−1.9	29
4334	B1p	4.87	17.0	34	319	+ 4	0.7	82	+ 8.6	78	−113	36	+ 8.1	......	......	......	......	......
4369	B3	5.50	17.2	33	321	+ 2	0.9	63	......	76	−132	44	......	......	......	......	......	......
4697	B3	5.38	18.4	33	329	−12	0.9	153	......	74	− 25	37	......	......	......	......	......	......

LIST 3—*Continued*

Boss No.	Sp.	Harv. Mag.	1900 α	1900 $\delta-$	Gal. Long	Gal. Lat.	100 μ	p	ρ	λ	p O−C	r_p	ρ O−C	Group	π_1 Unit 0.″0001	$\frac{r_\pi}{\pi}$	M	π_2 Unit 0.″0001
4560	Oe5	5.86	18h0	24°	334°	− 2°	0.6	180°	....	66°	− 4°	43°	....	....	......	......	......	......
4549	B	5.73	17.9	23	335	− 1	1.2	215	....	65	+ 30	23	....	....	28	0.50	−2.0	26
4604	B8pc	4.01	18.1	21	338	− 3	0.6	141	− 7.0†	64	− 42	18	− 3.3	....	......	......	......	......
4612	Ba	5.42	18.2	21	338	− 3	0.5	169	Bin.	64	− 14	36	....	....	......	......	......	......
4613	B1	6.02	18.2	20	339	− 2	1.0	180	....	62	− 3	23	....	....	30	41	−1.6	21
4668	B8	6.03	18.4	18	341	− 4	0.7	207	....	60	+ 28	60	....	....	......	......	......	......
4793	B8	5.89	18.8	23	340	−12	1.6	202	....	65	+ 28	17	....	....	40	37	−1.1	32
4865	B3	5.41	19.0	19	345	−13	0.1	90	Bin.	62	− 80	344	....	....	......	......	......	......
4934	B8p	4.58	19.3	16	350	−15	0.5	217	−10.0‡	60	+ 51	39	− 5.2	....	......	......	......	......
5240	B8	5.20	20.4	18	353	−30	1.6	142	−18.3	67	− 13	8	−15.5	....	45	20	−1.5	41
4548	B5c	3.92	17.9	3§	358	+12	1.4	172	− 4.0	39	− 15	10	+ 5.9	....	57	0.27	−2.3	48
4527	B8	5.73	17.9	1§	355	+12	0.6	45	−17.2‡	42	−142	50	− 7.9	....	......	......	......	......

* 1 observation.
† Velocity center of mass from orbit.
‡ Spect. Binary, estimated velocity of center of mass.
§ Declination north.

GRONINGEN
January 1914

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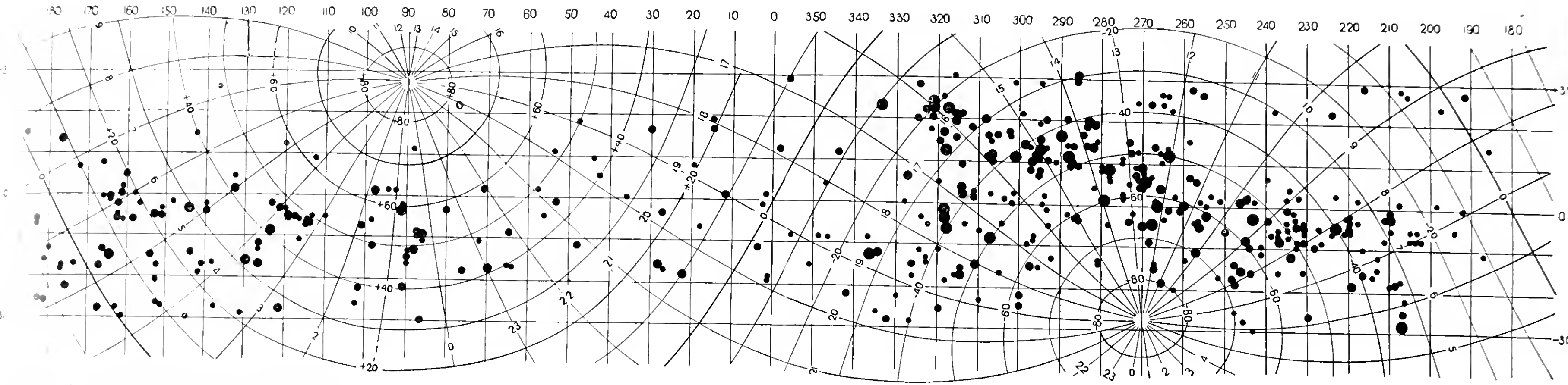

1.—Helium Stars Having Total P.M.≧0″.017

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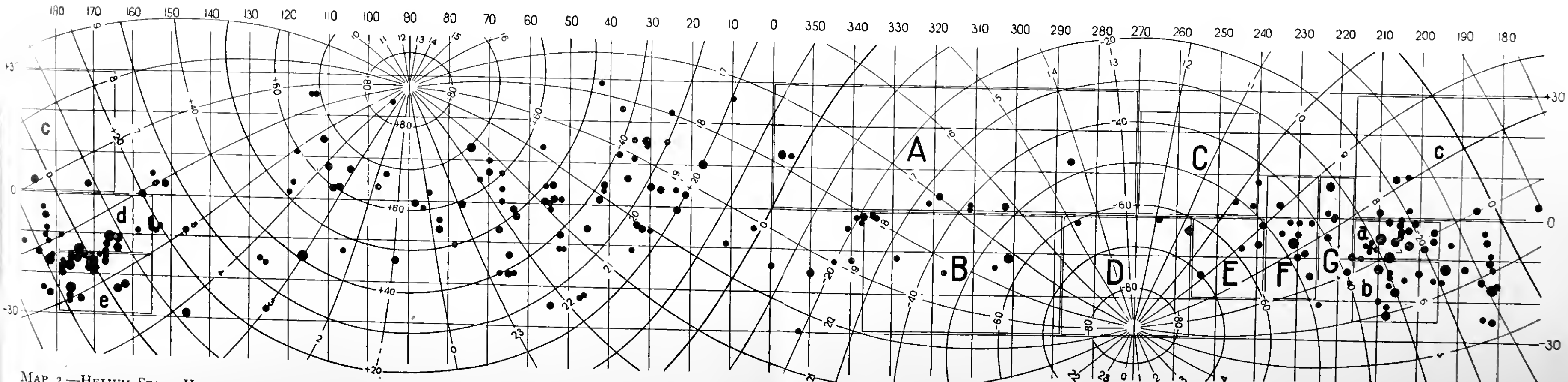

Map 2.—Helium Stars Having Total P.M.≦0″.016

IMPROVEMENTS IN THE OPTICAL SYSTEM OF THE STELLAR SPECTROGRAPH

By J. S. PLASKETT

It has long been my opinion that the dense silicate flint, the O 102 glass of the Jena Glass Works, which has been almost exclusively chosen as the prism material for the stellar spectrograph, is too highly colored and hence too absorbing in the violet for the best results. This opinion was confirmed by my experiments with the grating spectrograph[1] and also by some work with objective prisms of light flint glass. The O 102 is probably the most suitable among the dense flints and was apparently chosen because the high dispersion and resolving power, together with the great compactness and symmetry of form, demanded in the modern three-prism stellar spectrograph, seemed most easily satisfied by the use of dense flint glass.

It was not, however, at first recognized that the conditions required in one-prism instruments were not the same. Their principal usefulness has proved to consist in obtaining the spectra of early-type stars, where the lines are few in number, are each due in general to one element only, and are often broad and diffuse. Under such circumstances high dispersion and resolving power are not necessary and indeed are, when the lines are diffuse, a disadvantage. Furthermore, to increase the measureable material in spectra with few lines, it is desirable that as long a range of wave-length as possible be photographed at one exposure. Under these conditions O 102 glass is not suitable as it is not only highly dispersive but it is highly absorbing in the violet and ultra-violet, thus considerably limiting the range of wave-lengths available.

The only investigation bearing on the absorption of glasses suitable for prism material seemed to be the work undertaken at Potsdam by Vogel, Müller, and Wilsing[2] in their investigation of

[1] *Astrophysical Journal*, **37**, 373, 1913.

[2] *Berichte der Berliner Akademie*, November 1896.

the materials proposed for the 80-cm objective and the spectrographs to be used with it. Their values in the photographic region for three glasses, O 203 Ordinary Silicate Crown, O 340 Ordinary Light Flint, and O 102 Heavy Silicate Flint, are given in Table I. An approximate idea of the absorptions of other glasses can be obtained by comparing their relative dispersions in different parts of the spectrum. Consequently in the same table are given the dispersion constants of four other glasses selected from the Jena list as being the most promising.

TABLE I

CONSTANTS OF REPRESENTATIVE GLASSES

Kind of Glass	Trade No.	n	Δ	Ratios of Dispersion			Transmissions through 10 cm for Wave-Length				
				α	β	γ	4341	4000	3950	3900	3750
Ordinary Silicate Crown........	O 203	1.5175	0.00877	0.642	0.702	0.568	0.667	0.695	.....	0.583	0.583
Baryta Light Flint.........	O 722	1.5797	.01087	.632	.707	.577	.....	.....	.....	.....	.....
Baryta Light Flint.........	O 1266	1.6042	.01381	.616	.711	.594	.....	.....	.....	.....	.....
Ordinary Light Flint.........	O 340	1.5774	.01396	.614	.713	.600	.569	.614	.....	.456	.388
Baryta Flint.....	O 748	1.6235	.01599	.605	.713	.604	.....	.....	.....	.....	.....
Ordinary Silicate Flint.........	O 93	1.6245	.01743	.604	.715	.609	.....	.....	.....	.....	.....
Heavy Silicate Flint.........	O 102	1.6489	0.01919	0.600	0.714	0.615	0.502	0.463	0.167	0.025	.....

In the table n is the index of refraction for D, Δ is the dispersion from C to F, and α, β, γ the ratios of the dispersions between A′ and D, between D and F, and between F and G′ to the dispersion Δ. The last five columns give the experimentally determined values of the transmission for a thickness of 10 cm of the glass. The transmission of the other glasses is presumably intermediate between the given values, varying probably with the run of the relative dispersions, with α and γ.

The transmission through 10 cm of O 102 glass shows how unsuitable it is for prism material, not only on account of the general absorption all along the photographic spectrum but especially on account of the strong absorption around K. The O 722 glass was

selected as the most promising, as it gives considerably greater dispersion than the O 203 with probably very little more absorption.

Consequently a prism of fine annealed glass of Baryta Light Flint O 722 was ordered from the J. A. Brashear Co. In order that it could be placed in the one-prism spectrograph without alterations of the frame, the deviation of the ray at minimum must be 60°. As it was proposed to carry the measurements farther to the violet, the central wave-length was chosen as λ 4200, midway linearly between λ 4550 and λ 3934 or between λ 4862 and λ 3750. As the constants of the material of which the prism was to be made were $n=1.5782$, $\Delta=0.01078$, $\alpha=0.637$, $\beta=0.708$, $\gamma=0.576$, even more favorable for transparency than the tabular values, the angle required was 68° 57′, the length of the sides for a 51 mm (2 inches) aperture was 118 mm (4.64 inches) and of the base 133 mm (5.25 inches). The prism was made 57 mm (2.25 inches) high and is consequently a large block of glass. It is beautifully colorless and transparent, and, notwithstanding its large size, shows no trace of imperfect annealing. Careful tests by diaphragming different sections have shown that every part defines equally well and as a dispersing piece it is practically perfect.

In order to obtain the full advantage of the transparency of this glass, the isokumatic collimator objective, whose central component is decidedly yellowish, was replaced by a Brashear triplet and a triplet camera objective of slightly longer focal length than the "Single Material" was also obtained. In order to reduce the loss by reflection in the internal surfaces, which, as will be seen later, is a serious matter, amounting to over 30 per cent for the two objectives, it was desirable to cement them. Watch oil and glycerine were first tried but, though they answer admirably at ordinary temperatures, when the spectrograph gets much below freezing, they become crystallized or mottled in appearance and cannot be used. Some special balsam prepared for cementing together three-color transparencies which remains softer than the ordinary balsam was successfully used and up to the present no ill effects on the definition have appeared.

The spectrograph box was dismounted and the new prism and objectives installed and carefully tested with an artificial source

before being used on the stars. The definition given was excellent and some preliminary comparative tests of the two prisms showed striking advantages in efficiency, especially in the violet, of the light flint. A curious change in the character of the field given by the triplet camera was noticed, for, while with the dense flint prism the field was concave to the lens, with the light flint it was convex and of smaller curvature. With a prism of intermediate dispersion the field would probably be flat. The curvature of course is small, the difference between the center λ 4200 and the ends λ 4550 and K being about 0.1 mm. By accommodating, the focus of no part of the spectrum need be more than 0.05 mm from the plate. The dispersion given by the combination is 54.5 A per millimeter at H_γ, 48.3 A at λ 4200, and 37.5 A at K. This is almost exactly three-fifths of the linear dispersion given with the O 102 prism and can be made equal by increasing the focal length of the camera, although this is not readily accomplished with the present arrangement.

Before giving the results of the comparative tests of exposures, it will be of interest to obtain the intensities of the emergent pencils by computation. The intensities after reflection at the various surfaces are obtained from the well known formulae used for this purpose, while the intensities after absorption have been computed for the prisms, from measured and interpolated values of O 102 glass, and from interpolated values of O 722 glass. For the isokumatic collimator, its absorption was estimated by comparing its color with that of the O 102 prism. It was found that between 50 and 60 mm thickness of the latter gave about the same depth of color as the isokumatic, and its absorption was hence assumed as equal to that of the prism. The absorption of the material of the other objectives was taken as the same as that of O 722 glass. The mean thickness of the O 102 prism is 57 mm, of the O 722, 66 mm, of the triplet lenses about 20 mm, and of the single material 10 mm. The intensities, after losses by reflection and by absorption, and the final emergent intensities are given in the tables below.

The total emergent intensities, for the various wave-lengths in Table III, show that the new optical system transmits a considerably larger percentage of the incident light than the old, giving

a pencil 50 per cent stronger in the blue, 75 per cent stronger at H_δ, and four times as strong at K. The comparatively small

TABLE II

INTENSITIES AFTER REFLECTIONS

System	Optical Part	First Reflection Air to Glass	Reflection at Each Inner Surface	Method	Final Intensity
Old...	Isokumat Collimator.....	0.9535	0.99	$0.9535^2 \times 0.99^4$	0.8735
	O 102 Prism	.874	....	....	.779
	Single Material Camera...	.9535	.9535	$.9535^4$	.8247
	Total.............	....	....	....	.5607
New..	Triplet Collimator.......	.9535	.99	$.9535^2 \times 0.99^4$	.8735
	O 722 Prism	.868	....	....	.768
	Triplet Camera..........	0.9535	0.99	$0.9535^2 \times 0.99^4$	.8735
	Total.............	....	....	....	0.5861

TABLE III

INTENSITIES AFTER ABSORPTIONS

System	Optical Part	H_β	4600	H_γ	4200	H_δ	4000	K	H_ζ
Old...	Isokumat Collimator	0.842	0.761	0.696	0.658	0.658	0.639	0.400	0.108
	O 102 Prism	.842	.761	.696	.658	.658	.639	.400	.108
	Single Material Camera......	.983	.974	.960	.945	.945	.945	.925	.900
	Total intensity after absorption..........	.697	.564	.464	.408	.408	.385	.148	.011
	Total emergent intensity....	.391	.316	.260	.229	.229	.216	.083	.006
New..	Triplet Collimator........	.973	.962	.938	.925	.925	.925	.903	.884
	O 722 Prism...	.916	.881	.809	.775	.775	.783	.714	.666
	Triplet camera ..	.973	.962	.938	.925	.925	.925	.903	.884
	Total intensity after absorption..........	.858	.814	.712	.664	.664	.671	.582	.520
	Total emergent intensity....	0.503	0.477	0.417	0.389	0.389	0.393	0.341	0.305

values show also how large a percentage of the light is lost in the optical system of even the most efficient form of spectrograph, and

how important it is to look after apparently minor details. For example, cementing the triplet camera makes the difference between transmission of 0.8735 and 0.7515, a gain of 16 per cent, while cementing both gives increased transmission of 35 per cent. Similarly the loss by absorption in the isokumatic collimator is some 30 per cent greater than in the triplet, and this computed loss is fully borne out by the experimental results.

Experimental tests of the relative efficiencies of the new and old optical systems have been carried out in two ways: first, by comparing a number of plates of the same stars by the two systems, making allowance for the differences in seeing; second, by making direct comparative tests following one another on the sun and stars in exactly the same way as with the grating spectrograph.[1] The latter method gives results probably more reliable and certainly more directly comparable than the former. By the first method three spectra of one star with the new system, with exposures in the ratio of 1, 2, 3, were made side by side on one plate and this was repeated for a number of stars. Each of these plates was then compared with about ten plates of the same stars with the old spectrograph, and it was comparatively easy to get reliable estimates of the relative exposures. Similarly, by the second method, seven or eight exposures, with times increasing about 50 per cent on each, were made of the same stars by each form of the instrument, each set of exposures being on one plate and all the plates developed together. It is evident that numerous accurate comparisons of the relative intensities at any wave-length can easily be made from such a set of exposures.

Table IV contains a summary of the mean values obtained by both methods. In (6) of this table we get the relative exposures experimentally determined for the new and old optical systems, and it will be seen how marked a saving in exposure time is effected. The new system requires less than two-fifths the exposure of the old at H_γ, one-quarter at H_δ and only one-ninth at K. It must not be forgotten, however, that the dispersion of the new system is only three-fifths that of the old and to make them directly comparable the figures in (6) should be multiplied by five-thirds.

[1] *Astrophysical Journal*, **37**, 373, 1913.

This has been done in Table V, giving the relative experimentally determined efficiencies. If we divide the total emergent intensities of Table III we get the relative computed efficiencies.

TABLE IV

MEAN VALUES OF RELATIVE EXPONENTS AT EIGHT WAVE-LENGTHS

Method	No.	Optical Systems Compared, Last Unity	H_β	4600	H_γ	4200	H_δ	4000	K	H_ζ
First	1.....	O 722 Triplet:O 102 Isokumat........	0.381	0.402	0.363	0.285	0.225	0.156	0.108	0.059
Second ..	2.....	O 722 Triplet:O 102 Triplet...........	.570	.554	.471	.395	.313	.263	.177	.123
	3.....	O 102 Triplet:O 102 Isokumat........	.785	.754	.726	.712	.712	.687	.638	.586
	4.....	O 722 Triplet:O 102 Isokumat........	.508	.501	.406	.351	.270	.180	.115	.080
	5.....	O 722 Triplet:O 102 Isokumat (2)×(3)	.447	.418	.342	.281	.223	.181	.113	.072
	6.....	Mean of (1), (4), (5)	0.445	0.440	0.370	0.306	0.239	0.172	0.112	0.070

TABLE V

RELATIVE EXPERIMENTAL AND COMPUTED EFFICIENCIES

	H_β	4600	H_γ	4200	H_δ	4000	K	H_ζ
Experimental Old : New	0.742	0.733	0.617	0.510	0.398	0.287	0.187	0.117
Computed Old : New	0.770	0.666	0.624	0.589	0.589	0.550	0.245	0.019

There is good agreement in general in these figures and the deviation between λ 4200 and λ 4000 is probably due chiefly to insufficient data as to the absorptions of the glasses in this region. It is my opinion that the run of the absorptions in the O 102 glass must be much more gradual than given in the tables and furthermore that the absorption of the isokumatic collimator may be such as to account for part of this difference. A more gradual run of the absorptions would make excellent agreement between observed and computed values. Another factor will influence the magnitude of the experimental values for certain wave-lengths, the fact that all wave-lengths of the light forming the star image are not in sharp focus on the slit. Owing to the color-curve of objective and correcting lens there is a difference of over 3 millimeters in the position

of the star focus for H_γ light and for light at H_β and K. The slit was placed so that wave-lengths about λ 4000 and λ 4650 were in focus on it and the higher experimental value at λ 4600 may be due to this cause.

It may be pointed out that (3) in Table IV gives relative exposures when triplet and isokumat collimator are interchanged and that they agree well with the estimated values of the absorption of the isokumat in Table III until we get near K. Here evidently the absorption of the borosilicate flint of the isokumat differs from that of the O 102, producing a more gradual change. Both sets of figures form striking evidence of the unsuitability of this objective for stellar spectrographs.

In addition to the gain in efficiency, another great advantage of the new optical parts is the uniformity in the intensity of early-type spectra. With the O 102 prism, if the exposure was such as to make the K line measureable, the region around H_γ was so much overexposed as to block up the fainter metallic lines. Furthermore, spectra from the light flint prism contain four or five more measureable hydrogen lines than those made with the dense flint, thus considerably increasing the material available for measurement in stars with few lines.

The results of this investigation may be summarized as follows:

1. The dense silicate flint (O 102) glass, almost universally used as prism material in stellar spectrographs, has been shown to be too highly absorbing all along the photographic spectrum and especially toward and in the ultra-violet for the best results in radial velocity work. This is especially the case in single-prism spectrographs employed on early-type stars.

2. The substitution of a baryta light flint prism, O 722, for the O 102 has diminished the exposure times (when both are reduced to the same linear dispersion) by 22 per cent at H_γ, 48 per cent at H_δ, and 70 per cent at K, besides giving considerably more measureable material in the ultra-violet without overexposure around H_γ.

3. The substitution of a Brashear Triplet for the isokumat collimator objective has, owing to the strong absorption of the latter, effected a further saving of about 30 per cent.

4. The cementing of the contact curves of the triplet collimator and camera objectives diminishes the loss by reflection over what would occur with uncemented lenses by 35 per cent.

5. The ratios of the intensities of the emergent pencils from the old and new optical parts are 0.62 at H_γ, 0.40 at H_δ, and 0.19 at K. The actual ratios of the exposures required owing to the three-fifths dispersion of the new prism are 0.37 at H_γ, 0.24 at H_δ, and 0.11 at K. Although the smaller dispersion will probably entail proportionally larger probable errors, the fact that nearly all the stars within range of the present equipment have been observed, and that the new optical parts enable stars at least a magnitude fainter to be reached, form partial compensation for the diminished accuracy.

6. Finally, the investigation has shown the importance in stellar spectroscopy, where the light is always meager in quantity, of so selecting the materials and designing the optical parts of a stellar spectrograph that all the losses by reflection and absorption may be minimized; and the results indicate what a great saving in exposure time and consequent increase in output and range may be effected.

It gives me pleasure to acknowledge here the readiness of the director, Dr. W. F. King, to supply the apparatus needed in this investigation, as well as the interest he has taken in the work.

Since the above was written, measures of the transparency to violet and ultra-violet light of five optical glasses have been kindly sent me by the Jena Glass Works. These measures with the corresponding refraction-constants are given in the following tables.

The measures show that only a very general guide to the relative absorptions can be obtained from the ratios of the dispersions and this only when glasses of similar composition are considered. The ordinary silicate flints are evidently more suitable than the baryta flints for prism material, as O 118, which is recommended by the Jena people, has apparently less absorption than O 722 and at the same time 50 per cent greater dispersion. The choice seems to lie between this material and the ultra-violet flint, depending upon how far into the ultra-violet one wishes to go. It is a simple matter from the accompanying data to compute the dimensions of the prisms and the losses by reflection and absorption for any given dispersion and spectral region and hence to determine the most suitable material.

TRANSMISSION OF GLASSES

Wave-Length	U.V. 3248 Ultra-Violet Flint		O 722 Baryta Light Flint		O 748 Baryta Flint		O 340 Ordinary Light Flint		O 118 Ordinary Flint		O 919 Ordinary Flint
	1 cm.	10 cm.	1 cm.	10 cm.	1 cm.	10 cm.	1 cm.	10 cm.	1 cm.	10 cm.	1 cm.
4250	......	......	......	......	......	......	......	......	......	......	0.96
4050	......	......	0.99	0.86	0.98	0.84	0.98	0.83	0.99	0.88	......
3970	0.98	0.96	......	......	......	......	......	......	......	......	.94
3840	.98	.87	......	......	......	......	......	......	......	......	0.86
3650	......	......	.87	.25	.81	.11	.86	.22	.90	.33	......
3610	.98	.79	......	......	......	......	......	......	......	......	......
3460	.92	.45	......	......	......	......	......	......	......	......	......
3340	......	......	0.39	0.00	0.22	0.00	0.35	0.00	0.43	0.00	......
3250	0.78	0.08	......	......	......	......	......	......	......	......	......

REFRACTION CONSTANTS

Kind of Glass	n	Δ	ν	Ratios of Dispersion		
				α	β	γ
U.V. 3248 ultra-violet flint	1.5332	0.00964	55.4	0.634	0.705	0.573
O 722 baryta light flint	1.5797	.01078	53.8	.632	.707	.577
O 748 baryta flint	1.6235	.01599	39.1	.605	.713	.604
O 340 ordinary light flint	1.5774	.01396	41.4	.614	.713	.600
O 118 ordinary flint	1.6129	.01660	36.9	.606	.713	.607
O 919 ordinary flint	1.6315	0.01770	35.7	0.600	0.715	0.613

Dominion Observatory
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ON THE PRESSURE-SHIFT OF THE LINES OF THE ZINC SPECTRUM AT LOW PRESSURES

By V. F. SWAIM

INTRODUCTION

Previous studies of pressure-effect have, for the most part, been at comparatively high pressures. A part of the earlier work of Humphreys and Mohler[1] and of Humphreys[2] dealt with pressures as low as 4 atmospheres, but the more recent work of Humphreys[3] has been at much higher pressures. Similarly, the work of Duffield[4] upon the arc spectra of iron, gold, and silver contains some results obtained at pressures as low as 5 atmospheres, but the majority of his values are also for high pressures. The work of Rossi[5] upon the arc spectra of titanium and vanadium is based principally upon pressures between 25 and 100 atmospheres.

All the work referred to above showed that the lines of any spectrum were shifted toward the red as the pressure about the source was increased; also, that the amount of this shift for a given increase of pressure was greater for the long wave-lengths than for the short ones.

Later work by Gale and Adams[6] showed that many lines of the spectrum of iron could be classified into groups, according to their behavior under pressure; also, that the displacement for the lines of each group varied as the third power of the wave-length.

St. John and Ware[7] found that a certain group of iron lines was shifted toward the violet under pressure. This result was later confirmed by Gale and Adams.[8] Goos[9] has shown that the value of the wave-length of many of the iron lines depends on the part

[1] *Astrophysical Journal*, **3**, 114, 1895.

[2] *Ibid.*, **4**, 249, 1896; **6**, 169, 1897.

[3] *Ibid.*, **22**, 217, 1905; **26**, 18, 1907.

[4] *Ibid.*, **26**, 375, 1907; *Phil. Trans.*, A **208**, 111, 1908.

[5] *Proc. Roy. Soc.*, A **83**, 414, 1910.

[6] *Astrophysical Journal*, **35**, 10, 1912.

[7] *Ibid.*, **36**, 14, 1912.

[8] *Ibid.*, **37**, 391, 1913.

[9] *Ibid.*, **38**, 2, 1913.

of the arc used and on the strength of the current. Many of the iron lines show large displacements, when exposures near the negative pole are compared with exposures at the middle of the arc.

The interpretation of the pressure-shifts of spectrum lines, the relation of pressure-shift to wave-length and to other properties of the elements, is not at all evident. Any theory which is to explain the facts as they are known today must permit both a positive and a negative shift. As will be shown in this investigation, it must permit either an increasing or a decreasing displacement as the wave-length decreases. The wave-length of many lines is strikingly affected by current and by the portion of the arc used. This may mean that wave-length is a function of temperature. As a first step toward the formulation of a satisfactory theory of pressure-shifts we must know more about the behavior of lines of different substances, especially series lines.

In view of the work of Gale and Adams, together with the fact that previous work upon elements which have series lines has been done at high pressures, and not with a constant current, it seemed desirable to examine the series lines of some element, at low pressures, paying special attention to the points brought out by Goos and St. John. Hence the work described in this article has a twofold purpose: (1) to see whether a classification of the lines into groups according to their behavior under pressure coincides with their series classification; (2) to determine the relation between the wave-length and the displacement per atmosphere for the series lines of some element. Zinc was chosen as the element to be studied in this investigation.

APPARATUS AND ADJUSTMENTS

The grating used was a 6-inch Rowland concave of 21.5 feet radius, ruled with 15,000 lines to the inch. It was mounted in the usual way, as fully described by Ames.[1]

The pressure box used by Gale and Adams[2] at the Pasadena Laboratory of the Mount Wilson Observatory was used in this experiment. A quartz window was substituted for the glass window in the front of the brass hood, through which the light from

[1] *Phil. Mag.* (5), **27**, 369, 1889. [2] *Astrophysical Journal*, **35**, 10, 1912.

the horizontal arc passes. The light passing through this window was focused upon the vertical slit by means of a quartz lens placed so as to give a threefold magnified image. This image was kept the same width across the slit throughout the experiment. Special care was taken to see that the cone of light covered the grating with a good margin. To provide for accurate guiding during an exposure, when the arc shifts from one point to another upon the electrodes, the simple expedient of placing the lens upon a slide rack was adopted. However, the horizontal motion of the arc, due to the burning-away of the poles, was cared for by moving the pressure box horizontally. Hence the lens was never moved, except vertically.

The arc was produced by a 110-volt direct circuit, between horizontal brass poles, 7 mm in diameter. Several exposures were made in which the current and pressure were varied in order to determine the following points: (1) the greatest distance which the poles may be separated, and yet give a steady arc; (2) the largest values of the pressure and current which will give the lines of the series, narrow and distinct.

It was decided to use a 2.5-mm arc at 3.5 amperes throughout this entire investigation. Great care was taken to see that all exposures were made exactly upon the center of the arc. For the lines of the second subordinate series the maximum range of pressure was approximately from 5 cm to 2.2 atmospheres. For the first subordinate series the maximum range of pressure was from 5 cm to 5 atmospheres. However, many plates were taken at pressures considerably below the maximum. All these plates showed the displacement of a particular line to be directly proportional to the pressure within this range.

The camera took 2×19-inch plates and was provided with a shutter of the form usually used with the concave grating.

METHOD OF PHOTOGRAPHING

For the purpose of accurate comparison it was necessary to obtain side by side photographs of the spectrum of the substance in question as given by the arc under the two pressures used. It was also necessary to guard as far as possible against any accidental

movement of any part of the apparatus, and to be able to detect with certainty the exact amount of such a displacement, should it occur.

The first of these requirements was met by means of the shutter, mentioned above. In nearly every case the middle strip was exposed to the arc under the low pressure, after which air was pumped into the pressure box, the shutter rotated, and strips of the plate above and below the first exposure were exposed to the arc under the higher pressure.

The second requirement was satisfied by photographing, on some part of the plate, a few lines at atmospheric pressure, before and after both vacuum and pressure exposures. By this order of exposures, one is able to detect an accidental displacement, should it occur either during an exposure of the zinc lines or between the exposures of the zinc lines. In order to make these exposures separate from the lines for which the shift was being investigated, black paper curtains were hung in front of the camera, shielding the parts of the plate on which the respective lines appeared. Hence, either set of lines could be photographed by raising the corresponding curtain. There were generally lines appearing upon the plate other than those whose displacement was being determined, in which case they were used to determine the accidental displacement. However, if no such lines appeared, iron poles were substituted for the brass poles and the iron lines used to determine the accidental displacement.

The length of exposure for both the comparison and the pressure spectrum on Seed 27 and Cramer Crown plates was as follows: group IIN3, 20 seconds; group IN4, 2 minutes; groups IIN4, IIN5, and IN5, 20 minutes; groups IN6 and IIN6, 1.5 hours; non-series lines, from 10 to 15 minutes.

METHODS OF MEASURING

The plates were carefully measured with a small comparator constructed by William Gaertner & Co. The instrument reads directly to thousandths of a millimeter, and may be estimated to ten-thousandths.

The majority of the plates were taken in the second order, where the dispersion was a little more than 1 mm per angstrom. In

all about two hundred photographs were taken and the displacements determined from those lines whose positions were well defined by reason either of their sharpness or of their reversals. Each plate was measured twice, once in each direction. In each measurement, five settings were made upon the vacuum line and ten settings upon the pressure line.

EXPERIMENTAL RESULTS

The extraordinary variety in behavior of different spectrum lines under pressure has of course been noted by all observers.

The first photographs showed that all the lines of the second subordinate series of the zinc spectrum broadened very unsymmetrically toward the red, under high pressure, thus making the determination of their displacement a matter of personal judgment. This broadening was much greater for the violet lines than for the blue ones. For instance, the lines λ 2712.60 and λ 2684.29 could hardly be said to be more than blurs under a pressure of 5 atmospheres, while the lines λ 4810.71, λ 4722.66, and λ 4680.38 were fairly good for measurement. Hence, the lines of the second subordinate series of the zinc spectrum will be placed in class five of the Gale and Adams[1] classification of spectrum lines, with the added statement that the violet lines explode completely under high pressure. However, it was further found that by using pressures below 2.5 atmospheres, the position of the real line appeared, and that at a pressure of 1.5 atmospheres the time of exposure could be so regulated that the real line appeared with but very little shading on its side.

The lines of the first subordinate series of the zinc spectrum showed a tendency toward unsymmetrical broadening at the above pressures, as only one reversal appeared until a pressure of approximately 5 atmospheres was reached. The one reversed line was λ 2802.11 which appears in group IN5. It reverses nicely at a pressure of 1.25 atmospheres, while the other members of that group showed no tendency toward reversing, even at 5 atmospheres. The lines λ 3345.13 and λ 3302.67 reversed very nicely at a pressure of approximately 5 atmospheres, but the other members of

[1] *Astrophysical Journal*, **35**, 10, 1912.

group IN_4 only became very broad and slightly unsymmetrical. The close triple and the close double lines in group IN_5 could not be obtained as single lines (except λ 2802.11) at pressures above 1.2 atmospheres and then the two lines λ 2800.90 and λ 2771.05 were very diffuse. However, the other two lines were fairly good and afforded very good measurements upon a plate from which some of the most accurate measurements of group IIN_4 were taken. In general, the lines of the first subordinate series broaden immensely under pressure, showing no tendency toward reversal at moderate pressures, thus making the measurement of their displacement very difficult. Most of the lines of this series belong to class five of the Gale and Adams classification.

We may make the general statement that the violet lines are broader and much more diffuse than the blue lines in both the first and second subordinate series of zinc.

A number of special photographs, some of which were exposed for 1.5 hours, showed that it was impossible to photograph many of the lines of the zinc spectrum for the purpose of this investigation.

In Table I are given the results of the measurements of the displacements of the zinc lines in the spectrum of the arc between brass poles, taken within the range of pressures mentioned above, and reduced to one atmosphere. The wave-length of the line according to Kayser and Runge is given in the first column; in the second column is given the classification of the zinc lines under a pressure of 5 atmospheres, according to the Gale and Adams classification of spectrum lines; in the third column, the series group as given by Kayser and Runge; the mean displacement per atmosphere, in the fourth column; the number of plates measured, in the fifth column; and the displacements as found by Humphreys with a difference of pressure of 7 atmospheres (except λ 3075.99, and it was taken at a difference of 6 atmospheres) is given in the sixth column.

The groups IIN_3 and IN_4 were measured first, and found to agree pretty closely with the values obtained by Humphreys. However, it was a great surprise to find that the displacements of group IIN_4 were larger than those for group IIN_3. This was wholly unexpected, and much doubt was felt as to its validity.

Several times each and every adjustment of the apparatus was gone over, and new photographs taken, but each time with the same displacements. After taking about fifty photographs, all

TABLE I

Wave-Length	Class	Group	Mean Δ per Atm.	No. of Plates	Humphreys 7 Atm. Difference
Second Subordinate Series					
4810.71	5	IIN3	0.0099	4	0.056
4722.26	5	IIN3	0.0098	4	0.051
4680.38	5	IIN3	0.0099	4	0.061
3072.19	5	IIN4	0.0163	15	0.049
3035.93	5	IIN4	0.0165	15	0.044
3018.50	5	IIN4	0.0160	9	0.046
2712.60	5	IIN5	0.0204	4	....
2684.29	5	IIN5	0.0199	4	....
2670.67	5	IIN5	0.0176	3	....
2567.99	5	IIN6	0.0224	3	....
First Subordinate Series					
3346.04	5	IN4	0.0046	3	0.030
3345.62	5	IN4	0.0045	3	0.025
3345.13	2	IN4	0.0047	3	0.026
3303.03	5	IN4	0.0044	3	0.022
3302.67	2	IN4	0.0044	3	0.030
3282.42	5	IN4	0.0045	3	0.027
2802.11	2	IN5	0.0024	3	....
2800.90	5	IN5	0.0082	3	....
2800.17	5	IN5	0.0088	3	....
2771.05	5	IN5	0.0085	3	....
2770.94	5	IN5	0.0077	3	....
2756.53	5	IN5	0.0092	3	....
2608.65	5	IN6	0.0112	3	....
2582.57	5	IN6	0.0091	3	....
2570.00	5	IN6	0.0085	3	....
Non-Series Lines					
6362.58	4	....	0.0300	3	....
4630.06	3	....	0.0120	3	....
4058.02	2	....	0.0057	3	....
3740.12	3	....	0.0084	3	....
3572.90	3	....	0.0085	3	....
3075.99	2	....	0.0021	3	0.0120*

* Difference of 6 atmospheres.

of which appeared to have about the same displacement when reduced to one atmosphere, fifteen of the best plates were measured, and the mean adopted as correct. Since the groups IIN4, IIN5,

and IN5 all appeared upon the same plate and were subjected to the same instrumental corrections, it seems impossible that there should be any mistake concerning the relation of the displacements for these groups to one another. Also, it is wholly impossible that such a mistake should lie in the judgment of the maximum of the lines, since group IIN4 appeared very sharp and narrow up to approximately 1.25 atmospheres. The individual settings on this group could be made within one division on the micrometer head. However, the groups IIN5 and IN5 were not quite so good.

Table I shows a very good agreement between the values of the displacements obtained in this experiment and those obtained by Humphreys at higher pressures, with the exception of the group IIN4. Of course it is possible that there should be no difference in the values of the displacements obtained under the different experimental conditions, but it seems more probable that the value of the current and the part of the arc used for the exposures should be the source of difference in the two experiments.

It is to be noted that the displacements for all the lines of a particular group of a series is nearly a constant for a given pressure (except λ 2802.11). If the mean wave-length for each group of the second subordinate series be plotted against the reciprocal of the mean displacement for that group, one finds that the points fall almost exactly upon a straight line as shown in Fig. 1. The corresponding numerical values are given in Table II.

TABLE II

Mean Wave-Length	Group	$1/\Delta$
4738	IIN3	101
3042	IIN4	61
2689	IIN5	52
2568	IIN6	45

This graph (Fig. 1) shows that the displacement of the lines of the second subordinate series vary inversely with the wave-length, i.e., the displacement increases as the wave-length decreases. Very little weight should be given to the line λ 2567.99, since it was very diffuse and could hardly be measured at all.

If the mean wave-length for each group of the first subordinate series be plotted against the reciprocal of the cube root of the mean displacement for that group, it is found that the points fall upon a straight line, as shown in Fig. 2. The corresponding numerical values are given in Table III. This graph (Fig. 2) shows that the displacement of the lines of the first subordinate series varies

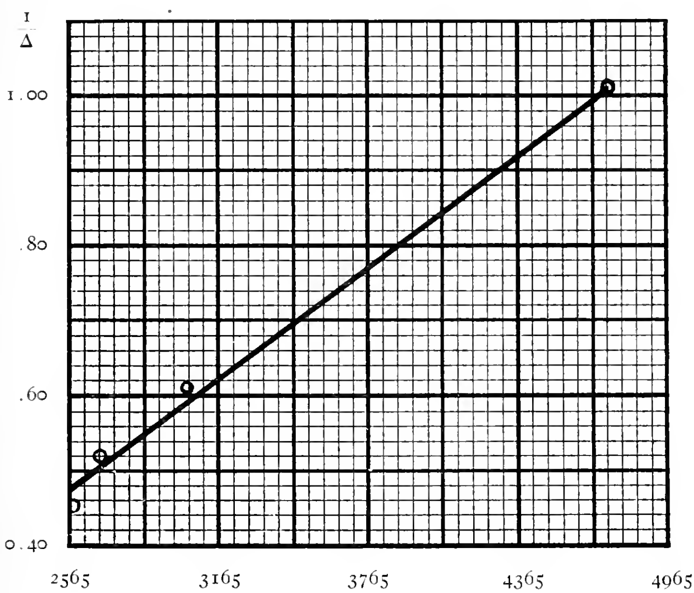


FIG. 1.—Second subordinate series

inversely with the cube of the wave-length. In making this graph, the line λ 2802.11 was not included. This line has such a different value for its displacement, and reverses so much more easily than the other lines of group IN5, that it seems probable that it should not be considered as a member of this series. Hence, it will be classed as a non-series line in this article.

TABLE III

Mean Wave-Length	Group	$1/\sqrt[3]{\Delta}$
3321	IN4	61
2780	IN5	49
2587	IN6	46

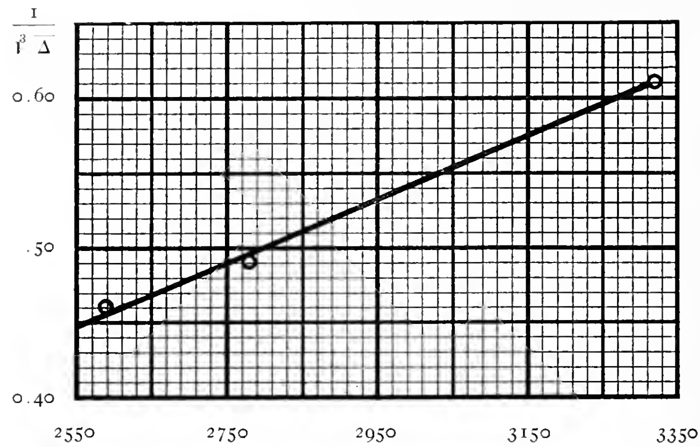


FIG. 2.—First subordinate series

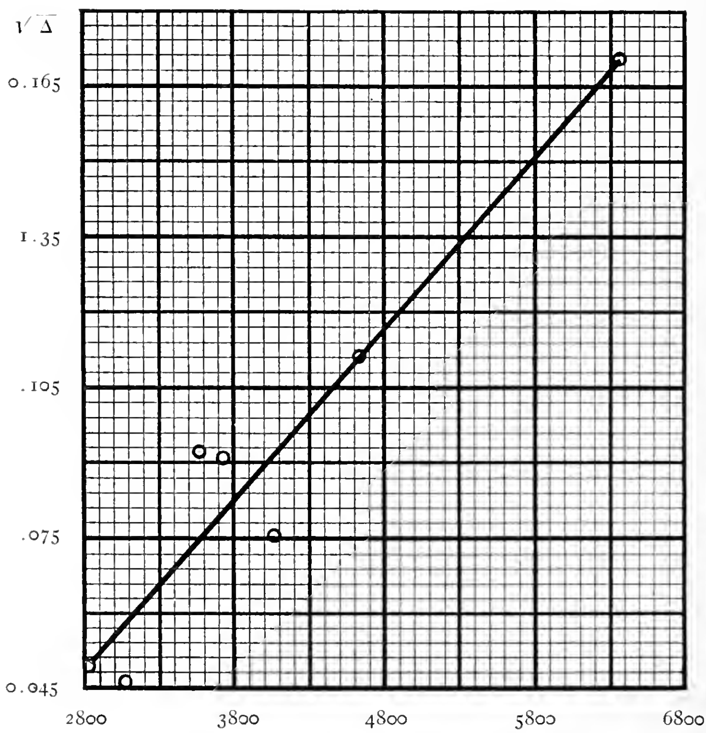


FIG. 3.—Non-series lines

All the foregoing results seem to indicate a connection between the diffuseness of the line and its susceptibility to pressure.

If we plot the wave-lengths of the non-series lines against the square root of their respective displacements, we obtain the graph as shown in Fig. 3. The corresponding values are given in Table IV. Fig. 3 shows that these displacements are well represented by an expression involving the square of the wave-length.

TABLE IV

Wave-Length	Group	$\sqrt{\Delta}$
6363	Non-series	.17
4630	"	.11
4058	"	.075
3740	"	.091
3573	"	.092
3076	"	.046
2802	"	.049

It is very striking that the displacements of the lines of the different series should vary inversely according to different powers of the wave-length, while the displacements of the non-series lines vary directly according to a still different power of the wave-length.

SUMMARY OF RESULTS

1. The classification of the lines according to their action under pressure agrees with their series classification (except λ 2802.11).
2. The displacement of the lines of the second subordinate series varies inversely with the wave-length, at a particular pressure.
3. The displacement of the lines of the first subordinate series varies inversely with the cube of the wave-length, at a particular pressure.
4. The displacement of the non-series lines varies directly with the square of the wave-length, at a particular pressure.
5. The line λ 2802.11 is better classified as a non-series line.

In conclusion, the author desires to express his appreciation to the staff of the Physics Department for their kindly interest in this work and especially to Professor Gale, at whose suggestion and under whose direction the investigation was carried out.

Ryerson Physical Laboratory
University of Chicago
April 4, 1914

THE EFFECT OF SELF-INDUCTION ON THE NITROGEN BANDS

By E. P. LEWIS

In a previous article[1] some effects produced by self-induction on the band spectrum of nitrogen were described. It was found that it enhanced the negative bands and caused them to appear in all parts of the tube. Hemsalech[2] had previously found that at atmospheric pressure self-induction caused these bands to appear prominently in the spectrum of the spark discharge between electrodes of certain metals. Recently, in the course of a study of the effects of changed conditions on the vacuum tube spectrum of nitrogen, other effects were noted which had escaped observation in 1903 on account of the small dispersion of the system used and the absorption of the ultra-violet by the glass prism. The quartz spectrograph previously described[3] was used. Evident changes were produced in the first group of positive bands by self-induction, but on account of the small dispersion in this region attention was concentrated on the second positive group. As before noted, self-induction caused the appearance of the negative bands everywhere, but more striking was the marked change produced in the structure of the positive bands. The general character of these changes is shown in Fig. 1, Plate III, and is most clearly seen in the group of bands with the first head at λ 3371. Comparing the spectrum of the simple discharge (*a*) with that when self-induction and capacity are used (*b*), it is seen that these effects are: (1) relative enhancement of the more refrangible lines belonging to the principal head, so that they may be easily followed to the head of the next group at λ 3159; (2) almost complete suppression of the subheads of the group and the lines belonging to them; (3) increased sharpness of the lines. Observations were made with the light from the capillary of a quartz tube, with end-on glass tubes with quartz windows ranging in diameter from 1 to 20 mm; and with a spark gap 1 cm

[1] *Astrophysical Journal*, 17, 258, 1903.

[2] *Comptes rendus*, 132, 1040, 1901.

[3] *Astrophysical Journal*, 23, 390, 1906.

long in a tube of 2 cm diameter. At high pressures it was noted that the effects produced by self-induction were somewhat increased by increase in self-induction; but at low pressures substantially as great an effect was produced by 50 turns as by 500 turns in the self-induction coil. At high pressures, from about 30 cm up, the effects were almost as marked with the simple discharge as with self-induction. At pressures above 35 cm (spark gap) or 10 cm (capillary) capacity alone produced a line spectrum; at lower pressures, bands, which showed in lesser degree the same characteristics as those due to self-induction. The spectrum of the discharge from a small Tesla coil at different pressures was similar to that of the simple discharge, the lines rapidly dying out in going from the head, and each subhead of a group was prominent. When the quartz tube was used, the capillary was heated to bright red, and it was found that this produced slight effects similar to those of self-induction.

No record of effects of such magnitude as those described here has been found by the writer, although effects of a similar nature have been described. Hagenbach and Konen[1] and Stark[2] found that the bands of nitrogen became more extended toward the violet as the pressure or temperature increased. Berndt[3] found, conversely, that at low pressures the bands became more concentrated near the heads, while at atmospheric pressure they were extended toward the violet by capacity and self-induction, with improved resolution.

Inasmuch as the current density, potential gradient, and temperature must have varied widely under the different conditions employed, it seems unlikely that any one of these factors alone can determine the effects most prominently shown when self-induction is employed. It would seem more probable that the cause must be looked for in some characteristic peculiar to self-induction, perhaps in connection with the oscillation period. Ladenburg[4] shows that not only is the logarithmic decrement of the oscillations decreased by self-induction, but that the current density is diminished. He explains the narrowing of lines by self-induction as the result of

[1] *Phys. Zeit.*, **4**, 227, 1903.

[2] *Ibid.*, **7**, 357, 1906.

[3] *Ann. der Physik*, **7**, 946, 1900.

[4] *Ibid.*, **38**, 248, 1912.

a reduced number of emission centers due to this cause. In the present case, however, observations taken with very small simple currents and the still smaller currents due to the Tesla discharge show quite the opposite effect from that due to self-induction—a narrowing of the bands, poorer resolution, and relative increase in the intensity of the subheads.

Fig. 1, Plate III, is the reproduction of a photograph (enlarged about three times) taken at a pressure of about 6 mm. The lines of the band beginning at λ 3371 can be distinctly followed to the head of the next group at λ 3159. The subheads of the first group are very faint and the lines belonging to them make scarcely any impression on the film. The heads of the negative bands at λ 3298 and λ 3296 show as a slight shading. On the less refrangible side of λ 3371 some lines can be seen which belong to a band which appears only when hydrogen or ammonia vapor is present, and which is further described on p. 154. Some of the films taken showed no trace of any of these bands, owing to slight hydrogen impurity, but this film was chosen as most suitable for reproduction. Near the head the lines of the band are not resolved.

Inasmuch as this band may be followed much farther toward the violet than in any case previously described, it seemed of interest to find whether Deslandres' formula applies. The wave-lengths of the lines were determined by Hartmann's interpolation formula, the iron spark being used for comparison. The wave-lengths of the comparison lines are those of the International System as given by Kayser in Vol. 5 of the *Handbuch der Spektroscopie.* In this region the values are about 0.14 of an angstrom less than those of the Rowland system. The wave-numbers were computed from the formula $n=n_0+Am^2$, where $n_0=296{,}617$ and $A=1.817$. Hermesdorf[1] has measured many lines of this band. For comparison, the third decimal in his results was dropped and each wave-length reduced by 0.14. Those corrected values which approximate most closely to the calculated values are given in Table I. There are at least twice as many more lines in his table which do not at all fit the computed values; so that it seems probable that there are several series in this band, all but the principal one being suppressed by self-induction.

[1] *Op. cit.*, 11, 161, 1903.

TABLE I

m	λ	$n=\frac{1}{\lambda}\times 10^9$		DIFFERENCE	HERMESDORF (CORRECTED)	
		Obs.	Calc.		n	λ
0........	3371.35	(Head)	296,617	....	296,617	3371.35
1........	....	....	619			
2........	....	....	624			
3........	....	....	633	....	296,640	71.08
4........	....	....	646			
5........	....	....	662	....	677	70.69
6........	....	....	683			
7........	....	....	706	....	728	70.08
8........	....	....	733			
9........	....	....	764	−15	779	69.50
10........	....	....	799	2	797	69.31
11........	....	....	837	− 1	838	68.84
12........	....	....	879	3	876	68.40
13........	....	....	924	−21	945	67.62
14........	....	....	973	− 3	976	67.27
15........	....	....	297,026	− 5	297,032	66.63
16........	....	....	082	7	075	66.15
17........	....	....	143	9	132	65.50
18........	....	....	206	− 2	208	64.64
19........	....	....	273	− 8	281	63.82
20........	....	....	344	−12	356	62.97
21........	3362.40	297,407	418	11, 21	397	62.50
22........	61.50	487	496	9, 20	476	61.61
23........	60.51	574	578	4, 12	566	60.69
24........	59.50	664	664	0, 22	642	59.73
25........	58.50	752	753	1, 22	731	58.73
26........	57.50	840	846	6, 26	820	57.69
27........	56.46	933	942	9, 21	921	56.59
28........	55.29	298,036	298,042	6, −12	298,054	55.00
29........	3354.10	298,142	298,146	4, − 8	298,154	3353.96
30........	52.87	243	252	9, 22	230	53.11
31........	51.70	356	363	7, − 7	370	51.54
32........	50.49	463	477	14, − 7	484	50.26
33........	49.16	582	595	13, − 6	601	48.95
34........	47.79	704	716	12, − 8	724	47.59
35........	46.40	828	842	14, − 2	846	46.21
36........	45.00	954	971	17, − 2	973	44.78
37........	43.44	299,093	299,104	11, 0	299,104	43.31
38........	41.80	240	240	0, − 2	242	41.77
39........	40.30	374	380	6, 1	379	40.24
40........	38.70	515	524	9	....	....
41........	37.05	661	671	10	....	....
42........	35.41	814	822	8	....	....
43........	33.66	970	976	6	....	....
44........	32.00	300,120	300,135	15	....	....
45........	30.20	282	295	13	....	....
46........	28.30	452	464	12	....	....
47........	26.45	621	632	11	....	....
48........	24.50	797	805	8	....	....
49........	22.50	978	982	− 4	....	....
50........	20.49	301,160	301,159	− 1	....	....
51........	18.50	341	345	4	....	....

TABLE I—*Continued*

m	λ	$n=\frac{1}{\lambda}\times 10^9$ Obs.	Calc.	Difference	Hermesdorf (Corrected) n	λ
52........	16.40	532	532	0	....	....
53........	14.34	719	717	− 2	....	....
54........	12.26	908	915	7	....	....
55........	10.10	302,105	302,114	9	....	....
56........	7.85	311	319	8	....	....
57........	5.53	524	524	0	....	....
58........	3303.26	302,731	302,732	1	....	....
59........	3200.85	952	942	−10	....	....
60........	98.54	303,164	303,158	− 6	....	....
61........	96.22	378	378	0	....	....
62........	93.71	602	601	− 1	....	....
63........	91.20	841	829	−12	....	....
64........	88.70	304,072	304,060	−12	....	....
65........	86.20	303	294	− 9	....	....
66........	83.60	544	532	−12	....	....
67........	81.00	783	774	− 9	....	....
68........	78.32	305,034	305,019	−15	....	....
69........	75.66	282	274	− 8	....	....
70........	72.96	538	520	−13	....	....
71........	70.15	796	790	− 6	....	....
72........	67.49	306,045	306,036	− 9	....	....
73........	64.60	316	300	−16	....	....
74........	61.82	377	567	−10	....	....
75........	58.90	852	837	−15	....	....
76........	55.98	307,127	307,124	− 3	....	....
77........	53.05	404	395	− 9	....	....
78........	50.10	683	673	−10	....	....
79........	47.14	963	957	− 6	....	....
80........	44.05	308,257	308,246	−11	....	....
81........	40.90	556	539	−17	....	....
82........	37.83	850	834	−16	....	....
83........	34.70	309,148	309,133	−15	....	....
84........	32.55	450	438	−12	....	....
85........	28.38	753	745	− 8	....	....
86........	25.18	310,060	310,055	− 5	....	....
87........	3221.93	310,373	310,370	− 3	....	....
88........	18.62	692	688	− 4	....	....
89........	15.28	311,015	311,009	− 6	....	....
90........	11.92	340	335	− 5	....	....
91........	08.61	660	661	1	....	....
92........	05.20	993	996	3	....	....
93........	01.70	312,334	312,332	− 2	....	....
94........	3198.12	684	673	−11	....	....
95........	94.66	313,022	313,015	− 7	....	....
96........	91.07	365	363	− 3	....	....
97........	87.57	719	715	− 4	....	....
98........	83.98	314,072	314,067	− 5	....	....
99........	80.40	427	426	− 1	....	....
100........	76.75	787	787	0	....	....
101........	73.06	315,154	315,152	− 2	....	....
102........	69.38	519	521	2	....	....
103........	65.64	892	894	2	....	....
104........	62.00	316,257	316,270	13	....	....

PLATE III

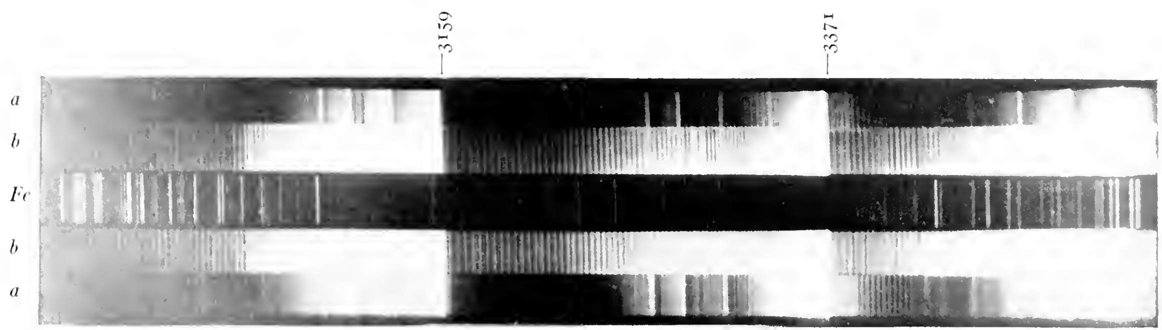


Fig. 1

Fig. 2

The differences between the observed and the calculated values of the wave-numbers are, except in a few instances, well within the limits of errors of measurement; but it may be significant that the corrections are almost uniformly positive at the two ends of the series and negative in the middle. This may indicate a systematic error in the determination of wave-lengths, possibly due to a warping of the film in drying; or it may indicate that the Deslandres formula does not apply with exactness.

Thanks are due to Mr. S. L. Quimby for assistance in measuring the film and calculating wave-lengths.

UNIVERSITY OF CALIFORNIA
March 30, 1914

THE ULTRA-VIOLET BAND OF AMMONIA

By E. P. LEWIS

Eder[1] describes the spectrum of the flame of ammonia burning in oxygen. A number of bands in the ultra-violet attributed by him to ammonia have since been found to be the well-known third group of "nitrogen" bands of Deslandres, and are probably due to an oxide of nitrogen. A band between the green and red observed by Eder has likewise been described by other investigators. Another band in the ultra-violet, extending from λ 3295 to λ 3432, was held by Eder to be the principal band of ammonia, but apparently it has not been described by others. Kayser[2] says: "Whether this band belongs to our third group [of nitrogen] is unknown." Recently, while studying the spectra of mixtures of nitrogen and hydrogen in vacuum tubes, I found this band at all pressures up to about 70 cm, and it becomes specially prominent when self-induction is used. It is found with all proportions of the two gases, from the time that the red line of hydrogen first appears until the nitrogen bands disappear. With nitrogen free from hydrogen or hydrogen free from nitrogen it is not found. Small traces of oxygen have no effect upon it, while considerable quantities destroy it. It seems to require the presence of these two gases alone, and might reasonably be attributed to ammonia.

There is, however, such a marked difference between the conduct of this band and of that between the green and red that further examination seems desirable. When ammonia gas is introduced into the vacuum tube the visible band is at first very prominent, but it rapidly dies out, whatever may be the nature or the intensity of the discharge. It can be maintained only by allowing a continuous stream of gas to pass through the tube. It would appear from this that the ammonia is rapidly dissociated by the current. The ultra-violet band is likewise found in the spectrum of the ammonia gas, but it is most persistent, and still appears after the

[1] *Denkschriften Wiener Akad.*, **60**, 1, 1893.

[2] *Handbuch der Spektroscopie*, **5**, 833.

discharge has been passed for hours. This strongly suggests that different emission centers are responsible for the two bands; either a different and more stable compound of hydrogen and nitrogen, or possibly a combination of these two gases with some impurity present in the tube, such as traces of oxygen or hydrocarbons.

A photograph of this band is reproduced in *d*, Fig. 2, Plate III; *c* is the comparison spectrum of nitrogen. The nitrogen contained about 10 per cent of hydrogen, the pressure was 2 mm, and the band was made more prominent by the use of self-induction. Its appearance is complicated by superposition over the nitrogen band at λ 3371. The resolution is better than that obtained by Eder, but not sufficient to make it worth while to attempt to separate the lines of this band from those of nitrogen. An early attempt will be made to secure a photograph of this spectrum with larger dispersion, as this band is so peculiar in structure that it seems desirable to make a more detailed study of it.

University of California
March 30, 1914

Minor Contributions and Notes

NOTE ON RADIAL MOVEMENT IN SUN-SPOTS

In his very valuable contribution to the study of radial movement in sun-spots, published in the *Astrophysical Journal* for June 1913, Mr. St. John states that the usual course of the displaced lines over spots in a solar image 170 mm in diameter "shows no sharp break and the displacement does not suddenly cease at the periphery of the penumbra, but the line gradually returns to its normal course."

This differs from the statement I have published, viz., that there is "an appreciable break or jolt in the lines at the points where they pass from the penumbrae on to the surrounding photosphere."[1]

The deduction from my plates would appear to be that the motion outward ceases abruptly just at the point where the maximum velocity is obtained, and Mr. St. John's observation "seems to remove one great difficulty in explaining the displacement as due to motion."

The question is of course to some extent one of degree, and it is of importance to determine how far outside the penumbral limits the motion can be traced. It appears to me that the photographic evidence is by no means conclusive, because of the inevitable movements of the solar image on the spectrograph slit while the plate is being exposed; and I am inclined to believe from my own results that the limiting distance within which the velocity apparently changes from its maximum value to zero may be very much smaller than is implied in the sentence I have quoted from Mr. St. John.

I have found that the clearness with which the jolt in the lines is brought out in the photograph depends mainly on the exposure time, but also on the state of the "seeing" during the exposure and on the accuracy of guiding. With very short exposure times the guiding factor does not come in appreciably, and if the seeing is

[1] *Monthly Notices of the Royal Astronomical Society*, January 1910, p. 219.

reasonably good the limits of umbra and penumbra in the photographed spectrum will be very clearly defined. In such plates the displacements are greater and end more abruptly than in plates which have had a long exposure, especially if guiding has been necessary and the seeing has been poor. The reason for this is sufficiently obvious, for with long exposures the unsteadiness of the image on the slit tends to spread the displacements at each point over an appreciable length of the lines, and this has the effect both of reducing the amount of the maximum displacement and of spreading the displacement beyond the point of maximum outside the penumbra.

As I am at the present time on a visit to North India I am unable to refer to my records, but I believe that in my best plates the exposure times did not exceed about 30 seconds; and in those plates especially in which the displacements were first detected the definition is unusually fine, the limits of umbra and penumbra being very clearly marked. In these the displacements certainly seem to end exactly at the dividing line between penumbra and photosphere, and I should say from memory that the movement could not be traced so far outside the penumbra as one-fiftieth of the diameter of the spot.

It is not perhaps fair to judge of the definition of Mr. St. John's plates from the reproductions given in his paper, but it is almost inevitable from the form of spectrograph used that the exposures must have exceeded mine many times over, and Mr. St. John himself states that "with the utmost possible care in guiding the slit cannot be . . . . rigorously held upon the same point with respect to the edge of the penumbra." The Mount Wilson plates have an advantage over mine in the somewhat greater linear dispersion, and the much larger scale of the spot image, but I think this may be more than offset by the longer exposure times. Evidence of the longer exposures seems to be discoverable in the generally smaller values of the maximum radial motion which Mr. St. John's results show when compared with my own.

In the Kodaikanal spectrograph high dispersion is obtained by inclining the camera to a high angle with the collimator, actually 60°, and not by the use of a very high-focus lens. This method

magnifies the spectrum in one dimension only, viz., dispersion, instead of in two dimensions as with increased focal length. The intensity of the spectrum, therefore, is considerably greater than in an autocollimating spectrograph giving the same linear dispersion.

It is of great interest to settle the question of the limits of the radial motion. If it is eventually found that the displacements really end abruptly at the edge of the penumbra, the difficulty of the sudden stoppage of the motion might be explained by supposing the photosphere surrounding the spot to be heaped up, so that the reversing layer is at a higher level outside the spot. The moving gases would then be hidden at the point where they would penetrate the raised photosphere.

J. EVERSHED

KASHMIR
August 15, 1913

As Mr. Evershed says, the question of how suddenly the motion outward from spots and tangential to the solar surface passes from its maximum value to zero is of much importance. In the case of the high-level calcium vapor our observations agree in attributing a smooth curve to the displacement of the K_3 line,[1] there being no "jolt or break" at the outer boundary of the penumbra; and in respect to the lines of the lower reversing layer, the apparent difference between our observations is to some extent one of degree. I remark that "in the type of vortex in which the data of the present paper seemed to find their interpretation, there is an enormous flux of energy from the vortex into and below the reversing layer. The mechanical energy of motion is rapidly transformed into other forms, as the velocity decreases rapidly with distance. The question of how rapidly will be taken up in a later investigation";[2] while the deduction from Mr. Evershed's plates, especially those taken under the most favorable observing conditions, is that the motion outward ceases abruptly just at the point where the maximum velocity is attained.

[1] *Monthly Notices*, **70**, 220.

[2] *Mount Wilson Contr.*, No. 69, p. 29; *Astrophysical Journal*, **37**, 350, 1913.

There is force in the point to which Mr. Evershed calls attention, that of necessity the exposures were longer with the instrument employed at Mount Wilson, in fact, from two to six times as long as he used, and hence any unsteadiness of the solar image and any irregularity in guiding would tend to smooth out an abrupt break in the course of the line. The spectrum was often observed visually with the slit in position for maximum effect and even then no abrupt change in the course of the line was detected.

In the program arranged for the approaching spot maximum, it has been planned to make a special examination of this question and particularly of the variation of velocity with distance from the center of the spot and to determine how far the line displacement can be traced beyond the boundary of the penumbra. In doing this, a simple and apparently very efficient guiding device suggested by Mr. Evershed will be used, and it is hoped that with the large image given by the 150-foot telescope and by taking advantage of the best seeing, a more definite determination can be made of the limits to which the velocity can be traced and of the rate of change of this velocity, if it is found that the displacements do not end abruptly at the outer boundary of the penumbra. If, as Mr. Evershed suggests, a stoppage of the motion is due to the heaping-up of the photosphere around the spot, it might well be that the suddenness of the stoppage would vary with different spots, the piling-up of the photosphere depending upon the solar activity. It is to be noted that Mr. Evershed made his observations at a time of great solar activity and was able to observe more spots than I did, and that those I did observe appeared at a time when the solar activity was nearing its minimum.

CHARLES E. ST. JOHN

MOUNT WILSON SOLAR OBSERVATORY
November 13, 1913

Reviews

Annuaire astronomique et météorologique pour 1914. Par CAMILLE FLAMMARION. Paris: Ernest Flammarion. Pp. 427. Figs. 125. Fr. 1.50.

It is no small distinction to have edited an astronomical annual for half a century, and it is a pleasure to offer our congratulations to the ever-active editor on the appearance of this handy little volume for the fiftieth year. It contains all the astronomical data for the year in a popular presentation, with small maps of movements of the planets, and numerous pictures of these bodies, charts of the constellations for each month, special notices on recent discoveries, a review of the meteorology of the year preceding, and a chapter on wireless telegraphy. May M. Flammarion long continue this excellent means of popularizing science!

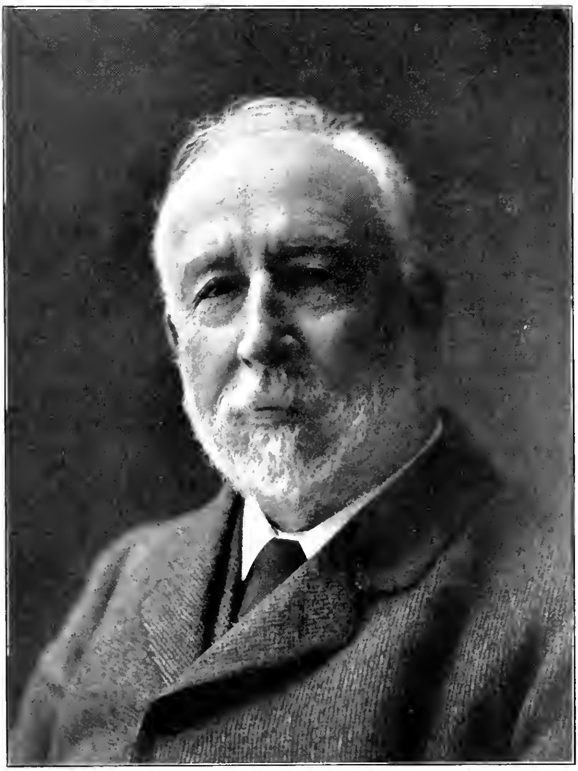
F.

Milton's Astronomy. The Astronomy of Paradise Lost. By THOMAS N. ORCHARD. London: Longmans, Green & Co., 1913. Pp. 288, with six plates and seven figures. $2.50 net.

This interesting book will serve a useful purpose in two directions: in giving to the "general reader" a brief history of astronomy and in introducing him to the beauty of Milton's poetry through numerous quotations. It will be useful also as a reference book for classes in astronomy and in literature for its exposition of the status of astronomical knowledge in Milton's time and of his cosmology as found in *Paradise Lost.*

A word must be added as to the illustrations. The subjects chosen are admirable, but it is greatly to be regretted that better half-tones were not secured to do them justice and to conform to the otherwise high quality of the volume.

F. B. L.



SIR DAVID GILL

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SIR DAVID GILL

By J. C. KAPTEYN

Such are the salt of the earth.[1]

On January 24 died in London, at the age of seventy years, Sir David Gill, formerly Her Majesty's astronomer at the Observatory at the Cape. In him science loses the foremost practical astronomer of the age. Only a few months before his death he crowned his life's work by the completion of his *History and Description of the Royal Observatory, Cape of Good Hope*. This publication gives the fullest existing description of his life and his work. About his youth we read:

The eldest surviving son of David Gill of Blairythan, Aberdeenshire, I was born at Aberdeen on the 12th of June, 1843, and attended the Bellevue Academy at that city till about the age of fourteen, when I went to Dollar Academy and came under the inspiring influence of Dr. Lindsay, at whose house I boarded. His teaching filled me with the love of mathematics, physics, and chemistry.

From Dollar I proceeded to Marischal College and University, Aberdeen, where I was a student under the celebrated Clerk Maxwell, and his teaching influenced the whole of my future life. My father had married late in life, for at the time I was twenty years of age, he was seventy-four years old. He was a successful merchant in Aberdeen, as had been his father before him, and he not unnaturally wished me to succeed him in business. I very unwillingly yielded, and, after some years, my father retired, leaving his business in my

[1] These words were applied to Gill by the famous General Gordon.

hands. My heart and my thoughts, however, had always been set upon things scientific. From the time that I entered college, I had a little laboratory in my father's house where I made chemical experiments, and, later, under Clerk Maxwell's influence, carried out preliminary essays on the determination of physical constants.

Gill then goes on to tell how in 1863 it occurred to him that Aberdeen was very much in need of a standard of accurate time and how the desire of satisfying this need brought him to Edinburgh, where he was for the first time introduced to an astronomer and to an observatory, and how on his return he, together with Professor Thomson, unearthed, mounted, and adjusted a portable transit instrument at what had been called "an observatory" at King's College.

Already, at this early date, the qualities which stand out so prominently in his future career become apparent. He cannot rest before at least a couple of clocks at Aberdeen are electrically controlled to show accurate Greenwich Mean Time; and no sooner has this end been attained than he is looking out for a wider field of action. A twelve-inch silver-on-glass speculum is purchased, for which he designs an equatorial mounting. The driving clock, too, was made by his own hands and turned out to be "as satisfactory a driving clock as I have ever known." The instrument was employed

in measuring double stars, examining nebulae, making photographs of the moon, and generally in satisfying my curiosity as to the wonders of the heavens, and I had just begun to plan attempts to determine the parallaxes of some stars with a micrometer by Steinheil of Munich, when I received a visit from Lord Lindsay (now the Earl of Crawford). He was then considering the question of creating an observatory at his father's seat, Dun-Echt, about thirty miles from Aberdeen. He had seen some of my photographs of the moon, and desired to examine the means I employed for the purpose. Our acquaintance rapidly ripened, and he became aware of my desire to devote my time exclusively to science. One fine day in 1872 I received a letter from his father, the Earl of Crawford and Balcarres, offering me charge of the observatory which his son was about to erect at Dun-Echt.

It seems almost a pity that the visit from Lord Lindsay did not come a couple of years later. It might have given us the spectacle—unique in the annals of science—of a business man measuring star-

parallaxes in his leisure hours. Meanwhile, even so, he had before him a task which might have deterred almost any experienced astronomer—the task of planning and discussing the details for the several instruments and their respective observatories, for their testing, mounting, and adjusting, besides making preparations for Lord Lindsay's expedition for the observation of the transit of Venus at Mauritius in 1874. When the time for the expedition had come, we find him traveling all alone with fifty chronometers.

> Before my visit to Greenwich I had no misgiving as to the success of the arrangements. But when Sir George Airy and his assistants came to wish me goodbye and when they saw me go off with all the chronometers on the top of two cabs, and no one but myself to look after them, it was evident that they regarded the whole matter as an experiment of very doubtful success and did not envy the task before me. A first suspicion of these difficulties dawned upon me at the time, and they were fully realized before the expedition was over. Incessant watchfulness was necessary, and the anxiety connected with every move on shore and every embarkation or landing of the instruments was excessive, especially at places like Suez, Alexandria, Aden, and Mauritius, where only colored labor was available. But all ended well.

We may pass over the results obtained for longitude, which were fully successful. The main interest attaching to the expedition is the heliometric determination of the solar parallax by means of observations of Juno. Unfortunately the late arrival of the heliometer, and cloudy weather, permitted observations only on 12 evenings and 11 mornings, all of them some time after the opposition. The value of the determination, therefore, lies not so much in the result obtained for the solar parallax as in the fact that it furnishes "sound data on which to found calculations as to the value of the opposition of any minor planet for the future determination of the solar parallax."

On the way back from Mauritius we find Gill, at the request of the chief of the military staff of the Khedive, measuring a base line for the projected survey of Egypt. Concerning the particular difficulties of this measurement, owing to the unfitness of the assistants, we must refer to the *History of the Cape Observatory*. Still time was found to make some observations, together with Dr. Döllen, in order to determine the deflection of the plumbline produced by the attraction of one of the pyramids.

In 1876 Gill left Dun-Echt. In 1877 he made his celebrated expedition to Ascension for the heliometric observation of the opposition of Mars. On this expedition he was accompanied by his wife, to whom he had been married in 1870. The costs of the expedition were borne by the Royal Astronomical Society and the Government Grant Fund of the Royal Society. There is a delightful description of the expedition by Mrs. (now Lady) Gill in *Six Months in Ascension*, from which a few sentences may be here quoted.

On July 17 the observatory had been ready for duty; but no sooner were the instruments adjusted and some preliminary observations made, than the face of the heavens darkened and we began to fear. . . . . Many hours, indeed whole nights, this went on, and sometimes the clouds followed each other so rapidly that no measures could be secured at all. . . . . Fearful of losing one hour of starlight during the night, we watched alternately for moments of break in the cloud, sometimes with partial success, but more frequently with no result but utter disappointment, and the mental and physical strain, increasing every night, grew almost beyond our strength . . . . when one day David spoke and took away my breath. He said, "Let us prove how far this cloud extends and find out whether there is any accessible part of the island *not* covered by it."

As the clouds appeared only at night, the investigation had to be made in the darkness. Gill could not leave his post, and no choice was left him but reluctantly to accept the persistent offer of his wife to go on an exploration herself. Simultaneous observations of the clouds were arranged, and the result of the trying expedition was the conviction that the cloud was systematic and that the chances of clear weather would be much better on the southwest point of the island. But the point would be hard to reach either by sea or by land.

Both routes seemed dangerous—the land route indeed impossible, while the surf and rollers which beset the Ascension coast gave little hope of the sea. No landing had ever been made at this bay. . . . . Oh, the sickening responsibility of making up one's mind and choosing between two evils. . . . . Either way looked gloomy. On the one hand my husband felt, if I stay here and fail, I shall have failed also in my duty, not having done my utmost. On the other hand, every night is now of importance, and a week is lost *certainly* if I pull down the observatory, while the slightest accident to an instrument here, with no one to repair it, will be fatal to the expedition.

Yes! both "ifs" were unpleasant, but the first was intolerable, and after a day of anxious thought David made up his mind that an attempt to reach South Point must be made. . . . .

It was on the 31st of July that the important decision was made, and, strange to say, that same night in Garrison, my husband was able to make his first complete determination of the parallax of Mars.

The sky was cloudless from sunset to sunrise, and I wavered, wondering, as many others did, whether the new hope would shake the new decision; but when I questioned, the answer was: "The man that hesitates is lost."

The removal of the observatory was achieved, not without great difficulty, but without serious accident. On August 5, five days after the last successful observations at Garrison, good evening and morning observations were secured at Mars Bay. The expedition was saved from failure.

The few lines quoted, besides throwing light on Gill's character, give a glimpse of his married life. All who knew him know what marriage meant to him. In doing homage to the man, they see, by his side, his highly gifted wife—the wife who made his happiness; who by her infinite, unselfish devotion encouraged and sustained him in every difficulty, and who, though in later life failing in health, still knew how to cheer him in moments of discouragement. To her is due not a little of the homage paid to her great husband.

In 1879 Gill became Her Majesty's astronomer at the Cape, a post which he filled for twenty-eight years. It is there that his real life's work was done. To give a satisfactory account of it, even as briefly as we did for his earlier work, is of course out of the question here. We dwelt somewhat longer on this early work only because it already shows clearly the mettle of the man apart from the co-operation of any assistants. The indomitable energy, love of work, and respect for duty which characterize the beginner leave little doubt as to what—given the necessary health—may be expected from the man when a wider scope is given to his activity. How this expectation is fulfilled is seen at once, if we compare what the observatory and the range of its activity were at Gill's appointment in 1879 and at his resignation in 1907.

In 1879 the only astronomical instruments available for observation were the non-reversible transit-circle, the 7-inch equatorial, and the photo-heliograph. Accordingly the work of

the observatory consisted in the observation of comets, a few double-stars, but in the main in meridian determinations.

Under Gill were added: the 4-inch heliometer (obtained by private purchase), the 6-inch refractor, the 7-inch heliometer, the astrographic telescope, the 24-inch Victoria telescope (gift of Mr. Frank McClean), the reversible transit-circle, the 3-foot altazimuth (loaned permanently to the Cape Observatory by the Trigonometrical Survey of India), and the zenith telescope.

Together with this increase of the astronomical outfit, the staff was gradually extended. The observatory had become an astronomical institution of the first rank. Hardly a problem of practical astronomy was left untouched, and no effort was spared to bring the knowledge of the southern sky more nearly on a par with that of the northern. Of course the meridian work was not neglected. On the contrary, not only were the observations made, the results of which are embodied in the *Cape Catalogues* for 1885, 1890, and 1900, but there were, besides, the *Cape Catalogue of Astrographic Standards*, containing 8560, and the *Cape General Catalogue* for 1900 with 4464 stars. Moreover, the arrears in the reduction of the older observations, already partly cleared off by his predecessor Stone, were finally brought to an end, furnishing the materials for the *Cape Catalogues* of 1850 and 1865. And last, but not least, Gill worked out the plans for and erected the new reversible transit-circle with its observatory and accessories, which places the Cape Observatory in the front rank for refined meridian work.

But along with the meridian work what a series of other labors! A few words may be devoted to two of them.

Researches on stellar parallax.—Twenty-two stars have been measured for parallax, either at the 4-inch or at the 7-inch heliometer. They are the only reliable determinations of stellar parallax ever made in the Southern Hemisphere. It might almost be said that they are the first parallaxes, or at least the first extensive series of parallaxes, which command the entire confidence of the astronomers. The gain in probable error may not be so considerable. The gain in real reliability is very great. In fact, in the domain of stellar parallax, as indeed also in that of the solar paral-

lax, Gill has given us back our belief in probable errors, a belief which, among astronomers, had given way to a pretty general skepticism.

Why this is so is not a matter of doubt. No one can study Gill's work without feeling that he has to do with the born observer, the man with the intuitive faculty of finding out every possible source of systematic error and with the unerring judgment in devising means for its removal; the man with the instinctive feeling for perfect symmetry by which all errors known or unknown must be eliminated. As a consequence we find Gill never satisfied with his work, as long as in any part of it the agreement of the several results is markedly inferior to what might be expected from the probable errors. It cannot be doubted that by the example thus given of a perfect arrangement of the observations and their exhaustive discussion, Gill has contributed to the advancement of science quite as much and more than by the results of his observations themselves.

Researches on the solar parallax.—The problem of the sun's distance has seen some curious vicissitudes during the nineteenth century. In the first half of that century the best values (excepting perhaps that of Henderson) ranged between the narrow limits 8.55 and 8.63, so that the value 8.571 found by Encke from the transits of Venus in 1761 and 1769 seemed to solve the problem of the sun's distance in a satisfactory way. Then in 1854, Hansen, from the parallactic equation, found 8.87 and Leverrier shortly afterward, from the lunar equation of the earth, 8.95. After that, till the time of Gill's first determination, one determination was published yielding the value 8.77, two giving 8.79, all the rest ranged from 8.83 to 8.97.[1] Toward the end of the period under consideration a great effort was made by most of the civilized nations to put an end to this discouraging uncertainty in one of the most fundamental constants of astronomy, by numerous expeditions for the observation of the transits of Venus, in 1874 and 1882. As a matter of fact, however, the problem has been solved, not by these expeditions, but by Gill's labors.

[1] According to Newcomb, "Synopsis of Papers on the Solar Parallax, 1854–1877," *Popular Astronomy* (1878), p. 538.

The determinations by means of his heliometer observations of Juno and Mars, already mentioned, are in a certain sense only preparatory.

In an admirable paper written in 1877 (*Observatory*, Vol. 1), Gill reviews all the available methods. He comes to the conclusion "that only the observation of minor planets entirely fulfils the conditions . . . . as containing within itself the means of eliminating all possible sources of error." The experience gained by the heliometric observations of Juno enables him to conclude that "four reasonably favourable oppositions of favourable minor planets will give the solar parallax with all the accuracy required by the present state of science, provided they are observed with a first-rate heliometer, by an experienced observer, in a good station."

It was not in Gill's nature to wait till others should work out the program thus laid down. "Whenever he set his hand to any bit of work," says the president of the Royal Astronomical Society (H. F. Newall) in presenting to Gill the gold medal of the society in 1908 (the second gold medal of the society awarded to him), "it became the dream of his life to see that particular bit of work completed in the most comprehensive way that he could attain."

Besides if for any man, then for Gill, held the definition of Molenschott, "difficulties are things that are overcome." So it was in the present case. No sooner had Gill been appointed Her Majesty's astronomer than he set to work to plan for the acquisition of the necessary instrument.

In March 1887 I had the pleasure of accompanying him to Hamburg. After a fatiguing journey, we arrived only a little before midnight. Repsold was there to meet us. He told us that early on the next morning everything would be in order, so that Gill might inspect the 7-inch heliometer which had just been completed. Gill would not hear of such a thing. "I can but give you the time necessary for reading my letter. After that we *must* see the heliometer." And we saw it, and when he had inspected every detail, turned every handle, read every microscope, he burst out: "Well, aren't you jealous? Why, I wouldn't be half as happy as I am, if you weren't." Not many weeks later the instrument was mounted at the Cape, the most efficient instrument of the sort in existence. Of course, regular experimental observations were

begun "without delay." In February 1888 it was in regular use for stellar parallax. Before the end of the year it was employed in observations on the minor planet Iris; by the middle of the next year on Victoria and Sappho. "One very noteworthy and delightful feature connected with these researches was the enthusiasm and goodwill with which astronomers of all parts of Europe and America responded and took part in the program of the observations."

The labor involved, both in the observations and in the reductions, was enormous, the success complete. The results obtained by the three planets agree within their probable errors. They agree with the value derived by Auwers from the meridian observations of the planet, and with the values yielded by the Juno and Mars expeditions in 1874 and 1877. They have been found later to agree perfectly with the results of the observations of Eros. Still Gill's enthusiasm for the problem was in no way exhausted.

Already in 1891 he was revolving in his mind a plan of deriving the solar parallax by the observations of the radial velocities of bright stars somewhat near the ecliptic. The method has been applied for the first time by Küstner in Bonn in 1904–1905. At the Cape a series of determinations was begun during the time of Gill's directorate. The investigation was carried to completion in an admirable way under his successor. The result is in complete agreement with that of the minor-planet campaign. In 1909 the thing which, perhaps more than anything else, made him insist on having another meeting of the Comité Permanent of the astrographic chart, was the preparation for the opposition of Eros in 1931!

It is with reluctance that we must confine ourselves to saying so much—rather so little—about only two of Gill's great undertakings. An enumeration of some of the other works must suffice, several of them of no less importance and of far greater extent: *The Cape Photographic Durchmusterung;* the revision of the *Cape Photographic Durchmusterung; Determination of the Mass of the Moon; Determination of the Mass of Jupiter and the Elements of the Orbits of the Older Satellites; The Parallax of the Moon;* investigation of the positions of close circumpolar stars; *Double Stars;* astrophysical observations. *The Heliometric Determination of the Positions of the Major Planets; the International Astrographic*

Chart and Catalogue, and, last in order, what is probably first in importance, *The Geodetic Survey of South Africa*. Only a few words must be said about the *Astrographic Chart and Catalogue*. The initiative for this great undertaking is due to the joint action of Gill and Admiral Mouchez, the director of the Paris Observatory, aided by the brothers Henry. What the whole undertaking, not only at starting, but during the whole of its progress, owes to Gill's untiring energy, all will know who attended the meetings of the Comité Permanent. Up to the last, his was the great driving force.

In the very last meeting of the committee, the most important resolutions taken are undoubtedly those relative to the fundamental, intermediate, and standard stars. It is well known that Gill is really father to these resolutions, though his name does not appear. How different everything will be at the future meetings, when Gill will not be there! How different would be the outlook now, if he could have carried through his plan for a central bureau, perhaps the only important measure which he failed to see brought about!

Outsiders who have seen him at work at these congresses may have been under the impression that it was the geniality of his person, his infectious enthusiasm, and strong self-reliance which carried the day. But those who had followed matters closely would know how carefully he had studied every detail of the matter to be discussed, how long beforehand he had extensively corresponded with the most capable and most interested persons, and how he brought many of them together a few days before the date of the congress, not only to arrange the program for the proceedings, but also to discuss informally all the main points. All during the congress, too, he would bring the ablest men together for these informal discussions. In these Gill would always play a prominent part; sometimes his impetuosity would make it far from easy for those opposed to his views to explain their standpoint. It might be some time before Gill would really give attention to what they had to say, but that moment having come, they could wish for no better listener, and if they succeeded in showing that their point of view was more nearly correct, no man would be

quicker to recognize his error than Gill. No man could be long with him without feeling that here was a man to whom the real interest of science was paramount, a man who was always ready to sacrifice any pet plan of his own to the real interest of astronomy. A favorite expression of his, in giving up his opinion, would be: "The man who never makes a mistake is he who does nothing." I cannot help thinking that such personal qualities—his indomitable energy, his broad-mindedness, love of his work, kindness—his manliness in the best sense of the word, in short the charm of his strong personality, had almost as much to do with his achievements as his qualities as a scientist.

The ready response to any appeal of his for international co-operation must, of course, in great part be ascribed to the soundness of the plan, and the confidence inspired by the qualities of the observer, but the personality of the man must have contributed not a little. It is hardly too much to say that in the minor-planet campaign all the active heliometers and meridian circles of the world took part.

More clearly, probably, we see this influence in the voluntary aid given by so many individuals. Well known is the part that Elkin took in the stellar parallax work. So is Innes' work on the double stars and on the revision of the *Cape Photographic Durchmusterung;* Auwers' timely and invaluable aid in the taking of the heliometer observations for Victoria and his reduction of the Meridian observations; Jacoby's part in the triangulation of the Victoria stars; De Sitter's and Cookson's highly important contributions to the observation and discussion of the Jupiter satellites, as also the work of the Groningen Laboratory on the *Photographic Durchmusterung*. The list must be incomplete. Still we ought perhaps to add the work done under the hospitable roof of the Cape Observatory by McClean and Franklin-Adams—work, indeed, which formed no part of the labors undertaken by the observatory, but which still greatly benefited our knowledge of the southern sky, and might not have come about without Gill's help and encouragement.

As a scientist Gill is best comparable in my opinion to F. G. W. Struve. The terms in which Argelander sums up his

character-sketch of Struve in the *Vierteljahrsschrift der astronomischen Gesellschaft* (1, 52) are applicable almost word for word to Gill. And might not the following words be applied to Gill's *History of the Cape Observatory:* "There is inspiration to be found in nearly every page of it, for its author had the true genius and spirit of the practical astronomer—the love of refined and precise methods of observation and the inventive and engineering capacity": As a matter of fact they were written *by* Gill *about* Struve. Even in the particulars of their careers there is the greatest parallelism.

In the annals of astronomy, Gill's name will take place with those of Bradley, Bessel, and Struve. In many a human heart his image will last as long as life itself.

GRONINGEN
May 1914

PHOTOGRAPHIC DETERMINATION OF THE COLORS OF SOME OF THE STARS IN THE CLUSTER M 13 (HERCULES)

By E. E. BARNARD

The great globular star clusters are among the most impressive objects in the sky when seen with a powerful telescope. Photography, especially with the present great reflectors, has added immensely to this impressiveness by increasing vastly our knowledge of the number of stars that compose them. The most accurate measures seem to show little or no relative motion in the individual stars forming these clusters, which would lead us to believe that they are very distant from us. This would place them on a greater scale of magnificence than the casual telescopic view would imply. The individual stars of these clusters are too faint for the spectroscope to give us any information as to their physical condition. Even when observed visually with a powerful telescope they are far too faint to show any difference of color which would be an indication of differences in spectral types. This difficulty does not hold, however, with the sensitive photographic plate, to which the effect of color is a most striking attribute. This fact became evident to me many years ago when comparing a photograph of the cluster M 13, taken with the Potsdam astrographic telescope, with the object itself observed with the 40-inch telescope. It was soon seen that some of the stars of the cluster were very much bluer than the rest because of their relatively greater brightness on the photograph (*Astrophysical Journal*, **12**, 176–181, 1900). A few of the abnormally blue stars were picked out visually. Later the work was greatly facilitated by comparing the Potsdam picture with a photograph taken with the 40-inch refractor and yellow color-filter, which gave results essentially identical with those obtained visually with the same instrument. It was then noticed that there were also stars in the cluster that were relatively very much brighter on the 40-inch plate and whose light must therefore

be strongly yellow (*Astrophysical Journal*, **29**, 72–75, 1909). The fact, apparent from these results, is that there exist in this cluster stars of extremely different types and hence, by inference, that there are probably stars of all the different spectral types in M 13. I have found indications of this diversity of color in some of the other globular clusters. This fact once more enlarges our ideas of the magnificence of these great star clusters.

It has been my intention to follow out the investigation of color in the clusters, and I had hopes of using a blue filter for photographing with the 40-inch refractor in this work, but so far I have not secured a suitable one, though I expect to do so before long. In the meantime, it has seemed that the subject might be of sufficient interest to print what I have already done in the matter.

As a direct comparison of the two photographs used by me may be readily made without any instrument, I have prepared the two pictures to illustrate this paper. To facilitate their use, the accompanying chart has also been prepared, on which the various stars showing the strongest peculiarities are numbered. More readily to identify these objects on the two photographs, circles were drawn which include some of the more interesting regions. These circles, which are given Roman numerals, are the same on the map and on the two photographs. A glance will therefore quickly identify any star by its map number on either photograph. The stars have been numbered successively from left to right, in the order of right ascension. The map was found necessary to avoid the confusion that would arise from placing the numbers on the photographs.

To accompany these plates, I have prepared a catalogue of 146 stars in the cluster which show difference of color from the general average. [Three of these were later dropped from the list as they were not of special interest.] These stars are separated into two lists under the headings "blue stars" and "yellow stars." What these lists really mean, however, is that when the two photographs are compared the "blue stars" are those relatively bright on the Potsdam plate, while the "yellow stars" are those relatively bright on the plate with the 40-inch telescope. Or, in other words, the star is bluer or more yellow in one case than in the other, so that the words "blue" and "yellow" are relative terms.

TABLE I

YELLOW STARS

No.	Scheiner No.	Δα cos δ	Δδ	No.	Scheiner No.	Δα cos δ	Δδ
1....	16	−4′ 35″	−1′ 36″	72 ...	....	−0′ 27″	−2′ 20″
2....	19	−4 27	−1 8	73 ...	311	−0 19	+2 45
3....	24	−4 15	−0 34	74 ...	....	−0 17	−4 1
4....	....	−4 10	+2 39	75 ...	323?	−0 17	−3 50
6....	38	−3 46	+0 7	76 ...	327	−0 13	+2 3
7....	41	−3 37	+0 17	79 ...	443	+0 18	+2 54
9....	43	−3 29	+0 16	80 ...	484	+0 28	+2 5
10....	44	−3 26	+0 20	81 ...	487	+0 28	−2 8
11....	45	−3 25	−2 1	83 ...	496	+0 31	+3 4
12....	....	−3 12	−3 16	86 ...	....	+0 35	+1 45
13....	....	−3 10	+3 0	89 ...	568	+0 52	−2 21
14....	54	−3 6	+2 25	90 ...	582	+0 58	−2 15
15....	....	−3 2	+2 36	91 ...	....	+1 0	+3 58
17....	57?	−2 51	+0 49	92 ...	591	+1 1	−5 3
18....	....	−2 51	−4 26	96 ...	607	+1 5	+2 40
19....	58	−2 49	−1 34	97 ...	611?	+1 6	+3 54
20....	57?	−2 46	+0 48	98 ...	612	+1 6	−5 11
21....	61	−2 45	−2 21	101 ...	635?	+1 15	+1 30
23....	824	−2 41	−0 59	102 ...	....	+1 16	−2 55
24....	64	−2 39	+0 55	103 ...	637	+1 16	+2 12
25....	66	−2 36	+1 2	104 ...	....	+1 19	−2 53
28....	....	−2 20	−3 56	105 ...	....	+1 19	+1 40
30....	81	−2 19	−0 14	106 ...	655	+1 26	−2 11
31....	82	−2 17	−1 22	107 ...	661	+1 30	−4 18
32....	84	−2 16	+0 20	108 ...	664	+1 31	−2 48
33....	89	−2 8	+0 16	112 ...	677	+1 35	−4 20
35....	....	−2 3	−4 18	113 ...	686	+1 39	−3 25
36....	101?	−2 0	+0 48	114 ...	....	+1 44	−0 7
37....	102?	−2 0	−0 43	115 ...	698	+1 44	+0 26
39....	108	−1 55	−1 36	116 ...	....	+1 47	−0 38
40....	114	−1 51	+0 56	118 ...	710	+1 49	+1 18
41....	120?	−1 46	−1 35	121 ...	....	+1 53	−0 50
42....	124	−1 46	+0 54	122 ...	718?	+1 54	−2 33
43....	....	−1 40	+0 32	123 ...	....	+1 59	+0 44
44....	128	−1 39	−0 46	126 ...	752	+2 25	−2 36
45....	138	−1 32	−1 41	127 ...	....	+2 26	−5 7
46....	141	−1 28	+1 0	128 ...	757	+2 28	+0 33
48....	152	−1 22	−0 58	130 ...	760	+2 36	−3 16
49....	....	−1 19	−1 21	131 ...	....	+2 42	−4 6
50....	....	−1 18	+2 37	132 ...	765	+2 43	−1 38
51....	168	−1 15	+1 1	135 ...	769	+2 48	−0 40
56....	186	−1 7	+2 2	136 ...	772	+2 55	−0 12
58....	....	−1 6	−3 55	137 ...	....	+2 57	−3 25
61....	213	−0 55	+1 51	138 ...	781	+3 17	+2 8
63....	217	−0 54	+3 28	139 ...	....	+3 32	−2 16
67....	235?	−0 44	+1 14	140 ...	790	+3 38	+0 36
68....	....	−0 41	+2 34	141 ...	832	+3 40	−0 8
69....	253	−0 38	+2 46	143 ...	793	+3 44	+1 15
70....	....	−0 28	−2 28	144 ...	803	+4 17	−0 8
71....	....	−0 28	+1 59	146 ...	809	+4 37	−0 1

TABLE II

BLUE STARS

No.	Scheiner No.	Δα cos δ	Δδ	No.	Scheiner No.	Δα cos δ	Δδ
5.....	34	−3′ 53″	+0′ 9″	84...	500	+0′ 32″	+3′ 9″
16.....	55	−3 1	−0 41	85...	509	+0 35	+2 16
22.....	62	−2 44	+1 0	87...	524	+0 38	+2 26†
26.....	73?	−2 27	−0 58	88...	529	+0 40	−5 19
27.....	77	−2 23	−0 56	93...	592	+1 1	+0 46
29.....	80	−2 20	+0 22	94...	....	+1 3	−0 28
34.....	94	−2 4	−1 37	95...	599	+1 3	−2 38
38.....	101?	−1 59	+0 50	99...	829?	+1 13	+2 20
47.....	148	−1 22	−1 43	100...	630	+1 13	−0 24‡
52.....	171?	−1 15	−1 20	109...	667	+1 33	+1 16
53.....	179	−1 11	+0 1*	110...	670?	+1 34	+0 30
54.....	181	−1 10	+2 13†	111...	676	+1 35	+0 27
55.....	182	−1 9	−1 14	117...	709	+1 48	+0 46
57.....	188	−1 7	+1 23	119...	716?	+1 50	+1 12
59.....	194	−1 5	−4 10	120...	830	+1 53	+0 38
62.....	216	−0 54	−0 3‡	124...	749	+2 21	−1 25
64.....	229?	−0 48	+0 59	125...	750	+2 21	−2 23
65.....	229?	−0 48	+0 57	129...	759	+2 33	−2 31
66.....	234	−0 45	+3 0	133...	766	+2 43	−1 24
77.....	382	0 0	0 0	134...	767	+2 44	−0 22
78.....	393	+0 4	−0 26	142...	792?	+3 41	+1 30
82.....	493	+0 31	+1 44	145...	833	+4 17	−1 25

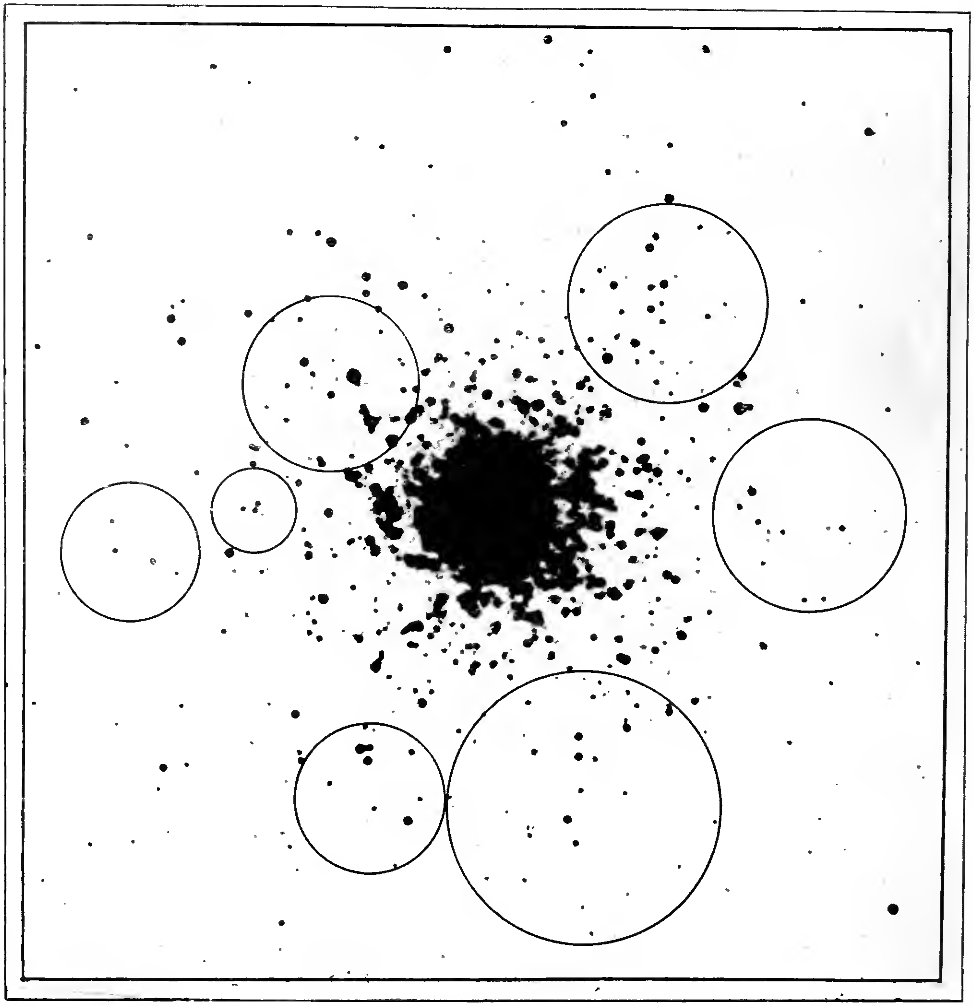
* Color uncertain. Two stars at this point.
† Variable?
‡ Variable.

It would be easy to enlarge this list, but I have thought best not to add to it until the work can be done more properly with different color-filters.

For the comparisons two photographs have been used. The first of these was made with the 13-inch astrographic refractor of the Potsdam observatory, which is of course corrected for the violet or ordinary photographic light. The other is a photograph taken with the 40-inch refractor of the Yerkes Observatory with a yellow color-filter and an isochromatic plate, which gives an image closely resembling that seen with the eye in the same instrument. The Potsdam plate was made by Eberhard and Ludendorff, July 20, 1900, with an exposure of 2^h31^m. The Yerkes Observatory plate was made by Ritchey with an exposure of 3^h30^m. I have not been able to find the date of this last plate, which was perhaps made also in 1900.

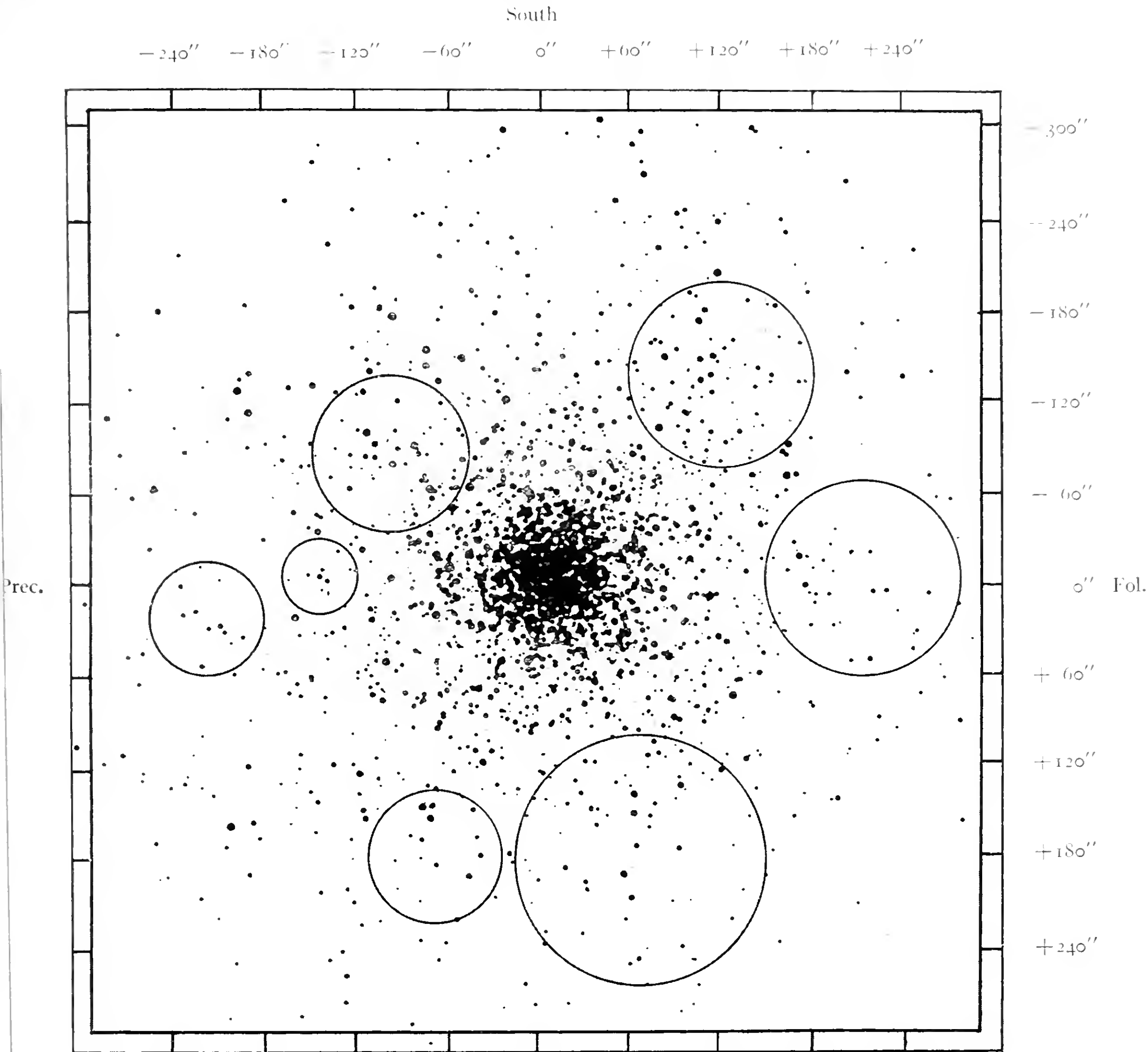
PLATE IV

South

Prec.  Fol.

M 13, Taken with the 13-inch (34-cm) Photographic Refractor of the Potsdam Observatory. No Filter Used.

PLATE V

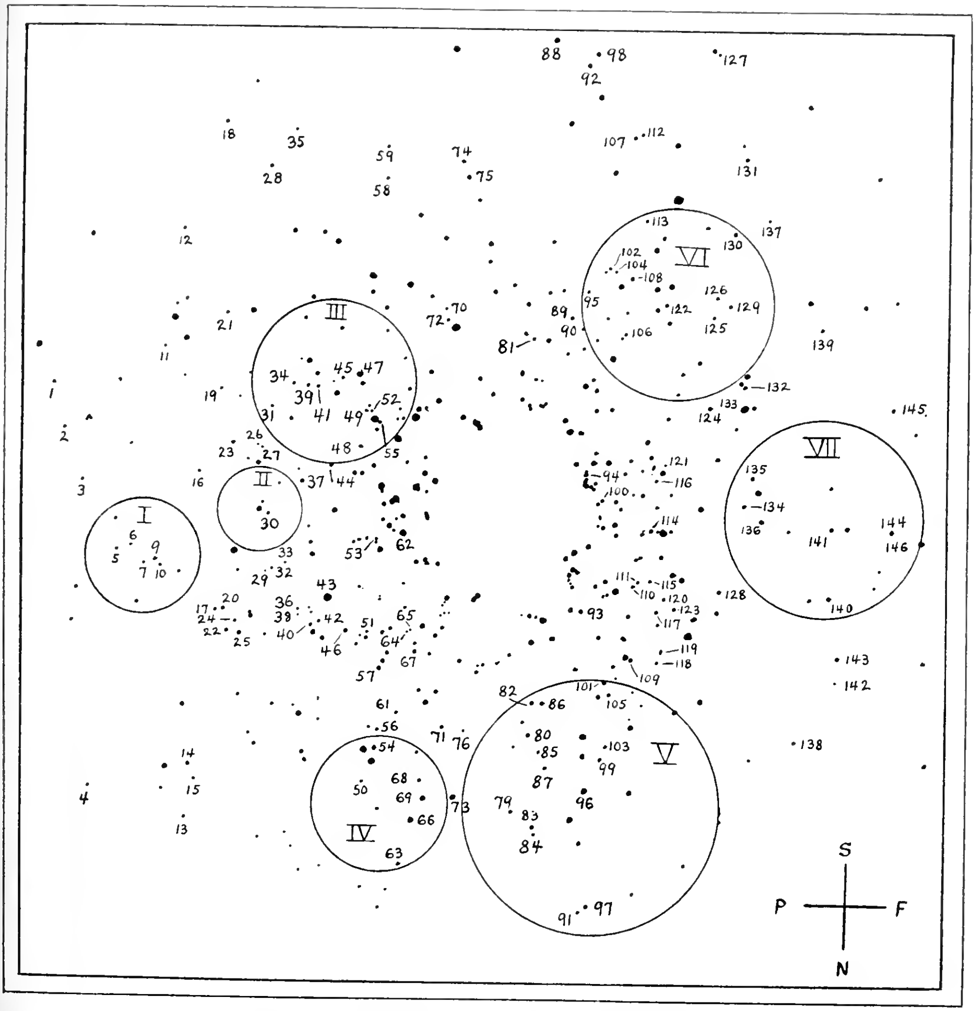


M 13, Taken with the 40-inch (102-cm) Refractor of the Yerkes Observatory
Yellow Color-Filter Used.

To refer any star from this chart to the printed lists, apply a correction in α of $+1\overset{\prime\prime}{.}6$, and in δ of $+20\overset{\prime\prime}{.}6$.

These photographs accompany this paper. A set of co-ordinates has been placed on the Yerkes Observatory plate, which is correctly oriented from my micrometer measures.

South



Index-chart of M 13, for use with Plates IV and V

It should be stated that for this work the Potsdam plate has received a very great enlargement which makes any comparison of

the relative qualities of the images on the two plates very unfair to it.

Though one would not fail to notice the marked differences in the relative sizes of some of the stars in comparing these photographs, especially in the fields inclosed in the circles, it is perhaps well to call attention briefly to a few of the most striking peculiarities.

The Roman numerals, of course, refer to the circles. The two plates are designated Y. O. and P. O. for the observatories where they were made.

I. Of the eight stars strongly shown on the Y. O. plate, only three appear strong on the P. O. plate and a fourth faintly. The three are evidently bluer than the other stars near.

II. Of the four stars on the Y. O. plate, one (No. 30) is evidently strongly yellow, as it is almost invisible on the P. O. plate.

III. The star 47, so conspicuous on the P. O. plate, is perhaps the most striking example of a blue star in the cluster. The Y. O. plate, however, shows that a number of the stars near it are yellow. The apparently conspicuous double star close inside the lower part of this circle on the P. O. plate is a defect.

IV. In the triangle of three stars of which No. 66 is at the right angle, it will be seen that 66 is very much bluer than the others. The visual observations confirm this great difference. With the telescope I have never been able to get No. 66 perfectly defined. It has always appeared hazy like a minute nebula, in this respect strongly resembling the present appearance of such stars as Nova Cygni (1876) and Mrs. Fleming's Nova Sagittarii of 1898. The best focus for it is about one-tenth of an inch (2.5 mm) farther from the object-glass than for the other stars of the cluster. Notice also the star 56 near the upper edge of the circle and the small star close by preceding it. The relative magnitudes on the two plates are reversed.

V. The eye will pick out many discordances in the relative sizes of the stars here on the two plates; perhaps the most striking are 82 and 86, and especially 99 and 103.

VI. This field shows many discordances, especially at 125, 126, and 129.

VII. There are several striking differences in this region, notably in the case of 135, 141, and 146.

In the lists, where it has been possible, I have identified the stars with those of Dr. J. Scheiner (*Der Grosse Sternhaufen im Hercules Messier 13*). Where such identification is certain, the Scheiner number and position are given. Where the identification is uncertain, or the star apparently was not measured by Scheiner, the approximate place (roughly taken from the photograph) is given. Scheiner's Nos. 216 and 630 are variable stars which were discovered by Professor Bailey of the Harvard College Observatory. I have found the period of these two objects to be 5.1 days and 6.6 days respectively. They seem to fall in the class of "blue stars," because apparently they are relatively brighter at maximum in the regular photographic telescopes than with the eye. I have previously called attention (*Astrophysical Journal*, **12**, 177, 1900) to the fact that Scheiner's normal star, No. 382, falls in the list of "blue stars." It is excessively faint visually. This faintness is perhaps due in part to its position in the bright part of the cluster.

From the great number of variable stars in some of the globular clusters it might be suggested that these differences in the relative size of the star images on the two plates of M 13 are due to variability in the stars themselves. In the principal cases given here I have visually assured myself that the stars are not variable. It is known that M 13 is relatively free from variable stars. There seem to be but the two stars mentioned above that are known positively to be variable, though there are several that I have suspected of variability. One of these, which seems to be certainly variable, lies midway between Scheiner Nos. 231 and 270. Its position is

$$1902.0 \qquad \alpha = 16^{h}38^{m}6^{s}.99 \qquad \delta = +36^{\circ}37'42''.4$$

Ordinarily, color-differences are not so apparent as those in the present plates. The Potsdam picture was made from a rather dense glass positive. This has considerably accentuated the contrast. I have, however, verified the principal differences by comparison with a photograph taken by Mr. Pease with the 60-inch reflector at Mount Wilson, which was kindly supplied me for the purpose by Professor Hale. It shows essentially all the stars that are on the 40-inch plate, though the exposure was only six minutes. In the case of faint stars the size of the image does not differ greatly, but the density may differ considerably. This density is a function

of the actinic energy of the light and is (when the photographs are taken with and without a filter) an index, to a certain extent, to the color of the star. By somewhat heavier printing the less dense image will give a relatively smaller star disk, and the color effect will be more apparent. Caution must, however, be exercised in such an experiment not to overdo the contrast, as this can produce effects that are not due to color.

In my early work on M 13 I found the following correction to Scheiner's *Der Grosse Sternhaufen im Hercules Messier 13*. His stars Nos. 98 and 782 do not seem to exist. The place of 782 ($+3'21''$, $-2'33''$) is evidently an error of reduction, for there is no star at that point on the plate measured by Scheiner. It is doubtless a measure of 785 ($+3'23''$, $-2'33''$) with an error of $2''\pm$ in the $\Delta\alpha$. They both have the same note, "Strich stört," which could not apply to 782, as its place is free of the reticle while No. 785 is on the reticle line. The given declinations are the same.

Number 98 ($-2'1''$, $-3'20''$), however, appears on Scheiner's Plate I, $5''$ south of No. 96. On a photographic copy of this plate which I have, its image is perfectly like that of 96 or any of the other stars. It is slightly brighter than 96. The note to 98, "Fleckchen stört," shows that there must have been some defect on the original plate (not visible, however, on the copy) which affected the image. The star does not seem to have shown on Plate II made the next night, nor on a Lick plate of the same year, nor on any of the other photographs that I have of the cluster, and many years' observations with the 40-inch telescope do not show anything at its place. If it is a false image it resembles a true star better than any defect I have seen before.

Yerkes Observatory
Williams Bay, Wis.
September 4, 1912

The foregoing paper was prepared in its present form nearly two years ago, and it is thought best to let it stand as it is. In the meantime some successful photographs of the spectra of a few of the individual stars of M 13 have been obtained with the 60-inch reflector at Mount Wilson. In the *Publications of the Astronomical Society of the Pacific* for October 1913, p. 260, Mr Adams gives a short paper on two photographs of this cluster (M 13) which had recently been taken by Mr. Pease with a small slit-spectrograph at the primary focus of the 60-inch reflector, with exposures of 21 and 22 hours.

These show the individual spectra of some nineteen stars. He classifies these spectra thus:

Class	No. of Stars
A0	2
A5	5
F0	2
F5	8
G0	2

From the two star lists in my paper we have the following results:

Yellow stars	100
Blue stars	44

The foregoing would indicate a larger percentage of yellow stars, but too much reliance must not be placed on these numbers. The want of an exact standard of comparison makes a thorough investigation difficult with the means at my command. Dr. A. Van Maanen has, however, lately taken up a similar investigation of the stars in this cluster with the 60-inch reflector at Mount Wilson, by taking plates through a yellow color-filter and without any filter, for color comparisons. This is a much better plan than the use of two different telescopes as in my investigation, and one that I had hoped to carry out with the 40-inch refractor; but I was prevented by the want of a proper blue filter or a correcting lens. When Dr. Van Maanen's work is complete a more definite value for the relative prevalence of yellow or blue stars in the cluster will be available. My investigation was made to prove that different stellar types exist in the cluster by showing striking examples of color differences in the stars.

In my micrometer measures of the stars of the various clusters I have adopted in each case a normal star to which, either directly or indirectly, the other stars have been referred. Dr. Scheiner, in his photographic measures of M 13, used for this purpose a star, No. 382 of his list, which because of its faintness could not be successfully used in my visual measures. Also, this star is not near the center but nearer the north edge of the brightest part of of the cluster. Quite close to the center of the figure is a distinct and considerable star, No. 373 of Scheiner's list. This star, because of its distinctness in the telescope and its central position, I have used as a normal star to which all my measures are referred. It is 20″6 south of No. 382 and has nearly the same right ascension as that star.

The co-ordinates that have been placed about the Yerkes plate refer to my normal star and not to Scheiner's. To use the chart, therefore, in connection with the star lists, it is necessary to apply a correction of $+1''6$ in right ascension and $+20''6$ in declination to any star whose position is taken from the chart for identification in the list, or to apply these same corrections with reversed signs to any star place in the two lists to find the star on the chart. This is simplified if we notice that for purposes of identification alone it is only necessary to shift the system of co-ordinates in declination on the chart $\frac{1}{3}$ of a division north.

May 22, 1914

WAVE-LENGTH SENSIBILITY-CURVES OF POTASSIUM PHOTO-ELECTRIC CELLS

By HERBERT E. IVES

In a recent paper[1] the writer has shown that the illumination-current relationship in the ordinary forms of gas-filled photo-electric cells is not rectilinear. This fact very seriously limits the use of the cell for light-measurement, even in the simplest case, that in which the light-sources under comparison possess the same distribution of intensity through the spectrum. Where the light-sources are different in color it is necessary not only to have a simple illumination-current relationship, but the wave-length sensibility-curve should be that of the normal eye, if actual visual photometric values are desired. Assuming that cells could be selected of such gas pressure that for a limited but useful range the illumination-current relationship is satisfactorily linear, it should be possible to make an absorbing screen which would reduce the cell's sensibility-curve to the desired shape. For this purpose and also where one is content to use the scale of magnitudes determined by the cell's natural sensibility, it is a matter of prime importance to determine what this natural sensibility-curve is, and as well whether it is the same for all cells of the same alkali metal.

A priori, from the work of Pohl and Pringsheim[2] it is to be expected that the sensibility-curve will depend upon the surface and the angle of incidence, since upon these factors will depend the relative amounts of normal and selective effect. Although, according to the theories of Einstein and Richardson, the long-wave limit of photo-electric action should be a definite characteristic of the metal used, recently published observations of Pohl and Pringsheim, as well as work of Elster and Geitel, have shown that this limit can be exceeded and actually appears to depend on the age and history of the surface. Richardson and Compton,[3] in their

[1] *Astrophysical Journal*, **39**, 428–458, 1914.

[2] *Berichte der Deutschen Physikalischen Gesellschaft*, **12**, 215–228, 349–360, 1910; **15**, 637–644, 1913.

[3] *Phil. Mag.*, **26**, 549, 1913.

work to test Richardson's theory, plot wave-length sensibility-curves showing a very great change with time after deposition of the metal.

In view of these facts it is hardly to be expected that all cells will have exactly the same sensibility-curve. But how much difference may be expected in cells of similar construction? Might it not be possible to prepare cells of similar surface characteristics which would be alike and reproducible? An answer to these questions was sought in the measurements here described.

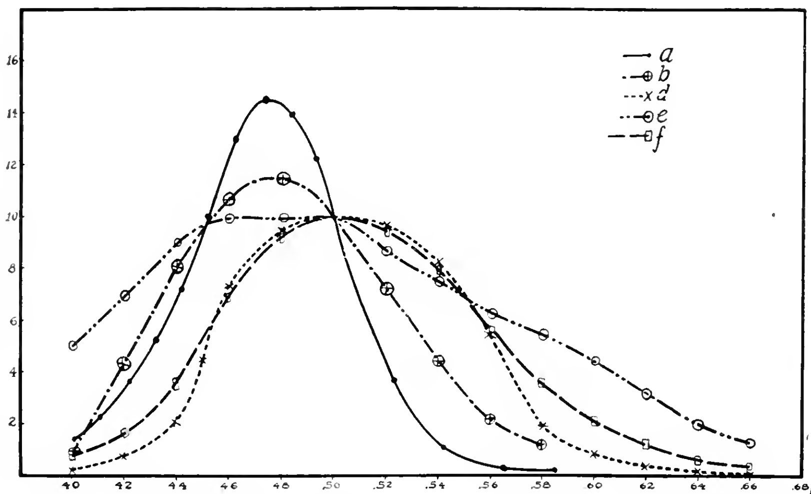


FIG. 1.—Wave-length sensibility-curves of five potassium photo-electric cells, using Nernst glower as source.

In all, nine cells were available at one time or another, and all were measured on the same apparatus. This consisted of a Hilger monochromatic illuminator, on whose slit was focused the image of a Nernst glower, operated at normal current. The photoelectric current was measured by the steady deflection method, as described in the previous paper. Owing to the non-linearity of the illumination-current relation the curves as obtained are not strictly accurate. As, however, the slit-width was kept quite small, the deflections were not large. Deviations from rectilinearity were consequently small and any errors due to this cause

were certainly much smaller than the large differences which show between the cells.

The wave-length sensibility-curves as obtained with the Nernst glower as source are shown in Fig. 1, drawn equal at 0.50 μ. Only five are given,[1] because the whole number of curves become almost inextricable if plotted together and those shown are sufficient to

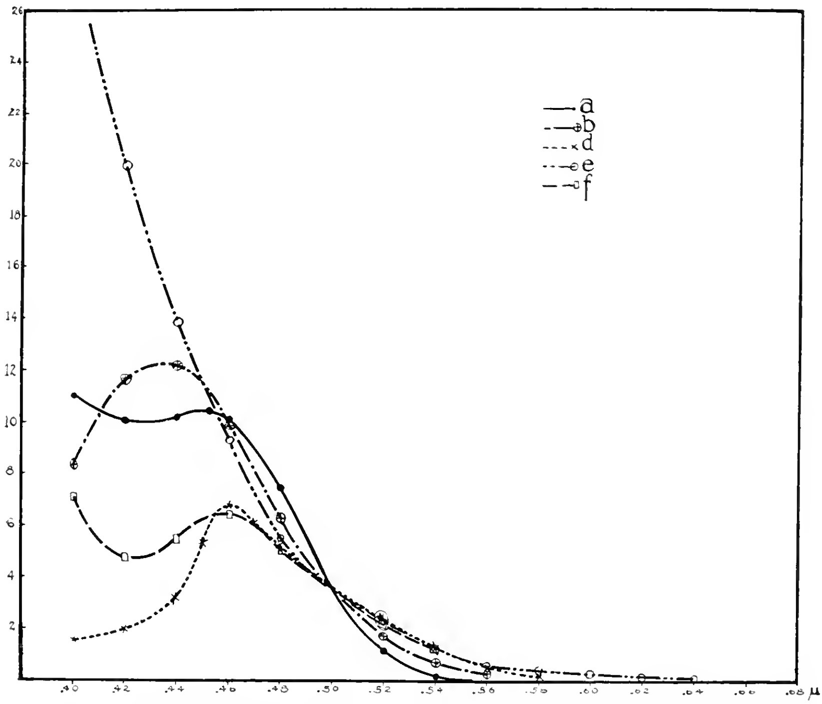


FIG. 2.—Wave-length sensibility-curves of five potassium photo-electric cells, reduced to equal energy spectrum.

reveal the essential facts. No two sensibility-curves are alike, either in position of maximum or in long-wave limit, and the differences can be characterized only as enormous.

In Fig. 2 the same curves are shown replotted for an equal energy spectrum. The calculation of these values was based on a spectrophotometric comparison of the light of the Nernst glower with that

[1] The cells are those designated by the same letters in the previous paper.

of a "4-watt" carbon lamp, previously found to be of the relative energy distribution of a black body at 2080° absolute.[1] From this comparison it appeared that the glower light corresponds in color to a black body at 2310°. Values calculated from Wien's equation then give the energy distribution.

Inspection of these curves shows that the different sensibility-curves differ in two respects: first, in the relative prominence of the "hump," due to the selective effect; second, in the long-wave limit to the effect.

The explanation of the relative amounts of normal and selective effect might be sought in the surface conditions were it not for the fact that the cells *b*, *d*, and *f* were all prepared by distilling the potassium on to the cold glass surface, as described in the previous paper, and possessed very similar matt surfaces. They were, in fact, as nearly alike in character as one could expect to make cells in practice. Yet they differ very materially in properties. In cell *e* the metal surface is quite mirror-like, and in another cell, not shown, a somewhat similar preponderance of normal effect is present. There is, in short, nothing in these results clearly indicating any other factor than the surface configurations, but they do discourage the idea that surfaces quite similar in appearance will have the same proportions of normal and selective effect.

As to the long-wave limits, these vary from approximately $0.6\ \mu$ to the infra-red. Four of the five cells shown were filled with the same "c.p." potassium, so that this peculiarity cannot be ascribed to differences in the metal before its introduction. Other possible causes are the gas pressure in the cell and the amount of gas occluded by the potassium. No certain connection appears to be shown between the gas pressure and the character of the sensibility-curves. There is some reason to believe, from knowledge of the history of the manufacture of the cells, that ones in which the potassium was most thoroughly boiled in vacua for a long time, and were afterward maintained at the best vacua, had the shorter wave limits.

The whole matter of wave-length sensibility-curves, their dependence upon the manner of preparing the cell, upon surface character, on past history, etc., is one demanding thorough

[1] *Phil. Mag.*, **24**, 862, 1912.

investigation. It is evident, however, from the data here presented that each cell is an individual, not only in its illumination-current relation, as previously shown, but in its wave-length sensibility. The use, therefore, of photo-electric cells as at present constructed, for stellar and other photometry, must be extremely cautious to say the least.

Physical Research Laboratory
United Gas Improvement Co.
Philadelphia
April 1914

ON THE CHANGE OF SPECTRUM AND COLOR INDEX WITH DISTANCE AND ABSOLUTE BRIGHTNESS. PRESENT STATE OF THE QUESTION[1]

By J. C. KAPTEYN[2]

In what follows I have brought together whatever evidence, published or unpublished, has come to my knowledge on the question indicated in the title of this note.

The evidence bears on the real existence of the two following observed phenomena:

I. OBSERVED PHENOMENA

(1) On the average the apparently fainter stars are redder than the brighter ones.

(2) Apparent magnitude and spectral lines being the same, the stars are redder the farther away they are.

II. POSSIBLE EXPLANATIONS

The phenomenon (1)—if real—is explainable in one of the three following ways:

(3) By the predominance of the later spectral types among the fainter stars.

(4) By an influence of the absolute brightness on the color index.

(5) By selective absorption (or scattering) of light in space.

For the phenomenon (2)—if real—there are only the two explanations (4) and (5).

III. INVESTIGATIONS REQUIRED

The three following investigations are therefore necessary to settle the reality of, and to determine quantitatively, the phenomena in question:

(6) Investigation of the *relative frequency of the several spectral classes* among the stars of the fainter magnitudes.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 83.

[2] Research Associate of the Carnegie Institution of Washington, Mount Wilson Solar Observatory.

(7) Investigation of the influence of *absolute brightness* on the spectrum.

(8) Investigation of the influence of *distance* on the spectrum.

IV. REMARK UPON (3)

It is important to note that a predominance of later-type stars among the fainter stars does not necessarily mean such a predominance among the more distant stars. If the spectral types are equally mixed throughout the stellar system, and if there is no space absorption nor any influence of absolute brightness, we shall find relatively more later types among the fainter stars. As a consequence, the fainter stars will be redder on the average than the brighter ones.

In order to see this at once,[1] imagine for a moment that the stellar system is sharply limited at a certain distance from the sun, and suppose, for the sake of simplicity, that we have to do only with stars of the first and second spectral classes, and that the proportion of the two classes does not change with distance. As the first-type stars are much more luminous, and as they are at the same time more nearly equal, the very faintest second-type stars must be much fainter than the least luminous first-type stars. Let the difference be a magnitudes. The very faintest stars that we see in the sky will therefore be the least luminous second-type stars which stand at the limit of the system. Let their apparent magnitude be μ. The stars of apparent magnitude μ will thus be exclusively second-type stars. So also will be the stars of magnitude $\mu-1$, $\mu-2$ . . . . up to $\mu-a$. At this last magnitude we begin to find some of the least luminous of the first-type stars and from this magnitude upward the number of first types, relatively to that of the second, will steadily increase. Evidently there is a predominance of second-type stars for the fainter magnitudes, without any thinning-out of the first-type stars with distance.

If we do not assume a sharp limit for the stellar system the same will still hold, at least if, as is found by all investigators,[2] the star-density diminishes with increasing distance. In order to show

[1] My attention was drawn to this consideration by Professor Hertzsprung.

[2] On the supposition, though, of the non-existence of appreciable space absorption.

this and at the same time to obtain at least some quantitative idea, I computed the relative numbers of the first- and second-type stars for different magnitudes on the suppositions: (*a*) that for the fifth magnitude the number of stars for the two types is the same; (*b*) that for the first type the color index is zero, and for the second type +1.0 mag. I adopted the luminosity-curves found in *Groningen Publication*, No. 11, p. 31 (Sol. B). They are well represented by the formula:

$$\text{Number of stars} = Ce^{-h^2(M-k)^2} \tag{9}$$

in which M = absolute magnitude and

$$\text{for the first type,} \quad h=0.243 \qquad k=\ \ 8.95 \text{ mags.} \tag{10}$$

$$\text{for the second type,} \quad h=0.247 \qquad k=10.30 \text{ mags.}^{1} \tag{11}$$

The computation was further carried out by means of the very convenient formulae given by Schwarzschild.[2] I found the results given in Table I.

[1] An attempt is now being made by Dr. Kohlschütter and myself to obtain the luminosity-curves for each of the spectral classes, B, A, F, G, K, M, separately. As these are not yet available I had no choice but to adopt the curves of *Groningen Publication*, No. 11, though well aware of the fact that they are less satisfactory than what could be obtained by the use of more recent data. It seems probable (as implicitly assumed a moment ago) that with these better data we should find the value of h for the first-type stars very appreciably greater than for those of the second type.

[2] *Astronomische Nachrichten*, No. 4557. According to Schwarzschild:

$$\log \text{number of stars} = \text{const.} - \alpha_1 m - \alpha_2 m^2 \tag{P}$$

where

$$\alpha_1 = \frac{a_2(b_1+0.4) - b_2(a_1+0.6)}{a_2+b_2} \qquad \alpha_2 = \frac{a_2 b_2}{a_2+b_2}\,.$$

The values of b_1 and b_2 must be found from the luminosity-curves. We have

$$\frac{b_1+0.4}{2b_2} = k \qquad b_2 = h^2 \times \text{Mod.}$$

It is necessary in formulae (10) and (11) to diminish the values of k by 5 magnitudes in order to comply with Schwarzschild's definition of absolute magnitude, which differs by 5 mags. from that adopted in the *Groningen Publications*. The quantities a_1 and a_2 depend on the star-density according to the formula:

$$\log D\ (r) = \text{const.} + 5a_1 \log r - 25a_2 (\log r)^2$$

The first constant, only, is supposed to differ for the two types. For the other constants Schwarzschild finds:

$$a_1 = +0.097 \qquad a_2 = +0.0088$$

The constants being thus found and substituted in (P), we get the numbers whose ratios are in Table I.

The numbers in this table may be appreciably in error owing to the confessedly defective data on which it rests. At all events, however, it is evident that the apparent magnitude approaching infinity, the average color index will asymptotically approach 1.000, the maximum change thus being half a magnitude.

TABLE I

m	$\frac{\text{First Type}}{\text{Second Type}}$	Average Color Index
5	1.000	0.50
10	0.778	0.56
15	0.609	0.62
20	0.477	0.68

We thus see that the average color index of the faint stars is almost of necessity different from that of the brighter ones. In consequence of this it will be practically impossible to define a scale of magnitudes for the stars too faint for special classification in such a way that on the average the photographic and the visual scales will agree. Furthermore, unless observation yields a change in the relative numbers of the several spectral classes or in the amount of the color index materially exceeding that shown in Table I, we shall not be justified in concluding, from the observed phenomenon (1), that there is a relative increase in the later types with distance, nor shall we be able to conclude that either causes (4) or (5) are active.

Meanwhile the assumption that "the spectral types are equally mixed throughout the stellar system" is probably wrong. In *Groningen Publ.*, No. 11, Table 8, it was found that the density of the first-type stars does not decrease nearly so quickly with the distance as that of the second-type stars. This cause acts strongly in the opposite direction of the Hertzsprung effect. The phenomenon (2) is of course entirely independent of the latter effect.

V. IMPORTANCE OF THE INVESTIGATIONS (6)–(8)

Suppose that the investigation (6) leads to the result that the change in the relative frequency of the various spectral classes varies more than can be accounted for in the foregoing manner.

We shall then conclude that the later types are more frequent among the more distant stars. But if this is so, the luminosity-curve for *all the stars together* will change with distance. For the luminosity-curve of the mixture of stars of different spectral classes is of course dependent on the proportional numbers in each class. These proportional numbers changing with distance, the luminosity-curve of the mixture must necessarily change with distance. This means that our present theories about the arrangement of stars in space will need revision, for they all start from the supposition that the luminosity-curve is constant. Such a state of affairs would emphasize the necessity of an investigation of the arrangement in space *for each class of spectrum separately.* Independently of such a change, and for many obvious reasons, such a separate treatment is, in my opinion, one of the urgent desiderata of science. The investigation (6) will furnish one of the most indispensable data for such a treatment.

A well-determined influence of absolute brightness (7), besides being important in a study of the physics of stellar atmospheres, would furnish the means of determining parallaxes, especially for the distant stars, where the other methods break down.

An influence of absorption, if it exists, must introduce elements in the spectra of the distant stars which have nothing to do with the chemical and physical properties of the stellar atmospheres. For a fundamental study of the star-spectra the knowledge of any distance effect is therefore urgently required. Besides, as with the former influence, it will furnish means of getting data for very great stellar distances. In fact, the simple result of the direct observation (2), if once well established and investigated, will do this independently of the question to what extent it is to be explained by either of the causes (4) and (5). For the rest, the determination of a selective loss of light in space would be the first step toward the determination of the total loss of light. A good illustration of the absolutely fundamental importance of this latter quantity in the investigation of the structure of the stellar system is furnished by the fact that a loss of 0.18 magnitude for every unit of distance (32.6 light years), which has been assumed by Comstock, leads to a star-density, at the distance of 3,000 light

years, hundreds of millions of times greater than the supposition that no such loss exists.[1] It must be clear from this that, unless we succeed in determining the absorption constant within at least something like a thousandth part of a magnitude, no really definitive results can be obtained for the arrangement of stars in space.

VI. INFLUENCE THAT WOULD BE EXERTED BY SELECTIVE ABSORPTION OF LIGHT

Before adducing the evidence already existing, I will give a little table showing to what extent selective absorption of light by an interstellar medium (if homogeneous) would make itself felt, on the average, for stars of different apparent magnitudes. (The mean parallaxes have been assumed in accordance with what, in my opinion, are the best data at present available.)

TABLE II

Vis. Mag.	Change in Color Index	Vis. Mag.	Change in Color Index
6.0	0.04 Mag.	14.0	0.42 Mag.
8.0	0.07	16.0	0.75
10.0	0.13	18.0	1.30
12.0	0.24	20.0	2.40

These numbers are in accordance with the "absorption coefficient" derived in *Astrophysical Journal*, **30**, 295, 1909,[2] where c was taken to be vanishing. For other values of the coefficient our numbers will have only to be multiplied by a constant factor. The table shows well how an absorption, even when small and hardly appreciable for the stars below magnitude 9, may still be all-important for the very faint stars—how, therefore, its accurate determination must almost of necessity be made dependent on these latter stars. For the dependence on absolute brightness we cannot make such a table because of our ignorance as to the law which would express such a dependence.

[1] *Astronomical Journal*, No. 566.

[2] *Mt. Wilson Contr.*, No. 42.

VII. EVIDENCE EXISTING ON THE PHENOMENON (1): THE FAINTER STARS ARE REDDER ON THE AVERAGE THAN THE BRIGHTER

a) Fath's result.[1]

The result of the comparison of 76 pairs of stars on the five plates showed that, in the mean, the fainter stars are distinctly redder than the brighter ones.

b) King's result.[2] King finds

$$d = +0.019 \pm 0.010 \text{ mag.} \tag{12}$$

d representing the change of color index over unit of distance (32.6 light years). His conclusion is:

> From the correspondence and agreement of the results as viewed from different standpoints, I believe that we are dealing here with an actual factor, which, if not real absorption, produces effects similar to absorption.[3]

There is a more recent paper of King's giving

$$d = +0.023 \tag{13}$$

I quote the result from *Observatory*, February 1914, because I have not as yet seen the original paper.

c) Turner's result.[4]—From various considerations the author finds:

$$d = +0.030 \tag{14}$$

d) Four clusters.[5]—The globular clusters offer a case promising a separation of the two causes (4) and (5). For on small-scale plates, or on larger-scale plates if we operate out of focus, we can compare the color index of the cluster with that of a star of equal average spectrum and magnitude, without introducing perturbing photographic effects. At the same time, if the star has a sensible proper motion, we may be sure that the difference in distance is very considerable. As there is little reason for assuming that the separate stars of the cluster are, on the average, of a luminosity much different from that of the comparison star, a possible

[1] *Ibid.*, No. 63, p. 5; *Astrophysical Journal*, **36**, 366, 1912.

[2] *Annals Harvard College Observatory*, **59**, 182, 1912.

[3] *Ibid.*, p. 183, 1908.

[4] *Monthly Notices*, **69**, 61–71, 1908.

[5] *Mt. Wilson Contr.*, No. 42, p. 33; *Astrophysical Journal*, **30**, 316, 1909.

effect of the absolute magnitude is avoided. The results are as follows:

N.G.C. 7076	Color index $=S+1.0$ mag.
N.G.C. 7089	$S+0.45$
Nucleus Great Andromeda Nebula	$S+1.0$
Hercules cluster	$S+0.9$

where S = color index of the comparison star, which is of the same magnitude and the same average spectrum (according to Fath) as the cluster. For the Hercules cluster I assumed the average spectrum to be F1, which is the average of the spectra recently found by Pease for 19 stars in the cluster.[1]

These determinations are rather rough, and the adopted average spectra of the clusters perhaps not unobjectionable. Still, as the differences are so large and the clusters almost certainly very far away, the qualitative evidence seems pretty strong.

e) Limiting photographic magnitudes as determined by Professor Pickering.—In *Harvard Circular*, No. 170, Professor Pickering gives for several instruments the magnitudes of the faintest stars obtained on the present rapid plates by an exposure of 60 minutes. I have myself derived the visual magnitudes of these faintest stars by the aid of the data in *Groningen Publication*, No. 18. In comparing the results we have to take into account the difference of the scales. Pickering's photographic magnitudes are on the International scale, the visual magnitudes on the Harvard visual scale. The International scale of photographic magnitudes is identical with the Harvard visual scale for the stars of spectrum A0, magnitude 6.0. For the stars of other spectra there is a difference. Let

$$\left.\begin{array}{r}\Delta = \text{average difference: Phot. Mag. (Int. scale)} \\ \textit{minus} \text{ Visual Mag. (Harv. scale)}\end{array}\right\} \qquad (15)$$

For the stars of visual magnitude 6 to 7 (Harv. scale) I found from Parkhurst's *Yerkes Actinometry*[2]

$$\Delta = +0.66 \text{ mag.} \qquad (16)$$

[1] "Annual Report of the Director of the Mount Wilson Solar Observatory," *Year Book of the Carnegie Institution of Washington*, No. 13, p. 219, 1913.

[2] *Astrophysical Journal*, **36**, 169, 1912.

If, therefore, the fainter stars are as red as the brighter ones, we must find for them the same average difference. If they are redder, the difference must be greater. As a matter of fact I find the results given in Table III.

TABLE III

FAINTEST STARS ON RAPID PLATES, 60 MINUTES EXPOSURE

	Phot. (Int. Scale)	Vis. (Harv. Scale)	Phot.−Vis. −0.66
12-inch refractor.	15.7	13.8	+1.24
16-inch refractor.	16.5	14.9	+0.94
36-inch reflector.	18.9	16.1	+2.14
60-inch reflector.	19.5	17.4	+1.44
Means.	17.65	15.55	+1.44

If the stars of these magnitudes were no redder than those of magnitude 6.5 (visual) the numbers in the last column ought to be *zero*. Still, they are probably not to be explained altogether by the increased color index of the fainter stars. Systematic errors in the scale of Pickering's photographic Polar Sequence and in the visual determinations of Parkhurst in his *Photometric Researches*—on which the determinations of *Groningen Publication*, No. 18, depend almost exclusively for the very faint stars—may have very materially contributed to the results. Seares has made a few direct comparisons of one of Parkhurst's fields with the visual Polar Sequence (considerably prolonged by Seares himself). These certainly indicate a very considerable divergence. But even if we allow for such divergences and if we take into account what has been said in Section IV, there must still remain a considerable part of the difference found between the color indices of the stars of magnitudes 15.5 and 6.5 which is to be explained either by a preponderance of the later types among the far-away stars or by one of the causes (4) or (5).

f) Limiting magnitude of the Franklin Adams plates.—Among the data of *Groningen Publication*, No. 18 (p. 11), occur fifteen Franklin Adams plates, which have been counted on an average down to visual magnitude 13.5, Harvard scale. The average limiting magnitude cannot well be fainter by more than half a magnitude, the plates being purposely counted close to the faintest

magnitude shown. As, however, there are three plates on which the counts extended to 14.1, 14.3, 14.5, Harvard scale, I will adopt

$$\left.\begin{array}{r}\text{Vis. Mag. faintest}\\ \text{stars F. A. plates}\end{array}\right\} = 14.3 \text{ Harv. scale} = 14.96 \text{ Int. scale} \qquad (17)$$

According to a private letter from Professor Dyson, direct comparison at Greenwich of 17 Franklin Adams regions with Pickering's Polar Sequence gave in the mean:

$$\text{Phot. Mag. faintest stars on F. A. plates} = 16.51 \qquad (18)$$

The difference noted for visual magnitude 14.3 (Harvard) is +1.55 magnitudes. The possible explanations are the same as in the preceding case.

g) *Barnard's estimates* (unpublished).—In a letter to Seares, Barnard gives some estimates made by himself at the 40-inch refractor. I have added (in brackets) the photographic magnitudes given by Pickering in *Harvard Circular*, No. 170:

The faintest star I could see on an ordinary night was 19s [18.30]. It could have been measured if necessary. I am sure I glimpsed 21s [18.67]. . . . . 17s [16.97] was very noticeable. My estimate of 17s was 14½ or 15. . . . . 34 [17.11] is very easy. Estimated 15^m. I hope to get a trial on a good night.

Parkhurst's determination of the limiting visual magnitude of the 40-inch refractor, made with "good seeing," ocular 750, is 16.8 Harvard scale.[1] It is in connection with this determination that we have to take Barnard's estimates. The possibilities are as under *e*) and *f*).

h) *Hertzsprung's results* (*unpublished*).—In the summer of 1912 Professor Hertzsprung took a great number of photographs with the Mount Wilson 60-inch reflector, stopped down to 40 inches in order to increase the field. A coarse grating was placed before the opening of the tube, which of course produced spectra on both sides of the main image. The distance between the first-order spectra, which are so short that they can hardly, if at all, be distinguished from ordinary star images, is about 1 millimeter. This distance must be different for different "effective wave-lengths."

[1] *Photometric Researches*, p. 189.

Careful measurement of this distance thus furnishes a measure of the color index. Up to the present only the photographs of a single region, that around N.G.C. 1647, have been completely measured and reduced. The total number of stars is 206. The color index increases very gradually and steadily with the magnitude. From magnitude 9 to about 14.5 the increase is about 0.7 or 0.8 magnitude. Both the stars in and outside the cluster show the phenomenon.

i) Seares's result.[1]—Seares gives results for two areas, one covering the region of Pickering's Polar Sequence, the other the region round S Cygni. For the stars of these regions both the photographic and the "photovisual" magnitudes (obtained by the use of isochromatic plates and yellow filter) were carefully determined. The number of stars included in the first area is 107, in the second about 200. The color index is found for both to increase very gradually with increasing magnitude.

> The real change in the minimum index (that is, of the color index for the whitest stars) between the 6th and the 17th magnitudes, is probably about one magnitude. Beyond the 15th magnitude there appear no stars with indices less than +0.5 mag.

The change of the average color index is 0.34 magnitudes.

j) Harvard result.—The Harvard results for the Polar Sequence give no systematic difference between the photographic and visual magnitudes of the white stars from magnitude 4.5 to 13.3. It is the only case I know in which the fainter stars were not found redder.

Meanwhile it seems quite possible that this is simply the consequence of the method of reduction adopted. A full explanation has not yet been published, but the following words occur in *Harvard Circular*, No. 170:

> An absolute scale of magnitudes was derived separately from each of about 60 plates taken by the above method, *the starting-point in every case being the mean photometric magnitude of such stars in the Polar Sequence, given in Table I, as were measured on that plate.*

[1] *Mt. Wilson Contr.*, No. 81. Read at the Astronomical and Astrophysical Society meeting at Atlanta, December 1913.

The words that I have here italicized seem to indicate that the agreement of the photographic and the photometric (i.e., visual) scales is a forced one. If this is really the meaning of the words, then of course the Harvard results yield no data for the present inquiry.

VIII. EVIDENCE ALREADY EXISTING ON PHENOMENON (2)

a) *Kapteyn's first paper on absorption.*[1]—Miss Maury divides her spectral class XV*a* (=K) into two subdivisions.[2]

In the first the general absorption is slight; in the second, it is more conspicuous both in the regions of the violet above mentioned and beyond wave-lengths 3889, where the photographic spectrum generally appears to be suddenly cut off.

It is found that with this difference in the spectrum corresponds a difference in distance, which is manifest by the difference in proper motion (μ):

	Average 100 μ	Percentage of Stars with 100 $\mu > 30''$	
α Cassiopeia stars (weak in violet)	11″4 (45 stars)	0	(19)
α Boötis stars (strong in violet)	47.1 (25 stars)	48	

b) *Kohlschütter's result* (*unpublished*).—In his classification of the Mount Wilson spectra Kohlschütter sometimes noted the fact that the violet is exceptionally faint. The nine stars thus marked are listed in Table IV.

TABLE IV

Boss No.	Mag.	Spectrum	μ	Aver. μ
2335	4.70	G	0″007	0″300
3039	6.17	K	0.018	0.100
3217	5.36	K	0.002	0.130
3336	5.67	G5	0.008	0.200
3542	5.71	G	0.009	0.230
4612	5.42	B	0.005	0.023
4613	6.02	B1	0.010	0.023
5003	5.04	B	0.004	0.023
5583	5.54	A	0.026	0.056

On consulting Boss's *Preliminary Catalogue* it was found that in every case the proper motion was much below the average proper

[1] *Mt. Wilson Contr.*, No. 31; *Astrophysical Journal*, **29**, 46, 1909.

[2] *Annals Harvard College Observatory*, **28**, I, 39, 1897.

motion of the spectral class to which the star belongs (see last two columns). In fact, in nearly all cases the proper motion is quite insensible.

c) *Kapteyn's second paper.*[1]—In this paper were treated the color indices of the stars whose spectra were determined by Miss Maury and Miss Cannon,[2] and whose photographic magnitudes could be taken from the *Draper Catalogue*. The visual magnitudes were from the *Revised Harvard Photometry*. A first attempt was made to separate the influence of the two causes: absolute brightness and distance. The data, however, proved inadequate for a satisfactory discrimination. If, therefore, we try to find only the change of color index with distance, and if we call

$$\left.\begin{array}{l} d = \text{change of color index for a change of distance} \\ \qquad \text{of } 32.6 \text{ light years } (\pi = 0\overset{''}{.}1) \end{array}\right\} \qquad (20)$$

we get

$$d = +0.0031 \pm 0.0006 \text{ mag.}^{3} \qquad (21)$$

If objection is taken to the distances assumed for the derivation in accordance with the table in *Groningen Publication*, No. 8, we may write the result in terms of proper motion. In this form at least the value (21), small as it is, seems well guaranteed within the limits set by the probable error. For it was derived from stars having the same apparent magnitude and the same spectral class and differing only in proper motion. It is hard to see how systematic error, either in the spectrum or in the magnitude, could creep in. Observers of spectrum and magnitude, who at the time of their observation were certainly not aware of the amount of the proper motion, could not have introduced such error, even had they wanted to do so. Here then is a case where, if ever, we may fully trust the verdict of the probable error, and this being the case, a value over five times its probable error deserves some confidence.

[1] *Mt. Wilson Contr.*, No. 42; *Astrophysical Journal*, **30**, 284, 1909.

[2] *Annals Harvard College Observatory*, **28** and **56**.

[3] The correction indicated in *Astrophysical Journal*, **30**, 398, 1909, has been applied.

d) Van Rhyn's results for Parkhurst's stars between $\delta = +73^\circ$ and $\delta = +90^\circ$ (unpublished).—The data here are somewhat scanty, but of much better quality than those available in the preceding investigation. As yet Van Rhyn has used only those of Parkhurst's stars which are in Boss. By adding those for which good proper motions are available in Groombridge and Carrington the material will be increased in the ratio of 2 to 1.[1] The results thus far are shown in Table V.

TABLE V

P.M.	Average P.M.	Average Mag.	Sp.	Average Color Index	No. Stars	Average π
<0".050.....	0".026	6.0	A3	+0^{m}12	33	0".0070
>0.050.....	0.100	5.8	A3	+0.09	24	0.0187
<0.150.....	0.078	6.2	F3	+0.46	10	0.0150
>0.150.....	0.263	5.7	F3	+0.36	8	0.0373
<0.100.....	0.047	5.8	G3	+0.96	20	0.0108
>0.100.....	0.218	5.5	G3	+0.89	10	0.0335
<0.068.....	0.034	5.6	K4	+1.56	21	0.0088
>0.068.....	0.103	5.1	K4	+1.39	6	0.0204

The values of π were found from the magnitudes and proper motions by the table for "all stars" in *Groningen Publication*, No. 8. From these data I find, assuming that the factor required for passing from the average parallaxes to average distances is the same as in *Astrophysical Journal*, **30**, p. 398:

$$d = +0.0050 \pm 0.0019^2 \qquad (22)$$

The result is decidedly confirmatory of (21). Combining the two we have:

$$d = +0.0033 \pm 0.0005^5 \qquad (23)$$

the probable error now being only a sixth of the amount.

[1] As this paper is going through the press, Van Rhyn communicates, as the provisional result of both Boss and Groombridge stars,

$$d = +0.0079 \pm 0.0023$$

[2] A more refined solution will lower this probable error. For it has been shown in my second paper that the values yielded by the spectra B to F must be increased, those found by the spectra G to M diminished. It is evident by simple inspection of Table V that a correction in this sense must greatly increase the agreement of the four separate results.

e) The results of *a* and *b* rest on spectra taken on different plates, at different times, which were separately developed. It is well known that under these circumstances photographs of the same star will often show great differences in the general absorption in the violet.

In a refined investigation it is necessary to compare stars of nearly the same magnitude, taken at the same zenith-distance, on the same plate. Some work in this direction was done by Van Rhyn at Mount Wilson. He photographed on one plate the spectra of two stars, of which the one has a large, the other a small, proper motion. Of the six pairs taken, three show the small proper motion star to be decidedly weak in the violet. The pairs are: Boss 3524 and 2750 (Sp. F8.); Boss 4042 and 4228 (Sp. Go); Boss 3922 and 4032 (Sp. Ko).

f) *Adams' work.*—The matter was taken up very thoroughly a few months ago by Adams. He compared the spectra of stars of great and small proper motions in the way just described. Of the results it is stated:[1]

> Out of 20 pairs of stars investigated, two pairs are of type B8, one Ao, one F4, one F7, two G5, two G6, one G8, seven Ko, one K2, one K4, and one K6. Of these, the pairs of stars of types B8, Ao, and F4 show no appreciable relative difference between the two ends of the spectrum, and the same is true of one pair of type G6 and one of type Ko. The remaining fourteen pairs all show a marked difference, which in some cases is very great. In every case the star which is relatively faint in the violet end of the spectrum is the star of small proper motion . . . . in no case is the more distant star relatively stronger in the violet portion of the spectrum.

The phenomenon is demonstrated *ad oculos* by the reproduction of the spectra of five pairs of the observed stars.

Afterward Adams made a comparison—as yet unpublished—at the two ends of the spectrum of those stars of large and small proper motion, which have been observed for radial velocity on Mount Wilson. Photographs taken at great zenith-distances and on hazy nights have been rejected. The results are as contained in the following table, which gives the differences: density for stars of large proper motion *minus* density for stars of small proper

[1] *Mt. Wilson Contr.*, No. 78; *Astrophysical Journal*, 39, 89, 1914.

motion at the wave-lengths indicated, the density at the red end of the spectrum having been reduced to equality for all.

TABLE VI

Type	Average Proper Motion		Difference in Density at		
			λ 4195	λ 4220	λ 4265
F0—F9	0″.66 (23 stars)	0″.012 (10 stars)	0.07	0.07	0.08
G0—G4 . . .	0.64 (30 ")	0.009 (8 ")	0.11	0.11	0.11
G5—G9 . . .	0.64 (22 ")	0.011 (14 ")	0.17	0.16	0.14
K0—K4 . . .	0.70 (20 ")	0.011 (24 ")	0.16	0.15	0.15
K5—K9 . . .	2.18 (5 ")	0.012 (5 ")	0.13	0.12	0.11

Adams thinks that the progression in the differences of density for F0 to K4 is probably real.

IX. FINAL REMARKS

a) Though many of the investigations summarized in what precedes are only in their beginning, the conclusion seems already very strong that both the phenomena (1) and (2) are real. Only in the latter it is provisionally necessary to read "are redder *on the average*, the farther away they are," the words in italics being added to the original formulation.

b) The various methods have their own peculiar advantages and disadvantages. The most direct and at the same time the most sensitive method appears to be that followed by Adams. It has the disadvantage of being, provisionally at least, restricted to the brighter stars, that is, to those in which in general any distance effect must be, relatively speaking, little pronounced. Meanwhile we may hope to get down to much fainter stars by diminishing the dispersion and using an objective prism or an equivalent arrangement on the reflector.

With the method followed by Seares very faint stars are reached. The interpretation of the results, however, is complicated by a lack of knowledge of the spectra.

If the relative frequencies of the several spectral classes among the fainter stars were known, the two methods would help each other in discriminating between the two causes (4) and (5). For suppose we found that this frequency is the same for the stars of

all magnitudes. Then from the two phenomena (1) and (2) we should conclude that absorption (scattering) must be the main cause and not absolute magnitude. To see this, suppose for an instant that absolute magnitude were the only factor. From the phenomenon (1) we should conclude that the absolutely *fainter* stars must be the redder. For it cannot well be doubted that the apparently fainter stars are also, on the average, the less luminous. From the phenomenon (2), on the other hand, it would follow that the absolutely *brighter* stars must be the redder. As the two are contradictory, we conclude that the premise must be wrong. Absolute magnitude could not, therefore, be the only, or even the main, cause.

c) For this separation of the effects of distance and absolute magnitude little has been done up to the present. Adams is working on the problem;[1] his results are still inconclusive, owing partly to the short time as yet devoted to the matter, and partly to the difficulty of finding suitable objects in sufficient number.

d) May not another phenomenon exist, which, side by side with other advantages, will in many cases permit an immediate separation of the two causes? In *Contributions from the Mount Wilson Solar Observatory*, No. 31, p. 3,[2] the question was put whether there might not be a gaseous absorption. In a private letter Adams states that there seem to be indications of a strengthening of the hydrogen lines in the spectra of many of the distant stars of the later types. If further investigation confirms these indications there will be another case that may be conceived as an effect either of distance or of absolute magnitude. The two will, however, now be separable, at least in many cases. For any distance effect must be explained by the presence of hydrogen in interstellar space. Now, as generally the radial velocity of the gas and the star will be different, there will be an unsymmetrical widening of the hydrogen lines, which must give rise to a displacement of the centers of these lines, different from that of the purely stellar lines; conversely such a difference will prove the distance effect, that is, the absorption or scattering of light in space.

[1] *Loc. cit.*

[2] *Astrophysical Journal*, **29**, 48, 1909.

In this summary of what has been done up to the present, I have deemed it unnecessary to enter into details of investigations already published. For the still unpublished data somewhat more detail was obviously desirable. It must be evident how greatly the latter have contributed in establishing the reality of the phenomena under consideration. I feel the deepest obligation to the astronomers who have helped me in my task by permitting me to use their results.

GRONINGEN
February 1914

Addendum to KAPTEYN, "On the Individual Parallaxes of the Brighter Galactic Helium Stars, ..." *Astrophysical Journal*, **40**, 43, 1914.[1] After line 6, p. 77, insert:

For 7 of the remaining stars in List 3 ($100\ \mu \leqq 1''.6$), viz., Boss 2187, 2342, 2217, 2267, 2408, 2575, 2880,[2] for which the divergence $p_o - p_c$ exceeds 50°, the probability is very great that the excessive values are caused in great part by observation error. It is for this reason that, before computing the parallax, I diminished the value of $p_o - p_c$ by one-third of its amount. This certainly is a somewhat arbitrary way of dealing with the matter, but it is to be remarked:

a) That the adopted change in the position angle is, in all cases but one, smaller than its probable error.

b) That, most likely, the procedure must bring us nearer to the truth.

c) That the value of the parallaxes of the stars in question—whether we take the corrected or the uncorrected values—is next to nothing, as the probable errors range from 0.48 to 1.06 times the whole amount of the parallax.

[1] *Mt. Wilson Contr.*, No. 82.

[2] For Boss 2880 instead of $p_o - p_c = -40°$, read $-60°$.

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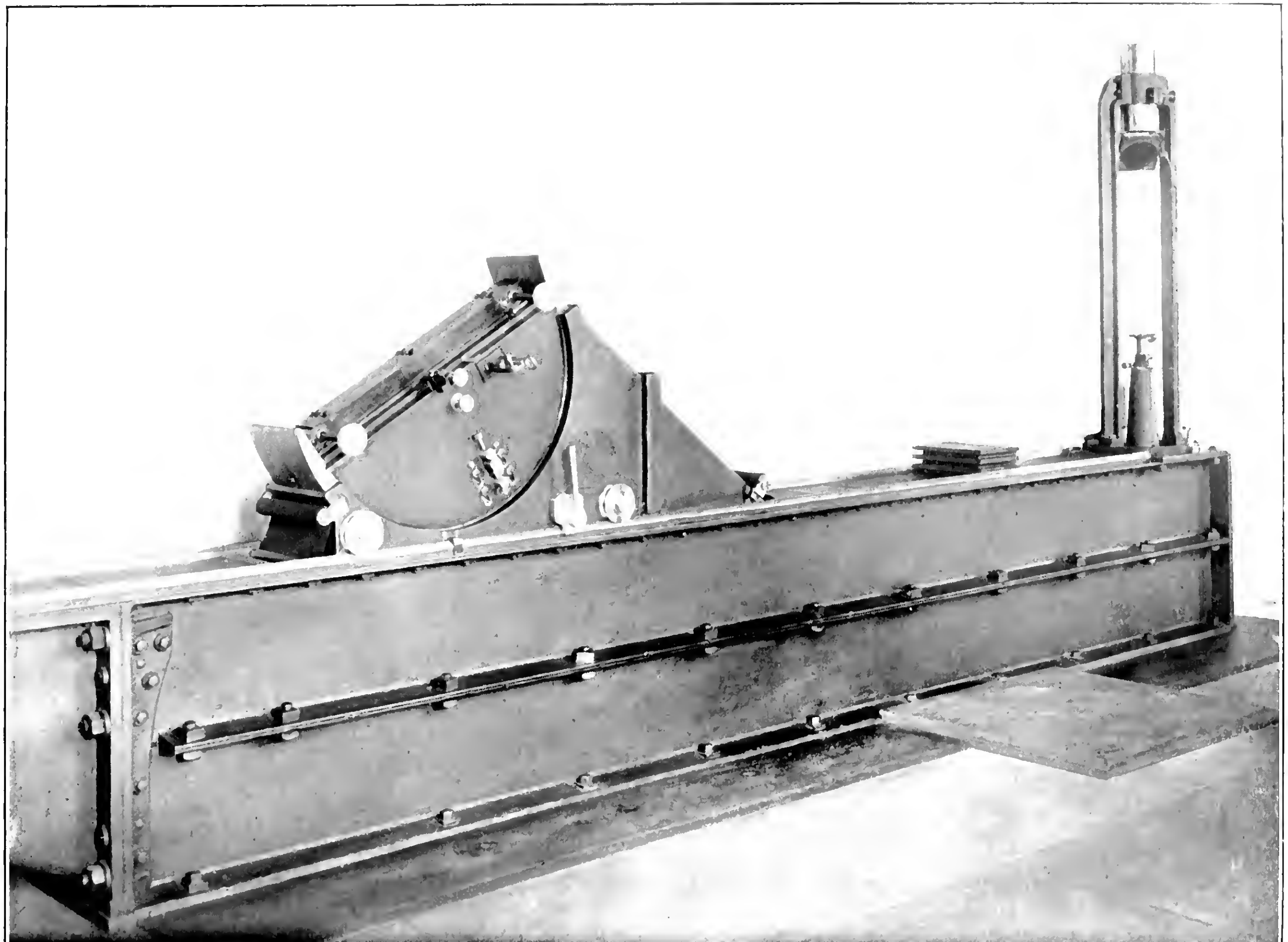

Vertical Spectrograph for Concave Grating

A VERTICAL ADAPTATION OF THE ROWLAND MOUNTING FOR A CONCAVE GRATING[1]

By ARTHUR S. KING

A concave-grating spectrograph recently mounted in the Pasadena laboratory of the Mount Wilson Solar Observatory embodies some new structural features which will be described in this paper. It provides for a grating of 15 ft. (4.57 m) radius, and, optically considered, is the Rowland mounting with the plane of the focal circle vertical. This permits placing the grating in a pit beneath the laboratory floor, thereby obtaining the constancy of temperature which has proved highly advantageous with plane-grating spectrographs, while the slit and the plate-holder are at a convenient height above the floor. The apparatus is so constructed that it may be operated in a fully lighted room and occupies a minimum of floor space for an instrument of this kind.

The portion of the spectrograph above the laboratory floor is shown in Plate VI. The plate-holder moves on a horizontal track, 50.5 cm above the floor, supported by a frame of channel iron placed over a slot in the cover of the pit used for the vertical Littrow spectrograph. When the plate-holder carriage is as near the slit as possible, the spectrum may be photographed as far as λ 2000 in the first order. When at the extreme end of the track, λ 7000 in the second order is reached, the third order being thus covered for the range to which ordinary plates are most sensitive.

The instrument is shown in elevation in Fig. 1. The frame carrying the two vertical rails is of angle-iron fastened firmly to the concrete wall of the pit. The grating-holder is a cast-iron box, with an extension at one side which is bolted to the web of the I-beam girder, 10 cm wide, connecting with the plate-holder carriage. This girder extends in a line with the side of the grating box for 185 cm, then offsets 6.5 cm inward in order that the web may align with the inside of the plate-holder carriage to which the upper end of the girder is fastened. Within the grating-holder is an iron plate 18.5×27 cm on which the grating rests, the latter

[1] *Contributions from Mount Wilson Solar Observatory*, No. 84.

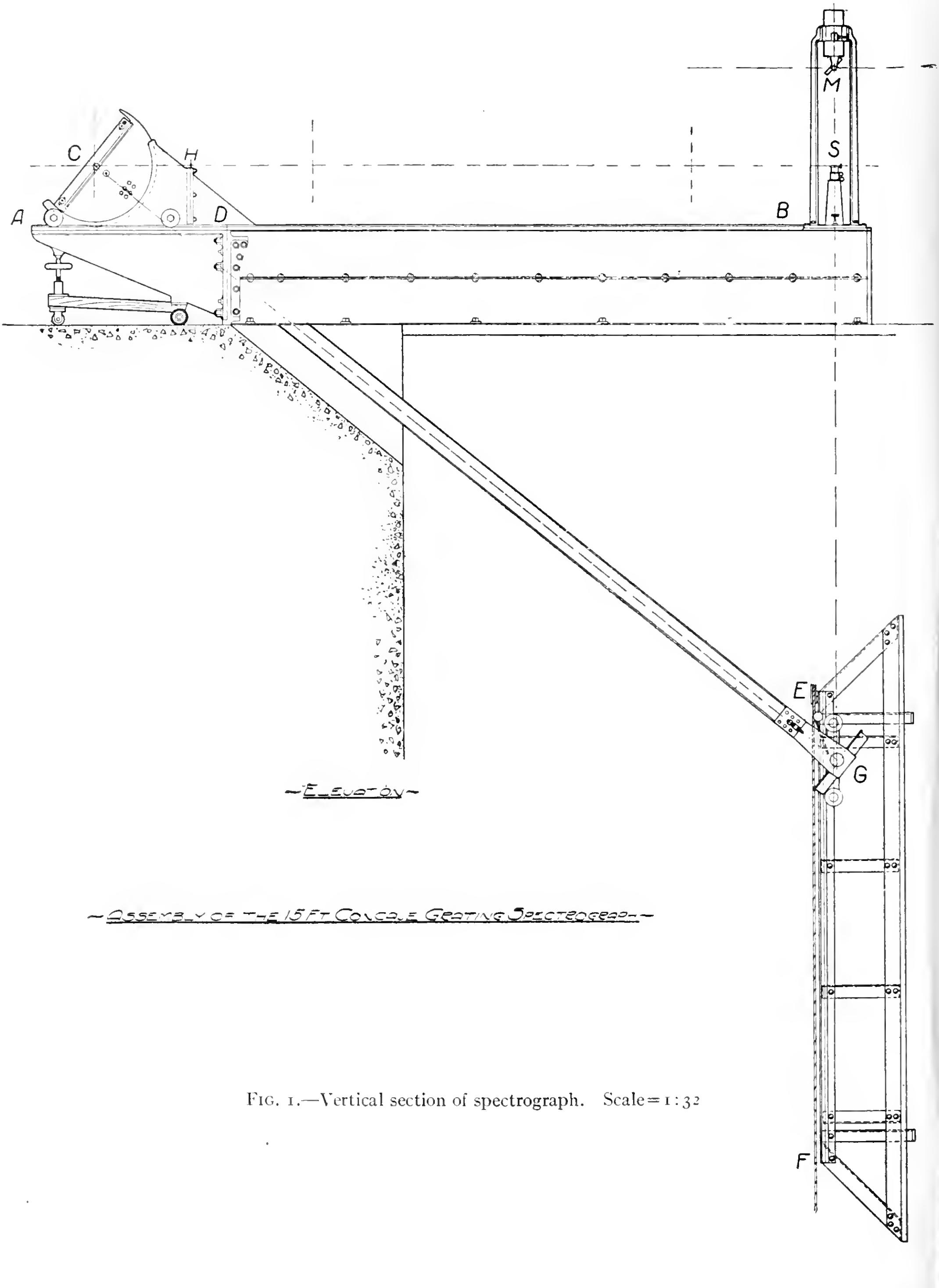

FIG. 1.—Vertical section of spectrograph. Scale = 1 : 32

being held in place by four brass strips screwed to the plate. At each corner of the plate are push-and-pull screws, which serve to adjust the inclination. The grating is centered and adjusted for orientation by screws passing through the inclosing brass strips, their ends resting lightly against the edge of the grating.

In this mounting, a counterweight for the grating carriage is required, and must be of variable weight, since, as the plate-holder carriage is moved outward from the slit, the pull exerted in raising the grating becomes more nearly normal to the vertical track. Steel cables are fastened to each side of the grating box (see Fig. 1), pass at an angle to small idlers and thence over pulleys at *E* to the counterweight below *F*. The lower portion of the system is shown in Fig. 2. *W* is a mass of iron forming the constant weight, and below this is suspended a cylindrical tank *T*. From the bottom of *T*, a flexible metal tube passes to a fixed reservoir *R*, containing a quantity of heavy oil and suspended below a platform used for the adjustment of the spectrograph. The height of this reservoir is so adjusted that when the grating is on the lower portion of its track, where only the weight *W* is needed, *T* is so high that all of the oil is in *R*. When the grating rises to the point where a larger counterweight is required, the descent of *T* causes oil to run into it from *R*, the weight increasing in the proportion needed to facilitate the movement of the grating as it is pulled to the top of its track.

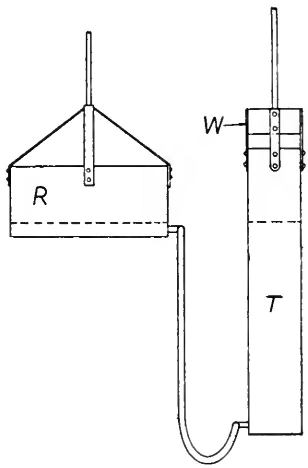

FIG. 2.—Variable counterweight system for grating-holder.

The slit is at the end of the horizontal track and vertically above the grating. A hollow iron casting has at its upper end a brass collar which in turn holds the slit-tube and permits the regulation of the height of the slit. A key-way prevents turning within the collar, the orientation of the slit with respect to the grating rulings being adjusted by turning the collar within the iron casting. This

movement is controlled by a tangent screw, and the angle, marked by a scale and pointer, may be read to 0°05. The slit was made by Hilger. Its divided head reads to 0.005 mm and its length is adjustable to 18 mm by means of a wedge-shaped opening. As the light from a source in the laboratory must in general be reflected to the slit, a holder for a mirror or a total reflecting prism is mounted as shown in Fig. 1.

The plate-holder carriage (Plate VI and Fig. 1) consists of a hollow iron casting moving on four grooved wheels. The sloping extension toward the slit, usually required in order that light may reach all parts of the plate, can be replaced by a vertical iron plate (at *H*, Fig. 1) when a close approach to the slit is desired. The opening between the tracks is rendered light-tight by overlapping boards and felt curtains attached to the ends of the carriage by means of spring rollers.

The side walls of the carriage are cut away along a circle as shown in Fig. 1, and a ledge at each side, 2.5 cm below the edge of this opening, supports a rotating section to which the girder is attached and on which the plate-holder lies. This is shown in section, with side and end elevations, in Fig. 3. The **I**-beam girder connecting the plate-holder with the grating carriage is attached to the inside of the rotating section by four bolts in slotted holes, and the distance from plate-holder to grating may be altered by means of the push-and-pull screws at *J*.

The plate-holder is carried on a sliding brass bed above an opening 4×52 cm in the top of the rotating section of the carriage and moved by means of the screw *F*. Ledges projecting from each side of the casting (see section at right of Fig. 3) carry a shutter used to limit the width of spectrum admitted to the plate. Hinged connections at the ends of the blades of the shutter (that at one end being shown in the top plan of Fig. 3) enable them to be brought into contact or separated to a distance of 3.3 cm by moving the handle *B* in or out. The whole shutter, with its blades at a fixed distance apart, may be moved across the spectrum by means of the screw *C*, so that in addition to regulating the length of spectrum lines in ordinary photographs, successive spectra for close comparison of wave-lengths may be taken by moving the shutter as a

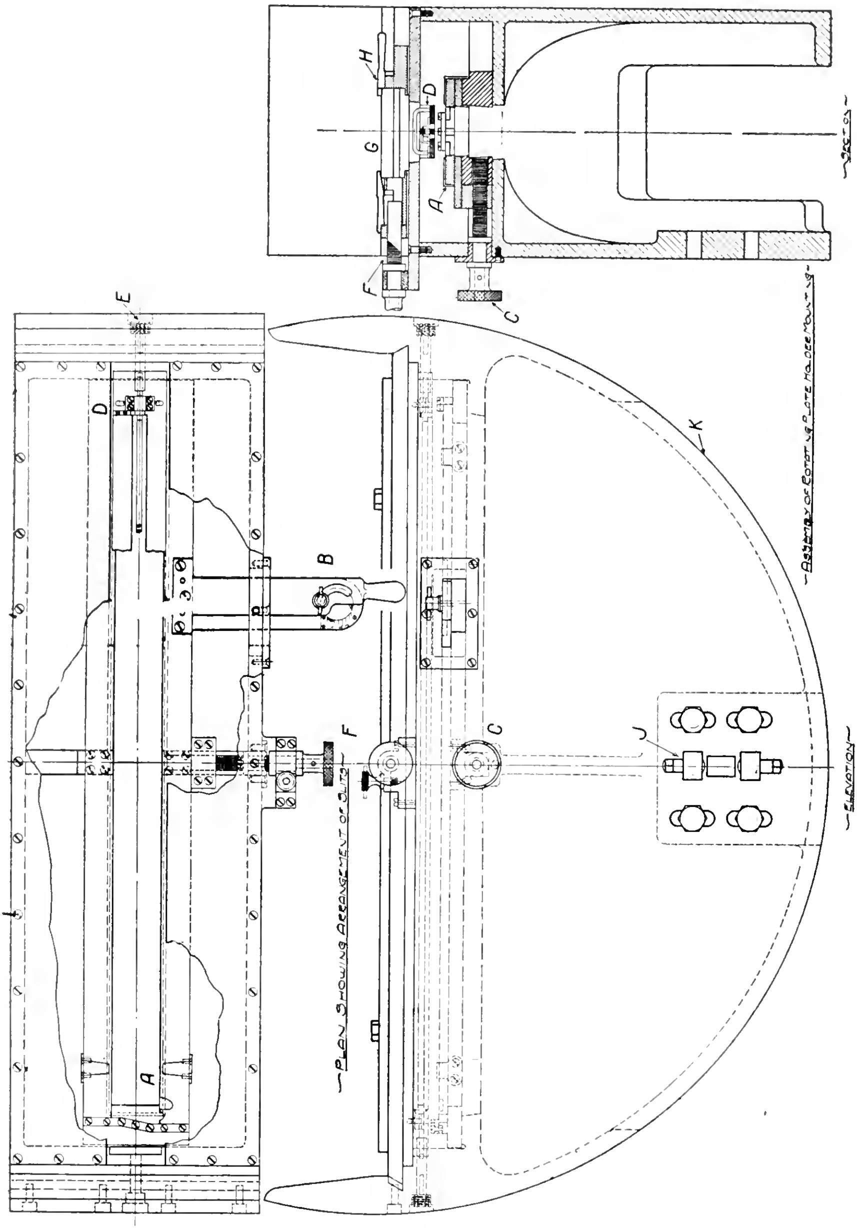

FIG. 3.—Sections of plate-holder carriage. Scale = 1:5

whole, or a narrow strip of spectrum may be superposed on any part of a wide strip.

A further attachment for photographing parts of a spectrum on each side of another spectrum without risk of instrumental shift consists in a brass plate which may be placed just above the shutter. A slot contains a rectangular brass frame which may be rotated through 90° about its longitudinal axis, admitting first a strip of spectrum in the middle and then one at each side. The rotation is accomplished by means of a key inserted at *E* through a hole in the rotating section of the carriage. If the hole at one end is covered owing to the position of the carriage on the track, a similar hole at the other end will be exposed. When the key is not in use, the hole is closed by a screw plug.

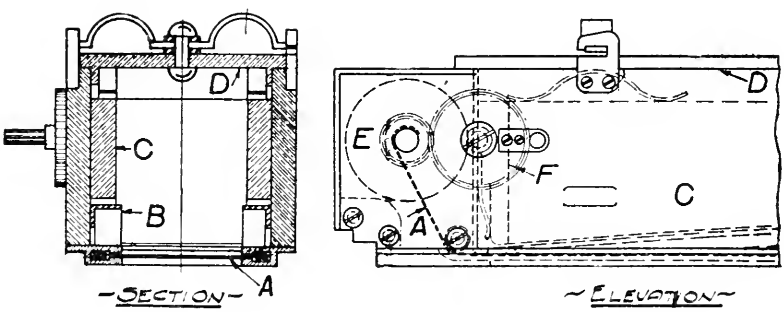

FIG. 4.—Design of holder for photographic plate. Scale = 1 : 2

The plate-holder is of brass and fits into the sliding bed already described. A ledge *B* (see Fig. 4), curved to a radius half that of the grating, supports a photographic plate 4.7×46 cm which is bent into position by the rectangular brass frame *C* held down by springs attached to the under side of the cover *D* when the latter is clamped in position. In case a celluloid film is used, the ends may be inserted under spring clamps at each end of *C* which hold the film stretched while *C* is put in position.

Instead of a sheet-metal slide in front of the plate, the movement of which would be interfered with by the construction of the carriage, a strip of opaque black cloth is used, whose edges run in deep grooves in the face-plate of the holder (*A*, Fig. 4). Tapes attached to the edges of the cloth connect with spools, *E*, at the

ends of the plate-holder. A geared wheel *F*, attached to a pin rotated by hand, permits each spool to be turned rapidly in opening or closing the plate-holder.

The mounting retains all of the good features of the regular Rowland form. In addition there is a great gain in the excellent temperature control given by the pit. The grating and the connecting girder, the two parts most sensitive to temperature changes, are protected to an extent difficult of attainment in a laboratory room. During the two months following the completion of the instrument the temperature at the level of the grating changed but 1°5 C. Thermometers hung at the upper and lower ends of the grating's run have never shown a difference of more than 0°2 C. and often agree within less than 0°1. The rigidity of the apparatus has been tested several times by furnace exposures lasting three hours or more, and by one photograph of the iron arc spectrum which was divided over a seven-hour period, extending from morning till evening, the exposures being made at the beginning, middle, and end of the time. No difference in definition could be detected between this divided exposure and short exposures on adjacent portions of the plate taken at the beginning and end of the test.

As a minor advantage, this form of mounting requires but little floor-space, a narrow space against a wall, or a hallway, being sufficient. As no darkening of the room is required, the operation of the instrument need not interfere with other work being carried on in the laboratory.

The reflection of light to the slit from a source under examination, while usually convenient, is not always necessary. Thus a vacuum tube or an arc or spark used horizontally may be placed directly over the slit. When this is not feasible, the observer will select the reflecting surface giving the greatest efficiency for the region of spectrum under examination. Mirrors of silver and of speculum metal and right-angled prisms of glass and of quartz have been used thus far.

The grating used is one of exceptional quality, ruled by Dr. J. A. Anderson on the Rowland machine. The ruled surface is 5.1×10.6 cm, with 590 lines to the millimeter. While the light

is largely in the first order on one side, the second and third orders are sufficiently bright to give good efficiency for the range allowed by the spectrograph.

For aid in planning the instrument, the writer is greatly indebted to Mr. Pease, who designed the main structural features, and to Mr. Nichols, of the draughting department, who introduced numerous devices which have added to the convenience of operation. The construction in the machine-shop and the mounting in the laboratory have been under the care of Mr. Ayers and Mr. Shumway, who have spared no pains to make the parts of the spectrograph accurate and easy of operation.

Mount Wilson Solar Observatory
January 26, 1914

SOME ELECTRIC FURNACE EXPERIMENTS ON THE EMISSION OF ENHANCED LINES IN A HYDROGEN ATMOSPHERE[1]

By ARTHUR S. KING

The following experiments have been carried out in order to test the hypothesis that the presence of hydrogen may facilitate the emission of those lines which are intensified in the spectrum of the electric spark. That hydrogen may have such an influence was suggested by the experiments of Crew,[2] confirmed later by Hartmann,[3] on the arc in a hydrogen atmosphere, which has a distinct effect in strengthening the enhanced lines of metals. It is known, however, that the arc, with either metallic or carbon terminals, is maintained with much more difficulty in hydrogen than in air, the same length of arc requiring higher potentials.[4] In a later work, Crew[5] showed by graphical methods that a higher potential gradient was produced by the presence of hydrogen around the arc, which thus furnished an approach to the conditions of the spark discharge. Although the evidence has thus indicated from the first that the action of hydrogen was to alter the discharge conditions, occasional reference has been made, the latest being in a paper by S. A. Mitchell,[6] to the possibility that hydrogen in stellar atmospheres affects by its presence in some unknown manner the emission of enhanced lines by metallic vapors.

The arc is obviously unsuited to test this question, on account of the changes mentioned. The electric furnace, on the other hand, seems especially adapted, since the graphite tube, when raised to a given temperature, may be expected to perform its functions as an exciting source independently, to a large degree, of the surrounding

[1] *Contributions from Mount Wilson Solar Observatory*, No. 85.

[2] *Astrophysical Journal*, **12**, 167, 1900.

[3] *Ibid.*, **17**, 273, 1903.

[4] See C. D. Child, *Electric Arcs*, p. 83.

[5] *Astrophysical Journal*, **20**, 274, 1904.

[6] *Ibid.*, **38**, 407, 1913.

gas, leaving the effects of mixed vapors, density, and total pressure to produce any effects that may be observed.

The experiments have included the production of the enhanced lines with the furnace in a partial vacuum and in hydrogen at varying pressures up to one atmosphere, and the use of greatly differing amounts of titanium. Thirty-three spectrograms were made, each of the important conditions being tested at least twice and in some cases several times.

EXPERIMENTAL METHOD

The tubes used in the electric furnace were of specially purified graphite, 12.5 mm inside diameter, 19 mm outside diameter, and 30.5 cm long, and were charged with pulverized titanium carbide. The heated portion of the tube was inclosed by a protecting tube of graphite of 3.2 cm internal diameter, which prevented a rapid loss of heat by the furnace tube. A potential of 25 volts with a current of about 1500 amperes brought the tube to a temperature close to 2600° C. in one minute, with little change in temperature thereafter, even when the hydrogen was at atmospheric pressure. A Gaede oil pump reduced the furnace chamber to a pressure of less than 1 mm. Observations were made for this pressure when the residue was air and also when the furnace had previously been filled with hydrogen. In both cases, a considerable quantity of occluded gas was driven off, and at low pressure the pump was not able to remove this fast enough, the pressure gradually rising during the run of the furnace to about 10 mm, which is the lowest final pressure used in the experiments. Various initial pressures of hydrogen up to one atmosphere were used, and, provided the pressure desired was as much as 10 mm, it was possible to secure constancy by frequently opening the connection with the pump. The hydrogen was prepared from zinc and sulphuric acid and passed through vessels containing sulphuric acid and caustic potash. Before a trial with hydrogen, the furnace was usually flushed three times with the gas, and again pumped out, after which hydrogen was admitted to the desired pressure.

The spectra were photographed with the second order of a concave grating of 15 ft. (4.5 m) radius, mounted in the vertical

spectrograph.[1] The region covered was from λ 3900 to λ 4700 and all adjustments of the spectrograph remained the same throughout the series. Seed "23" plates were employed, giving sufficient contrast to bring out the faint enhanced lines in spite of the rather strong continuous ground given at the high temperature. The exposure times ranged from one to two minutes according to whether or not the tube and graphite protector were hot from a previous run.

The titanium enhanced lines brought out most distinctly by the furnace were λλ 4300.211, 4395.201, 4443.976, 4468.663, 4501.448, 4534.139. These are the strongest of the enhanced lines in this region and their behavior may be taken as typical of the group. Several plates were first taken with the furnace chamber pumped out, which confirmed the previous observations[2] as to the appearance of the enhanced lines in the high-temperature spectrum. The series of photographs with a hydrogen atmosphere was then begun with initial pressures at less than 1 mm and at 5 mm, the pressure rising during the run as has been noted. These were followed by plates for which the pressure was held constant at 10, 20, 40, 100, 200, 400 mm, and finally at atmospheric pressure. In case several pressures were used successively with the same tube, the chamber was pumped out and fresh hydrogen admitted between successive exposures. Tests with the same pressures in a different order were then made with a new tube. Pyrometer measurements were taken regularly and during the latter half of the exposure the readings under all conditions indicated a temperature differing not more than 50° from 2600° C.

RESULTS

1. *Effect of hydrogen at various pressures.*—No distinct effect on the intensity of the enhanced lines resulted from the use of hydrogen up to a pressure of 100 mm. At 200 mm, a weakening was perceptible, which became quite distinct at 400 mm, while five photographs in hydrogen at atmospheric pressure showed that the limiting pressure had been almost reached for enhanced lines

[1] *Mt. Wilson Contr.*, No. 84; *Astrophysical Journal*, **40**, 1914.

[2] *Mt. Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

to appear at 2600° C., only faint traces being seen of the two strongest lines in the group. The efficiency of the furnace in emitting the arc lines appeared to be practically unchanged by the hydrogen atmosphere. The same exposure times were used throughout, and the titanium arc lines came out strongly, even increasing in general intensity as the pressure of hydrogen rose, an effect due largely to a general widening of the arc lines at higher pressures and to more numerous reversals. The enhanced lines do not seem to be especially sensitive to changes in the amount of hydrogen present as long as the pressure is low, but increasing the pressure from 100 mm up to atmospheric pressure exerts a progressive effect in suppressing them.

2. *Effect of vapor-density.*—The influence on enhanced lines of the amount of metallic vapor present has frequently been considered in studies of the arc and spark; but in these sources a change in density of the vapor results in altering the character of the discharge. The conditions in the furnace should remain more nearly constant with varying amounts of vapor.

With this point in view, experiments were made with less than 0.01 gm of powdered titanium and with the regular charge of from 1.5 to 2 gm. The chamber was pumped out and held by the pump to a low pressure in each case.

As nearly as could be judged, the enhanced lines maintained the same intensity relatively to the arc lines whether a large or small quantity of titanium was used. Certainly there was no difference in favor of the smaller amount. The whole spectrum was stronger at higher vapor-density, with more continuous spectrum, but, allowing for this, the intensity of the enhanced lines appeared to be little influenced by changes in the amount of vapor.

DISCUSSION

The experiments indicate that neither the presence of hydrogen at low pressure nor the density of the radiating vapor materially affects the strength of enhanced lines, but that the total pressure is very important. The lines of the arc spectrum are not thus affected by the pressure, as they maintain their strength, with increasing ease of reversal, up to atmospheric pressure in hydrogen,

and experiments[1] have shown that a rich spectrum of titanium arc lines is given by the furnace with pressures as high as 16 atmospheres. The evidence thus indicates that if the vapor is raised to a sufficiently high temperature in the furnace the arc lines will appear, while experiments with the furnace and with the tube arc and the ordinary arc at low pressure have shown that the enhanced lines are brought out best in a partial vacuum, a condition favorable to all of the phenomena of electro-luminescence.

The probable manner of the production of enhanced lines in stellar atmospheres appears much clearer since the discharge of electrons from hot bodies has been investigated for the relatively low temperatures of the electric furnace, this action having been shown by Harker and Kaye[2] to be strong at atmospheric pressure and by the writer[3] to persist at much higher pressures. The high stellar temperatures, especially when combined with low pressure, may thus be expected to duplicate the electronic speeds obtained in the arc and spark by steep potential gradients, a condition established by much evidence as favorable to the enhanced lines.

The relative strength of enhanced lines in the chromosphere would thus seem to follow from the rarefied condition at these levels of the solar atmosphere, which allows a high speed to be retained by the electrons expelled from the heated matter below and striking any vapor particles which may be present above. The conditions of rarefied vapor and presumably lower temperature are unfavorable for the arc lines, which are relatively weak in the chromosphere.

This point of view leaves the strength of enhanced lines as a valuable criterion for the temperatures prevailing in stars and in different regions of the solar photosphere at the levels where the electrified particles are produced. Thus the reduced strength of enhanced lines in sun-spot spectra seems valid as evidence of a lower temperature for those regions, since such a reduced temperature, if the pressure is not materially different, would result in the production of lower-speed electrons and in reducing the velocity

[1] *Mt. Wilson Contr.*, No. 60; *Astrophysical Journal*, **35**, 183, 1912.

[2] *Proceedings of the Royal Society*, **86** A, 379, 1912.

[3] *Mt. Wilson Contr.*, No. 73; *Astrophysical Journal*, **38**, 315, 1913.

of those drawn in from the surrounding regions. The view involves no contradiction to the hypothesis advanced by Mr. Hale[1] that electrons are drawn in by solar vortices so as to be very numerous in sun-spots, as the production of enhanced lines appears to be governed by the speed and not by the number of the electrons.

SUMMARY

1. The experiments here described have failed to show any effect of a hydrogen atmosphere in strengthening enhanced lines. They appear in the furnace at low pressures with equal ease whether hydrogen is present or whether the furnace contains a residue of air.

2. Widely different amounts of titanium vapor at low pressure and the same temperature have shown no material effect on the relative intensities of enhanced lines.

3. Increasing the pressure of hydrogen, the temperature being held as nearly constant as possible, causes a progressive weakening of the titanium enhanced lines, until at atmospheric pressure only traces of the strongest are visible in the furnace spectrum.

Mount Wilson Solar Observatory
March 21, 1914

[1] *Mt. Wilson Contr.*, No. 71; *Astrophysical Journal*, **38**, 27, 1913.

INTERMEDIATE DEGREES OF DARKENING AT THE LIMB OF STELLAR DISKS WITH AN APPLICATION TO THE ORBIT OF ALGOL[1]

By HARLOW SHAPLEY

Examples have been given[2] by the writer of the solution for the orbital elements of eclipsing binaries on the hypothesis that the stellar disks are darkened to zero at the limb according to the cosine law given in a later paragraph. Such a degree of darkening for light-emission in the visual part of the spectrum is somewhat in excess of that measured on the sun. If we assume an intermediate degree of darkening, for instance, one that would make the brightness at the limb one-third or two-thirds that of the center, it is not difficult to derive the orbital elements from a light-curve when the orbit has already been computed on the limiting assumptions of uniformly luminous and completely darkened disks. In fact, it will be shown that the interpolation of "intermediate" solutions will be sufficiently precise when uniform and darkened[3] elements are known.

The method of treating this problem has been outlined in a former paper.[4] In this note the details of numerical solutions are omitted and the results only are presented in order to show in what manner the various sets of elements differ from each other, and to what extent the orbit of a well-observed star is uncertain because of our lack of knowledge of the amount of atmospheric absorption on stellar surfaces.

I have undertaken for this investigation a discussion of the orbit of Algol, partly because of the intrinsic interest of the system and partly because of the high quality of the light-curve by Stebbins.[5]

[1] *Contributions from Mount Wilson Solar Observatory*, No. 86.

[2] *Astrophysical Journal*, **36**, 269, 1912; **37**, 154, 1913.

[3] "Darkened" used in this sense means "completely darkened," that is, darkened to zero at the limb.

[4] *Astrophysical Journal*, **36**, 401, 1912.

[5] *Ibid.*, **32**, 189, 1910.

A number of more or less extensive memoirs have been written concerning Algol's light-curve and photometric orbit, notably those by Pickering,[1] Harting,[2] Scheiner,[3] Pannekoek,[4] Rödiger,[5] and Stebbins.[6] All of the early solutions for orbital elements naturally neglected the secondary minimum and the reflection effect, and consequently it is not surprising that the photometric orbit derived by Stebbins and those presented in this note should be radically different from all that precede. The orbit of the system is, however, as yet by no means definitely solved, notwithstanding the enormous amount of labor—photometric, spectroscopic, and computational—that has been devoted to it, and the high accuracy with which the light-curve is now known. Stebbins' solution was based on the assumption of uniformly luminous disks and gave as one result that the faint companion is larger than its primary. My uniform solution, derived by another method, gives, as would be expected, elements practically the same as those obtained by Stebbins; but making the better assumption that the stars are considerably darkened at the limb, the size of the two components is reversed, the fainter one becoming the smaller.

It will be seen in Table I, which contains the elements computed for various degrees of darkening, that the primary star increases in relative size as we increase the darkening coefficient x, becoming just equal to its faint companion for $x=2/3$, that is, when the distribution of luminosity is similar to that of the sun. The adopted law of darkening at the limb referred to above is

$$J=J_0(1-x+x \cos \gamma),$$

where J is the apparent brightness of any point on the disk, and γ is the inclination to the line of sight of the normal to the stellar surface.

[1] *Proceedings of the American Academy of Arts and Sciences*, **16**, 1, 1880.

[2] *Untersuchungen über den Lichtwechsel des Sternes β Persei*, Munich, 1889.

[3] *Untersuchungen über den Lichtwechsel Algols*, Bonn, 1882; *Populäre Astrophysik*, p. 625, 1908.

[4] *Untersuchungen über den Lichtwechsel Algols*, Leiden, 1902.

[5] *Untersuchungen über das Doppelsternsystem Algol*, Königsberg, 1902.

[6] *Op. cit.*

The recent spectroscopic work at the Allegheny Observatory indicates that the orbit of Algol is sensibly circular.[1] The value of the reflection effect found by Stebbins is adopted for this work, and the intensities given by him have been "rectified" to allow for this rarely observed phenomenon so that the light-curve, for the purpose of study, is of the normal eclipse form. The loss of light due to primary eclipse is 0.643 in the unit of the maximum light of the system; the value for the secondary is 0.055. In addition to those given in the summary two other solutions might be mentioned.

TABLE I

SUMMARY OF SOLUTIONS

Elements	Stebbins	Degree of Darkening			
		Uniform	Intermediate		Complete
Darkening coefficient	0	0	1/3	2/3	1
Ratio of radius bright star to radius faint star	0.877	0.915	0.961	1.008	1.052
Fraction of the light of bright star eclipsed at primary	....	0.709	0.703	0.698	0.695
Radius bright star*	0.210	0.208	0.219	0.230	0.241
Radius faint star*	0.239	0.228	0.228	0.228	0.229
Inclination of orbit	82°18′	83°2′	82°56′	82°51′	82°35′
Light of bright star	0.897	0.907	0.916	0.922	0.926
Light of faint star (bright side)	0.103	0.093	0.084	0.078	0.074
Relative surface intensity J_b/J_f	11.4	11.6	11.8	11.6	11.4
Semi-duration of primary eclipse	4^h54^m	4^h48^m	4^h55^m	5^h2^m	5^h10^m
"Equal-mass" density of bright star	(0.089)	0.091	0.078	0.068	0.059
"Equal-mass" density of faint star	(0.060)	0.070	0.070	0.070	0.069

* Unit of length is the radius of the relative orbit.

1. From the peculiar nature of the problem, a central annular eclipse on the hypothesis of complete darkening becomes a possibility and an orbit based on such an assumption was computed. The resulting light-curve, however, did not give a sufficiently good representation of the observed curve as it now stands, though it would not necessitate a very marked change in the observed points to make this computed orbit quite satisfactory.

[1] *Science*, N.S., **37**, 34, 1913.

2. The accurate photometric curve of the principal minimum by Müller at Potsdam,[1] as worked over by Pannekoek,[2] was made the basis of an independent solution for uniform elements, assuming a secondary minimum of $0^m.06$. The maximum light-observations are not definite, but if considered of much weight would show ellipticity rather than reflection. The elements derived from this solution are:[3] $k=1.00$, $a_0=0.717$, $r_b=r_f=0.23$, $\cos i=0.102$, $L_b=0.925$.

TABLE II

NORMAL MAGNITUDES NEAR PRINCIPAL MINIMUM AND RESIDUALS FROM COMPUTED CURVES

Phase	Magnitude Difference	Stebbins O−C	Sin θ	Rectified Intensity	Observed−Computed			
					$x=0$	$x=\frac{1}{3}$	$x=\frac{2}{3}$	$x=1$
-5^h18^m....	0.18	$+0^m.01$	−0.465	0.998	$0^L.00$	$0^L.00$	$0^L.00$	$0^L.00$
−4 22.....	0.20	− .02	− .388	.981	+ .01	+ .01	+ .01	+ .02
−3 54	0.28	.00	− .349	.915	− .01	− .01	− .01	− .01
−2 41	0.55	.00	− .243	.723	− .02	− .02	− .02	− .01
−1 39	0.88	− .01	− .150	.545	.00	.00	.00	.00
−1 08	1.09	.00	− .103	.458	.00	.00	.00	.00
−0 41	1.21	− .05	− .062	.415	+ .02	+ .02	+ .02	+ .02
−0 08	1.35	− .02	− .012	.370	+ .01	+ .01	+ .01	+ .01
+0 25	1.37	+ .04	+ .038	.364	− .01	− .01	− .01	− .01
+0 48	1.15	− .06	+ .073	.436	+ .02	+ .02	+ .02	+ .03
+1 13	1.08	+ .02	+ .111	.462	− .01	− .01	− .01	− .01
+1 34	0.97	+ .04	+ .143	.505	− .03	− .03	− .03	− .02
+2 03	0.69	− .06	+ .186	.641	+ .02	+ .03	+ .03	+ .02
+2 34	0.64	+ .05	+ .232	.669	− .04	− .04	− .04	− .04
+2 55	0.42	− .07	+ .263	.810	+ .03	+ .04	+ .03	+ .03
+3 16	0.42	+ .02	+ .294	.810	− .02	− .02	− .03	− .03
+3 50	0.30	+ .01	+ .343	.899	− .02	− .02	− .02	− .02
+4 14	0.23	.00	+ .377	.955	− .01	.00	.00	.00
+4 42	0.19	+ .01	+ .416	0.989	− .01	.00	.00	.00
+6 14	0.17	0.00	+0.539	1.007	+0.01	+0.01	+0.01	+0.01

The normal magnitudes for the principal minimum of Algol are given in Table II. The co-ordinates of the mean observed points, taken from Stebbins' paper, have been transformed for convenience from time and magnitude difference into $\sin\theta$ and rectified light-intensity, where θ is the phase angle counted from minimum. The sum of the squares of the residuals from my computed uniform curve is 0.0066; from Stebbins' computed curve, reducing the

[1] *Astronomische Nachrichten*, **156**, 177, 1901.

[2] *Op. cit.*, p. 223.

[3] For the meaning of the notation see *Astrophysical Journal*, **36**, 404, 1912.

residuals from magnitude to intensity, it is 0.0088. The last four columns of the table show that the representation of the observations is equally satisfactory for all degrees of darkening—in fact, the curves coincide for the greater part of the eclipse. From the light-curve alone we cannot hope to distinguish between these various sets of elements, and within these limits, at least, the orbit is not only entirely indeterminate, but must remain so until the question of darkening is answered.

Referring to the summary of solutions, we see that in this typical case satisfactory intermediate solutions can be obtained from the uniform and darkened elements by linearly interpolating for the radii of the stars, the cosine of the inclination, and the duration of the eclipse. The remaining elements are then readily computed without the necessity of using the light-curve.

NOTE ON A THIRD BODY IN THE SYSTEM OF ALGOL

From the spectroscopic work by Curtiss[1] and Schlesinger[2] there seems to be little doubt that there is a third body in the system of Algol with a period of 1.9 years. If the distant component has a sensible brightness, its light will affect the computation for the orbit of the closer eclipsing pair. It may be appropriate to show briefly in what manner such a condition may be met, and to what extent the unknown quantity of light would alter the orbital elements computed above.

Uniform solution.—We proceed from the following data which were obtained from the solution of the rectified light-curve:

$$1-\lambda_p=0.643,\ 1-\lambda_s=0.055,\ \chi(k,\alpha_0,\tfrac{1}{4})=1.912,\ \sin^2\theta'=0.1793$$

(usual notation; see *Astrophysical Journal*, **36**, 404 ff., 1912). The difference in brightness of the two sides of the close companion is 0.045 so that the light at principal minimum, in terms of the greatest light of the system just outside the secondary minimum, is actually 0.312. We may assume any value of α_0, derive k from the χ-function, and then L_1 and L_2 by the equations $L_2=\dfrac{1-\lambda_p}{\alpha_0}$, $L_1=\dfrac{1-\lambda_s}{k^2\alpha_0}$

[1] *Astrophysical Journal*, **28**, 156, 1908.

[2] *Publications of the Allegheny Observatory*, **1**, 20, 1908; *Science*, N.S., **37**, 34, 1913.

(the bright star is found to be the smaller); the remaining elements may then be derived in the usual manner. The difference $1-(L_1+L_2)=L_3$ will represent for the various assumed values of α_0 the

TABLE III

UNIFORM SOLUTIONS WITH CONSIDERATION OF THE LIGHT OF A THIRD BODY

α_0 (Assumed)	k	L_2	L_1	L_3	Radius		Inclination	Density	
					Faint	Bright		Faint	Bright
1.00	0.733	0.643	0.102	0.255	0.247	0.181	84°41′	0.055	0.14
0.95	.74	.677	.106	.217	.248	.183	84 52	.054	.14
0.90	.76	.715	.106	.179	.246	.187	84 15	.056	.13
0.80	.83	.804	.100	.096	.238	.198	83 27	.062	.11
0.709	0.915	0.907	0.093	0.000	0.228	0.208	83 2	0.070	0.09

light of the distant companion. Ordinarily, when such a contributing light-source is not suspected, the adopted values of a_0 and k are those unique values that give $1-(L_1+L_2)=0$. L_1 is the light of the bright side of the close faint companion; that of the side seen at principal minimum is $L_1-0.045$. The light remaining at that time is, then,

$$L_3+(1-a_0)L_2+L_1-0.045=0.312$$

which is a relation that can be used to check the computations. We thus derive in Table III various sets of elements, each one being the best uniform solution for the corresponding value of the light of the third component, L_3. If the principal eclipse is assumed total, the third body may have as much as one-fourth of the whole light of the system, but under no condition more than that amount. The last solution is the one that gives $L_1+L_2=1$, and therefore yields the same set of elements as given in Table I.

Darkened solution.—The equations in this case are

$$\chi(k, a_0', \tfrac{1}{4})=1.912,\ L_2=\frac{0.643}{a_0'}\ ,\ \ L_1=\frac{0.055}{a_0'Q(k,a_0')}\ ,\ \ L_3=1-L_2-L_1$$

if we assume that the small star is the brighter and is eclipsed at principal minimum; but if the larger star is the brighter, we have

$$\chi(k, a_0'', \tfrac{1}{4})=1.912,\ L_2=\frac{0.055}{a_0'}\ ,\ \ L_1=\frac{0.643}{a_0'Q(k,\ a_0')}\ ,\ \ a''Q(k,\ 1)=a_0'Q(k,\ a_0').$$

The derivation of the sets of elements is then similar to the preceding case.

SUMMARY

1. If the orbits for an eclipsing binary have been computed when the darkening coefficient is zero (uniform disks) and when it is unity (completely darkened disks), solutions based on intermediate degrees of darkening can be obtained by linear interpolation for some of the elements (Table I).

2. The system of β Persei has been studied from this standpoint with the result that a considerable amount of indeterminateness is found to be inherent in the solution. For uniform disks the bright star is the smaller, while for completely darkened disks it is slightly the larger. Quantitative knowledge of the degree of darkening is necessary to remove this uncertainty (Table II).

3. An outline is given of the method of computing orbits for partially eclipsing variables when consideration is made of the possibility that a third body contributes a portion of the light, and an application is made to Algol to allow for the possible light of the distant companion whose period is 1.9 years (Table III).

Mount Wilson Solar Observatory
June 20, 1914

ON THE PRESSURE DISPLACEMENT OF SPECTRAL LINES AND MOLECULAR CONSTITUTION

By G. H. LIVENS

In a previous communication[1] an explanation of the pressure-shift of the lines in emission spectra was suggested on the basis of an explanation of the analogous phenomena in absorption spectra, which is already involved in the ordinary theory of absorption as given by Lorentz. At that time, however, the theory did not appear general enough to cover the whole phenomenon; in particular, it failed to give any account of the displacement of the lines toward the region of shorter wave-length, which has been occasionally observed in the more complicated spectra. A more thorough investigation of the theory for absorption spectra, however, soon convinced me that the apparent discrepancies merely involved an oversight occasioned by the method of approximation adopted throughout the original memoir. I propose, therefore, to rediscuss the equations formerly obtained and then to discuss the results deduced from them in their bearing on some fundamental questions of molecular constitution.

According to the most general views now held, the radiation from a molecule originates in the rapid oscillations of the electrons contained in it, as they adjust themselves to a configuration of equilibrium after a violent disturbance produced in a manner that is as yet not quite evident. On a tentative theory we could, if the displacements are small, regard each electron as vibrating about a position of equilibrium to which it is attached by some force of a quasi-elastic nature whose amount is proportional to its displacement. The equations of motion of the typical electron would then be of a type

$$m(\ddot{s}_r+n_r^2 s_r)=F$$

where m is its mass, s_r its displacement (vectorial), n_r its frequency of free oscillation, and F the component of any applied force.

[1] *Philosophical Magazine*, 24, 268, 1912. In this paper a full discussion of many points slurred over in the present communication is given.

Regarding the medium as a whole, however, it is found that the molecules in any effective differential element of volume cannot radiate energy to external space unless a certain condition holds, which in the present case is equivalent to the fact that the vectorial sum

$$\sum_r e\ddot{s}_r$$

must be different from zero when taken over all the effective electrons (each with a charge e) in the differential element of volume. But

$$\sum_r e\ddot{s}_r = \ddot{P}\, dv$$

where P is a vector defining for the element of volume dv what is analogous to its polarization in the ordinary theory of absorption. We can thus say that the element of a medium must be polarized before it can radiate energy; but if it is polarized to intensity P, there is a force aeP on any electron inside it, representing an average estimate of the actions, on the particular electron under consideration, of all the other electrons in the element. The constant a in the expression of this force is a numerical constant approximately equal to $\frac{1}{3}$ in the usual Hertz-Heaviside system of units. Thus in the absence of any external force the equation of motion of the typical electron in the element is of the form

$$m(\ddot{s}_r + n_r^2 s_r) = aeP$$

And now we see that the electrons in the element, instead of being single independent vibrators, really form part of a connected dynamical system in which the motion of any one part depends to a certain extent on the motions of all the others. The possible frequencies of this connected system are obtained in the usual manner in such problems; we try a solution of the equations of type

$$s_r = s_{r_0} e^{int}$$

which satisfies if

$$m(n_r^2 - n^2) s_r = aeP$$

that is, if

$$Pdv = \sum_r es_r = \left(\sum_r \frac{ae^2/m}{n_r^2 - n^2}\right) P$$

or if

$$1=\sum_r \frac{ae^2/m}{n_r^2-n^2} \tag{1}$$

$\sum_r$ now denoting a sum taken per unit volume over all the electrons contained in the element dv.

If the frequency n_r is the same for all the effective electrons in the volume element and if there are $N_r dv$ of them, then the equation satisfied by n is

$$1=\frac{\rho_r}{n_r^2-n^2}$$

wherein

$$\rho_r=\frac{N_r ae^2}{m}$$

This is satisfied by

$$n=\sqrt{n_r^2-\rho_r}$$

which, since ρ_r turns out to be small compared with n_r, may be approximately written in the form

$$n=n_r-\frac{1}{2}\frac{\rho_r}{n_r}$$

If, however, there are other free frequencies than n_r present in the substance, it appears that there is a value of n, a root of the above equation (1), near each distinct value of n_r; and it is easily seen that the root near n_r has a very approximate value

$$n=n_r-\frac{1}{2}\frac{\rho_r}{1-aA_r}\cdot\frac{1}{n_r}$$

wherein

$$A_r=\sum_s \frac{e^2/m}{n_s^2-n_r^2}$$

the sum $\sum_s$ being taken per unit volume over all the electrons other than those included in the group N_r.

The displacement of the frequency from the free value n_r is thus given by

$$dn_r=n-n_r=-\frac{1}{2n_r}\cdot\frac{\rho_r}{1-aA_r}$$

or, as it is usually interpreted, in terms of wave-lengths,

$$d\lambda_r = \frac{\rho_r \lambda_r^3}{8\pi c^2(1 - aA_r)} \qquad n_r = \frac{2\pi c}{\lambda_r}$$

The value previously obtained involved a neglect of the factor $\frac{1}{1 - aA_r}$ in this expression.

It thus appears that the frequencies of the light-disturbance obtainable from any medium as a whole are in general slightly different from the frequencies of the free oscillations of the contained electrons, owing to modification by mutual interaction among these electrons. This slight displacement of the free frequency thus necessitated by the statistical treatment of the aggregate of the independent vibrating electrons is primarily a function of the average partial density of the electrons giving rise to the particular line in the spectrum, but may also be considerably affected and even reversed in sign by the presence of other electrons with free periods near that of the particular group considered. The theory thus appears to account for certain irregularities in the phenomena, which are observed in complicated spectra, and in particular does involve an explanation of the displacement to the region of shorter wave-lengths which is occasionally observed in the complex spectra. It is perhaps worth noticing that this reversed displacement would occur in parts of the spectrum where

$$A_r = \sum_s \frac{\rho_s}{n_s^2 - n_r^2}$$

is positive and greater than $1/a$, which is most likely to occur just before one or more prominent lines in the spectrum. This appears to be the case with most of the reversed displacements observed in the iron spectrum, the only one for which I can obtain information. Of course for such lines the displacement would certainly be a more complicated function of the density than that which is usually determined for the other lines, for which the law of proportionality seems to be sufficiently satisfactory. This suggests a possible test as to the validity of the present explanation.

The fact that the displacement of a line is mainly proportional to the density of the electrons giving rise to it should furnish valuable

evidence as to the relative number of electrons concerned in the production of each line. Interesting particular cases have already been noticed in this respect. For instance, in the spectra of gold and iron there appear to be three distinct groups of lines whose displacements are approximately as 1:2:4;[1] this would indicate that the partial densities of the electrons directly responsible for the lines in a group are the same, but for lines in different groups they are in the ratio 1:2:4. This of course suggests that each group is the result of a more complex oscillation of the single electron or of a small group of electrons, a view that is supported by the fact that the order of magnitude of the Zeeman effect is the same for almost all the lines in a group in the iron spectrum. Have we here a more complex type of series?

A similar and perhaps startling conclusion is provided by the fact that when more than one series occurs in the spectrum of a substance the displacements of the lines of the principal series are always the smallest, the displacements of the corresponding lines of the first subordinate series are usually twice as great as for the principal lines, and those of the corresponding lines in the second subordinate series are four times as great.

Again, the fact that the displacement of the lines of a given series in a spectrum are almost invariably proportional to the cubes of their wave-lengths provides important evidence as to the origin of series lines. In fact, a tentative application of the theoretical formula given above would indicate that the density of the electrons giving rise to each line in a series must be the same. Such a conclusion would support the view that the series lines arise as the result of a more complex vibration of the single electrons or small group of electrons rather than as the result of simple vibrations of a large number of independent electrons. It would also exclude on general grounds any theory which ascribes the different lines of the series to different possible states of a simple vibrating system unless

[1] Gale and Adams (*Astrophysical Journal*, 37, 391, 1912) have shown that this ratio, which was originally given by Duffield, is not exactly expressed by integers and they suggest 1:2·3:4·5. The present theory of course would support such a view, but I do not think that it affects the deduction made from the results and I have therefore preferred to use the figures as Duffield gives them, as an illustration of the principle involved.

it can be shown that all possible states of such a system are equally probable.

Of course the direct application of the foregoing formulae in cases involving more complex types of oscillation than those examined above is open to question. It can, however, easily be verified that the results deduced are still true and the conclusion stated is valid if certain groups (or series) of lines arise as the result of a complex oscillation of a connected group of electrons. The exact analysis for this verification will be fully discussed in another connection in a future communication.

It would thus appear that the evidence of fact in the phenomena of the pressure-shift of spectral lines furnishes very valuable clues in the more fundamental problem as to the exact origin of the series regularities. This of course naturally arises from the fact that in such a phenomenon we have a means of varying at will the frequencies of the vibrations in the electronic system in the atom. Kayser's advice that particular attention should be paid to this branch of experimental work is therefore worthy of special emphasis.

It may of course be objected that probably too great weight is attached to the accuracy of the deductions made from the experimental results which are obtained under great difficulties. In reply, however, it may be said that in the majority of cases where certain results are obtained the correct theoretical relation given above showing the dependence of the displacement on the density and wave-length is deduced empirically, and often in direct contradiction to theoretical laws of other types offered in explanation. Besides, the discrepancies which often present themselves are more apparent than real, being usually based on erroneous methods of computation.[1]

THE UNIVERSITY
SHEFFIELD
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[1] An éxample is provided by Gale and Adams' work on titanium. The various lines in this spectrum, being too numerous to separate out, were averaged up together to obtain the relation between displacement and wave-length. Such a method of averaging is of course likely to lead to erroneous results unless it can be shown that all the lines belong to a single group, i.e., are due to the same average density of electrons.

THE WIDENING OF THE HYDROGEN LINES IN THE SPARK SPECTRUM

By R. ROSSI

The present note deals with experiments which have been carried out by passing sparks through hydrogen at atmospheric pressure under various conditions of electric circuit (capacity, self-induction, length of spark gap, etc.), with a view to getting some quantitative data on the influence of the electrical factors on the width of the spectrum lines.

It was found that the width decreased exponentially with increase of self-inductance, and increased, at first linearly, with increase of capacity until a limit was reached when further increase of capacity did not affect it appreciably.

No definite relation, however, could be obtained linking width with the above electrical conditions, and some other factor was looked for.

The fact established by Milner[1] that current-density in a spark increases with increase of capacity until a limit is reached when the current-density is no longer affected by capacity seemed somewhat similar to the widening of the hydrogen lines with increased capacity; and experiments were consequently undertaken to see if current-density is not an important factor in the widening of the hydrogen lines.

The electrical part of the apparatus consisted of an induction coil, through the primary of which the same current was kept flowing in all experiments, various plate condensers and self-inductances consisting of iron wire wound round wooden cylinders. The spark was passed in hydrogen at atmospheric pressure between two iron electrodes, the distance between which could be altered. An image of the spark was thrown on the slit of a reflecting half-prism stellar spectroscope, the dispersion on the plate being 14 mm from H_{α} to H_{γ}, a small dispersion being favorable to measurements of widths of hazy lines. The slit-width was 0.036 mm, its image on the plate

[1] S. R. Milner, *Philosophical Magazine* (6), **24**, 709, 1912.

being about one-half that value owing to the different focal lengths of the collimating and camera lens.

Under the influence of capacity the lines are of a hazy character, with wings slowly shading away, and their width was roughly measured by setting the microscope wires in the middle of the shadings. The plates used were Wratten and Wainwright Panchromatic, and several sets of photographs were taken under various electrical conditions, care being taken to expose all the plates for the same time and develop the plates of one set all together, thus avoiding as far as possible photographic irregularities in development.

The current-density in the spark was measured by using Milner's method:[1] the cross-section of the core of the spark under the various electrical conditions was obtained from direct photographs of the spark; the average current in each case was assumed to be the ratio of the charge of the condensers to one-half period of oscillation of the system.

The following table gives the values of the widths of H_α, H_β, H_γ in angstroms for the various current-densities:

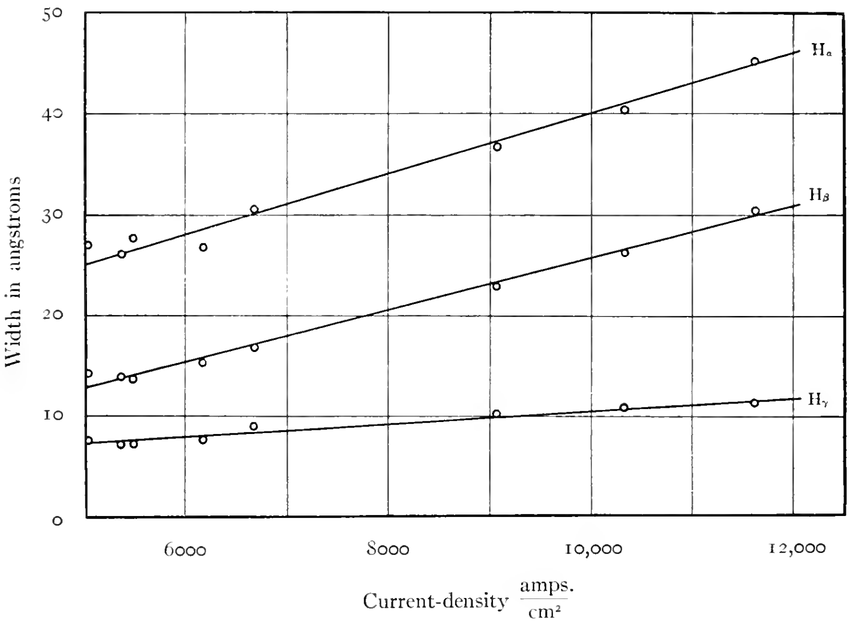
CURRENT-DENSITY	WIDTH OF LINES		
$\frac{\text{amps.}}{\text{cm}^2}$	H_α	H_β	H_γ
5,020	27.0	14.3	7.7
5,370	26.2	14.0	7.4
5,480	27.7	13.8	7.3
6,180	26.7	15.3	7.7
6,680	30.5	16.9	9.0
9,070	36.7	22.9	10.2
10,330	40.2	26.1	10.8
11,610	45.2	30.3	11.3

These values are plotted in the accompanying diagram from which it is seen that within the limits of experimental error (which, owing to the difficulty of both measurements, is probably over 10 per cent) the width increases linearly with current-density.

Owing to the different type of broadening exhibited by each individual line, which prevents their being measured in the same

[1] *Loc. cit.*

way, the widths of the three lines studied are not comparable to one another. With the dispersion used H_α remains always fairly sharp and the reading microscope wires were set on its fairly well-defined edges. H_β is a broad band, slowly fading off at the edges, and the wires were set on what appeared to be the middle of the shadings. H_γ has a well-defined intense narrow core and some very extensive wings extending far out and diminishing in intensity. Only the width of this core could be measured in the present case



and it is seen that it is less affected than the width of the other lines, but it was noticed that the winged portions of the line were far more widened than the less refrangible lines. There is no doubt that if we compare the different widened lines, without taking into account the nature of their structure, the more refrangible lines are the most widened by current-density. H_δ, although visible on nearly all the plates, was so wide, even with the small dispersion employed, that any attempt to get a definite value of its width would have been useless.

The widening of the hydrogen lines in the spark has been ascribed to various causes, such as potential gradient, intensity of spark discharge, but especially temperature. From the evidence afforded by the present work it would seem that current-density is a very important, if not the only, factor in the phenomenon. This would bring the cause of electrical widening of the hydrogen lines under the same heading as that of the widening due to pressure,[1] viz., both are due to the proximity of molecules vibrating with the same period. It is interesting to note to this effect that a single theory accounting for the widening due to both these causes has been sketched out by Sir J. J. Thomson,[2] who attributed the widening to the density of the vibrators.

My best thanks are due to Professor Newall for the interest he has taken in this research and for his constant and valuable advice.

Solar Physics Observatory
Cambridge University
May 1914

[1] *Philosophical Magazine*, **21**, 499, 1911.

[2] *Proceedings of the Cambridge Philosophical Society*, **13**, 318, 1906.

MINOR CONTRIBUTIONS AND NOTES

THE EFFECT OF HUMIDITY ON THE SENSITIVENESS OF PHOTOGRAPHIC PLATES

On p. 310, Vol. 39, of the *Astrophysical Journal*, Mr. Seares refers to the phenomenon noted by Pickering, that when several exposures are impressed on the same plate, the first and last being short and equal, the images of the first are often systematically brighter than those of the last, the differences amounting on the average to a quarter of a magnitude.

As it seemed probable that this effect was due to a change in humidity, we have made experiments to determine the effect of varied humidity on the sensitiveness and development factor of Seed 23 and Seed 30 plates.

The plates were brought to the requisite humidity by being kept in an atmosphere contained in a box where the humidity could be accurately controlled, and we found that when we used a range of humidity from 0.5 per cent to 85 per cent both sensitiveness and development factor decreased about 25 per cent when the humidity was increased from 0.5 per cent to 85 per cent, time being given in all cases for the emulsion film to come into equilibrium with the air.

The effect referred to by Mr. Seares, therefore, is probably due to a dry plate slowly gaining humidity during exposure in the telescope, with an accompanying loss of density for equivalent exposures.

This same phenomenon probably explains some of the observations which have been published as to variations of the sensitiveness of photographic plates due to changes of temperature, the effect being due not so much to the change of temperature as to an accompanying change of humidity. It would be desirable for all photographic materials used for photometric work to be brought previously into equilibrium with the atmosphere in which they are to be used.

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Reviews

Lehrbuch der Physik. By O. D. Chwolson. Vierter Band, zweite Haelfte, erste Abteilung. Braunschweig: Vieweg & Son, 1913. Pp. 446. Translated by H. Pflaum and A. B. Foehringer.

The fourth volume of Chwolson's well known *Lehrbuch*, containing the first chapters on Electricity and Magnetism, appeared five years ago, and many admirers of this masterly treatise have been hoping for an early publication of the remaining volume. The long delay and the fact that there appears now only a portion of it is easily explained by the untimely death, in the fall of 1912, of Professor Pflaum who had translated most of the earlier volumes. He completed only about one-half of the volume; the remaining chapters of the work are written by Miss Foehringer. The translation is excellent; the style is clear, and betrays nowhere that it is a translation.

For the first time we find that Chwolson no longer appears as the sole author of the work. While we are somewhat disappointed that he has felt the necessity of accepting the assistance of his friends in the completion of what he himself has called his life-work, his colleagues have carefully adapted themselves to his method of treatment and have thus avoided a criticism which can justly be made against some many-authored texts.

At the beginning of the fourth volume the author divided the subject of electricity into five large divisions: the constant electric field, the constant magnetic field, the variable magnetic field, the discharge of electricity through gases, and radioactivity. Only the first division and a portion of the second are contained in the volume of 1908; in the present continuation we find the concluding chapters of the second division and apparently the whole of the third. The first two hundred pages are devoted to the principles of measurement of electrical resistance, current, electromotive force, and of the intensity of the earth's magnetic field. While no claim is made that the subject is treated exhaustively, the reviewer has found all the fundamental methods fully and clearly described. There is no rubbish of time-honored experiments which are now out of date or of instruments which have been relegated to the museums.

The chapter on "Variable Magnetic Fields" contains the usual material, i.e., electromagnetic induction, measurement of inductances,

and an elementary treatment of alternating current phenomena. It should be mentioned with unqualified approval that two introductory chapters are devoted to the fundamental notions and laws of vector analysis of which constant use is made in the remaining portions of the book. In comparison with other subjects this chapter appears to have been unduly condensed.

Since the final portion of the work is still to appear the reviewer does not know if there will be a chapter on the applications of electricity, yet it seems there will be none. It is a matter of personal opinion how much of applied physics should be contained in a textbook of this kind; but certainly the principles underlying the telephone, telegraph, or the simple dynamo might well be included.

The greatest difficulty in the teaching of electricity lies in the existing chaos of theories attempting to explain electromagnetic phenomena. There is the action-at-a-distance theory with its life of nine cats, Maxwell's theory, several types of the electron theory, and finally, for good measure, the principle of relativity. If a textbook writer should arbitrarily choose among them in favor of a single one he may find any day that there has been an important change in our attitude and his book is out of date. Chwolson is perfectly aware of this difficulty, and therefore follows the plan of first presenting the experimental material and the theoretical deductions which are based exclusively upon experimentally tested laws. It is true that at the beginning he gives a short account of the principal theories which he uses, each in its place, but again and again he points out the hypothetical character of the theories as he proceeds with the description of the various electrical phenomena. In the book here reviewed, he completes the presentation of the fundamental concepts and laws, and then, in the second half of the book, gives a more detailed account of modern theories. Thus in the third chapter of the "Variable Magnetic Fields" an admirable elementary account of Maxwell's theory is given, and in the fourth chapter one of the electron theory. The book closes with an exposition of the principle of relativity which appears to have the great merit of having been written without partiality.

As in the earlier volumes full lists of references to original papers are given at the end of each chapter. It is hardly necessary to speak of the great value to physicists of a work of this kind. It fills the gap between the more elementary and somewhat antiquated works of Wüllner, etc., and more advanced mathematical treatises. It is being translated into French as well as into German. How long must we wait for a similar work written in the English language?

K. E. Guthe

The Constitution of Matter. By JOSEPH F. AMES. Boston: Houghton Mifflin Co., 1913. Pp. 242.

These are the six public lectures which Professor Ames gave at Evanston in the spring of 1913 before an audience composed of three different types of hearers, namely, the student, the instructor, and the man on the street. How excellently the speaker adapted himself to the heterogeneity of the company was attested by the enthusiasm of the listeners.

Nothing could be more obvious than the fact that any modern view of the structure of matter must deal largely with the electron theory. Accordingly, the author employs the first lecture to lead up, through the experimental facts of inertia, momentum, force, and energy, to the concepts of molecules and atoms, and to the experimental evidence for the existence of electrons—or corpuscles, as he calls them, preferring to adhere throughout the volume to the name originally assigned by Sir Joseph Thomson. This exposition of the Newtonian idea of mass is one of the clearest and best with which your reviewer is familiar.

The second lecture leads up to the metrical properties of the electron, the preliminary discussion being devoted to the phenomena of electrification, electric currents, and to two alternative views of electric mass. The experiments which yield values for the electronic charge, for Avagadro's number, for the radius of "sphere of action," and for other fundamental quantities are set forth with masterly clearness.

With the exception of the first ten pages, which are given to an introductory sketch of radioactivity, the entire third lecture is devoted to an exposition of the more important phenomena of gravitation and to various attempts at the explanation of weight. Here will be found the interesting experiments of Southerns with a pendulum carrying a bob of radioactive material, a device suggested by the association of mass with potential energy.

The mechanism of radiation in terms of electronic acceleration occupies a large portion of the fourth lecture; the remainder is given to chemical affinity, electrolysis of gases, and to another general property of matter, elasticity.

How beautifully thermal and electrical conduction fall into line on the electron theory of matter is forcibly presented in the fifth lecture. The elementary and preliminary presentation of the facts concerning the absolute scale of temperatures will be of value to all students. The new science of "thermionics," photo-electric action, opacity of metals, and magnetism are included, apparently with equal ease and naturalness, in this unitary corpuscular view of nature.

The purpose of the last lecture is to show how thoroughly the phenomena described in the five preceding lectures can be resumed in terms of the atomic structure proposed by Thomson, a model which really appears to become more and more efficient as time goes on, and as modifications are introduced here and there. Witness Thomson's latest change, made for the purpose of adapting his atom to the *quantum* hypothesis (*Phil. Mag.*, October 1913). The closing pages present a well balanced, modern, fair, and critical picture of the methods of physical science touching only briefly the *quantum* theory of radiation and the question of relativity.

To those who know Professor Ames it is needless to say that the volume is not made with scissors and paste; or that the English is marked with that clarity and precision for which he is justly celebrated. To anyone in search of a brief, elementary, and reliable sketch of modern physics in which the drawing is good and the perspective correct this volume is highly recommended.

HENRY CREW

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AN APPLICATION OF INTERFERENCE TO THE STUDY OF THE ORION NEBULA

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We published in 1911[1] an account of the principle involved in our investigations and of the preliminary attempts that we made to apply interference methods to the study of nebulae.

In 1912 and especially in 1914 we continued these researches and we were able to get a part of the results which we set out to obtain. It should be remembered that the method consists in producing, with the light from the nebula, interference fringes formed at infinity (rings of a thin sheet of air between parallel silvered planes). These rings are projected on a photographic plate which records them; on the other hand, it is arranged so as not to confuse the radiations sent out by different points of the nebula; that is, a clear image of the nebula is obtained on the same plate with the rings. The latter are therefore visible on the surface of the nebula.

DESCRIPTION OF THE DIFFERENT PARTS OF THE APPARATUS

The telescope.—The entire interferential arrangement was mounted on the reflecting telescope of the Observatory of Marseilles, of which the glass mirror, figured by Foucault in 1862, has

[1] Ch. Fabry and H. Buisson, *Astrophysical Journal*, **33**, 406, 1911.

a diameter of 80 cm and a focal length of 4.50 m. The telescope is furnished with a driving mechanism worked by electricity, as well as a slow motion in hour angle. To follow the star during the photographic exposure, an objective of 3 m focus and an eye-piece, forming a refracting telescope, have been attached to the tube of the reflector.

The interferometer.—The arrangement shown in section in Fig. 1, for producing and projecting the interference fringes, forms a compact piece which is placed at the open end of the telescope tube,

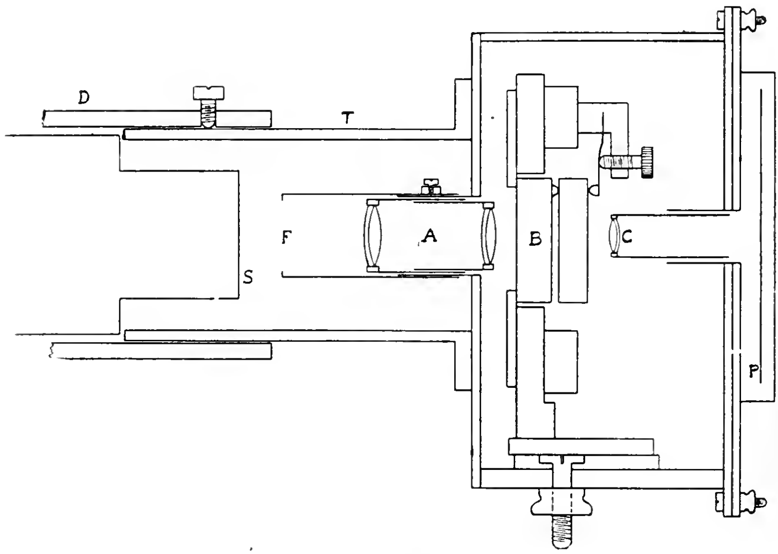


FIG. 1. (About 2/3 actual size)

centered on the axis of the mirror in such a way that the light reaches it without any additional reflection. This arrangement cuts off a little of the incident light, but only a rectangle 11×14 cm, or 1/30 of the surface of the mirror.

The étalon is composed of two plates of glass 4 cm in diameter and 1 cm thick, the adjacent surfaces of which are silvered and held parallel by three metal blocks of equal thickness. The exact adjustment to parallelism is made by the pressure of three springs, regulated by means of screws. We used thicknesses from 0.1 mm

to 3 mm. For the smallest thicknesses, the separation was produced by three short lengths of wire cut in succession from the same piece. Greater thicknesses were obtained either with small fragments of steel rod, or better, to avoid expansion, with blocks of invar.[1]

The quality of the silvering is of primary importance. We obtained it by cathodic projection, following a method which permitted us to obtain at once exactly and very rapidly the desired thickness of silver. As soon as one plate has been prepared, its transparency and its reflecting power are measured. According to circumstances, we used two sets of silvered surfaces to form the étalon. In the first pair each surface transmitted 0.15 and reflected 0.74 of the incident light; in the other pair, these numbers were 0.30 and 0.60, these values referring to the green radiation of mercury.

After reflection from the mirror of the telescope, the light coming from the nebula forms the image in the focal plane F. It next passes through a pair of lenses A, forming an optical system of short focal length, the focus of which coincides with F. This system is formed by two achromatic lenses of uviol glass in order to avoid the absorption of ordinary flint. Each one of them has a focal length of 86 mm and a diameter of 19 mm and they are 40 mm apart. This whole arrangement is calculated in such a manner that it may have a given focal length (56 mm) and that in the interior of a field with a diameter of 10′ the light reflected from the large mirror shall be completely utilized. The first lens performs the office of field-glass at a distance of only 30 mm from the real image, and diminishes the size of the pencil of rays on the second lens.

The combination consisting of the mirror and the lenses A forms a non-focal system, whose angular magnifying power is 80. In leaving A the light of the nebula seems to proceed from a star at infinity, enlarged angularly 80 times; in addition the bundle of

[1] The blocks were cut by M. Jobin, who gave them a very satisfactory form and who obtained in every case exactly the desired thickness. We shall offer an explanation in the near future of the technique of obtaining the silverings as well as the details of construction of the étalons.

rays passes through the ocular ring of which the diameter is only 1/80 of that of the mirror. It is there that the interference apparatus *B* is placed, immediately followed by the achromatic objective *C*, also of uviol glass, having a diameter of 10 mm and 45 mm focal length. In the focal plane of this objective is the photographic plate *P*. On it we get at the same time the sharp image of the rings (which have not been changed in any way by anything in front of the interference apparatus) and the image of the nebula 80 times larger than if the objective *C* were pointed directly at the sky.

There also appears on the plate the image of two cross-wires, placed at *F* in the focal plane of the mirror, to serve as reference lines. The étalon, the lenses *A*, the objective *C*, the reticle, and the plate-holder are fitted into a metallic box shown in the cut in Fig. 1. This box is attached to the tube *T* which fits into the socket *D*, supported by the frame of the telescope tube. This whole assemblage of parts, which weighs only 4.5 kg, and which takes the place ordinarily occupied by the totally reflecting prism in the Newtonian arrangement, can be taken out and replaced easily.

ADJUSTMENT. COMPARISON RINGS

Certain of the adjustments are made once for all; others should be made anew before each observation. The focusing of the objective *C* on the photographic plate, and the adjustment of the reticle, in such a way that its image is formed on the plate by the lenses *A* and *C*, are accomplished in the laboratory by artificial light.

On the telescope, the setting of the tube *T* (Fig. 1) is determined so that a clear image of the stars is also shown on the plate. This brings the focus of the mirror into coincidence with that of the system *A*. This arrangement, like the preceding ones, is independent of the interference apparatus and is made before the latter is put in place.

Before each observation, the parallelism of the silvered surfaces of the étalon is verified, and it is oriented in the box in such a way that the center of the rings coincides as exactly as possible with the crossed threads of the reticle. This arrangement and the

putting into place of the étalon is performed while the apparatus is taken out of the telescope and illuminated by monochromatic light from a mercury-vapor lamp. Before returning the apparatus to place on the telescope, observations are taken of the coincidences among the rings of the various mercury radiations, which give the exact order of interference of each ring.[1]

To measure the interferential photographs of the nebula, it is necessary to have a system of rings, obtained under identical conditions, but with a known monochromatic radiation. This system of rings plays a rôle analogous to the comparison spectrum in ordinary spectroscopes, but, while in the latter case the comparison spectrum and the spectrum to be measured must be made on the same plate, to avoid all displacement, on the other hand the system of comparison rings can be made on a separate plate. The interferential apparatus only must remain unchanged.

A photograph of the comparison rings is made before the exposure on the nebula and another after exposure, leaving the whole apparatus in place on the telescope. At the moment when we wish to make the comparison photographs, we place in the center of the tube of the telescope, about one meter from the opening, a screen of white paper 30 cm in diameter, which is lighted by a mercury lamp held by hand at the opening of the tube. Since the Observatory is supplied only with alternating current, the form of lamp constructed by M. Tian[2] was employed. The lamp is inclosed in a wooden box having a round opening 4 cm in diameter. A glass cell 2.5 mm thick is placed before the opening and contains a weak solution of chromate of potassium intended to absorb the radiations of short wave-length. Since the plates used are practically insensible to green and yellow, only the ray λ 4358 remains to produce the rings. The exposure is about ten seconds.

The necessary exposure for photographing the interference rings of the nebula depends on the radiation employed and the

[1] A. Perot and Ch. Fabry, *Annales de chimie et de physique* (7), **16**, 289, 1899. An explanation of the nature of interference rings by silvered planes will be found in Vol. **15** of the *Records* of the International Bureau of Weights and Measures.

[2] *Journal de physique* (5), **3**, 486, 1913.

thickness of the silverings. Our plates were made with exposures of from one to two hours.

After the preliminary trials, the photographing of interference rings was begun on January 27, 1914, and was continued until March 12; we were able to make 15 plates, some with the radiation H_γ using étalons 1 mm and 2 mm thick, others on the ultra-violet group λ 3728, with thicknesses increasing from 0.13 mm to 2.8 mm.

The accompanying Table I gives a list of all the plates; it gives the date, the radiation used, the thickness and kind of substances separating the silvered surfaces, and finally the exposure-time.

TABLE I

LIST OF PLATES

Date 1914	Radiation Used	Thickness	Blocks	Exposure
January 27	$H_\gamma+H_\beta$	1 mm	Steel	1½ hours
28	$H_\gamma+H_\beta$	2 mm	"	" "
30	H_γ	1 mm	"	1 hour
31	3728	1 mm	"	1½ hours
February 2	"	110 μ	Mica	" "
13	"	130 μ	Wire	" "
14	H_γ	1 mm	Invar	" "
16	3728	1 mm	"	1¾ hours
28	H_γ	2 mm	"	2 "
March 2	3728	0.64 mm	"	" "
3	"	2 mm	"	" "
5	"	2.8 mm	"	" "
7	H_γ	2 mm	"	" "
12	H_γ	1 mm	"	" "

Plate VII reproduces two of the photographs obtained, that of January 31, with the line λ 3728, the other of March 12, with H_γ. They are negatives, that is, the reproductions are identical with the original photographs, save for enlargement.

PRELIMINARY STUDY OF THE SPECTRUM. ABSORBING FILTERS

Before beginning the study of the interference rings, we wished to have a personal and direct knowledge of the spectrum of the nebula and of the relative intensity of the various lines. The publications which we found on this subject lack precision and are sometimes contradictory, which is accounted for by the unequal absorption of the different apparatus used and by the different

PLATE VII

South

H_γ

South

λ 3728

properties of the photographic plates. We constructed a spectroscope entirely of quartz, very compact and effective as a light-gatherer, the slit of which can be placed at the direct focus of the large mirror. Two 60° quartz prisms, one right-handed and the other left-handed, have square faces 3 cm on a side; the objectives of quartz are 2.5 cm in diameter and 9 cm in focal length. The whole apparatus is attached to a tube identical with that of Fig. 1, fitting into the socket of the telescope. It weighs only 2.7 kg and intercepts but 1/20 of the incident light.

On the Lumière Sigma plates the ultra-violet line λ 3728, not resolved by the spectroscope, is altogether the most intense; the next is the line H_γ; the hydrogen lines of shorter wave-length and the helium lines are much feebler. The group $\lambda\lambda$ 4861–4959–5006, composed of H_β and two lines of unknown origin, give a perceptible image on the plates.

In our experiments with the interference rings, we have isolated a radiation by absorbing filters as much as possible. The choice of these filters must be made with a great deal of care in order not to weaken the intensity of the line under examination and in order also to diminish as much as possible the intensity of the other lines. We have tried to obtain the most favorable filters by making exact laboratory measurements of the power of transmission. We

TABLE II

λ	Transmission
3728	0.00
4046	0.20
4340	0.40
4861	0.01

employed two groups of filters, one in our study of the line H_γ, the other for the double ultra-violet line. To isolate the line H_γ, the whole ultra-violet end and in particular the line λ 3728 must be weakened; it is necessary also to weaken the green group 4861–5006. The combination employed consists of a Wratten filter of esculin to eliminate the ultra-violet and a Wratten filter *D* (probably a methyl-violet one) which eliminates the green. The transmissive powers of this combination for certain radiations are given in Table II.

To isolate the line λ 3728, the whole visible portion must be eliminated. Violet and blue are eliminated by a screen of nitroso-dimethyl aniline. A fuchsine filter stops the green. These filters are obtained by bathing fixed photographic plates in aqueous solutions. We performed a series of experiments by using solutions of different degrees of concentration, and we chose those which yielded the best results. The transmissive powers are given in Table III.

TABLE III

λ	Transmission
3728	0.50
4046	0.14
4340	0.01
4861	0.00

The filters are put at S (Fig. 1) a little in front of the reticle. They are placed at the end of a tube which enters the socket D and which permits their interchange without modifying the rest of the installation. The same combination of filters which we used for H_γ was employed to photograph the comparison rings with mercury light.

METHOD OF MEASUREMENT

We are now in possession of three photographs, one from the nebula, and the other two obtained before and after, with the violet line of mercury. Since we know the wave-length of the mercury line, the problem is to determine the wave-length of the radiation which has produced the rings of the nebula. According to the circumstances, this radiation may be a known line like the lines of hydrogen, whose wave-length, however, is modified by the relative motion and will serve to measure the radial velocity, or else it may be a radiation of an unknown element whose wave-length must be determined. The problem of measurement is the same in both cases.

If the nebula has only a motion of translation, the wave-length of a radiation will be the same for all points, the rings are perfectly circular, and the problem is very simple. The diameters of the circles can be measured without reference to the location of their center. Matters are more complex if there are differences of

radial velocity from point to point, because then we can no longer speak of a single value of the wave-length, the rings are deformed, and every point must be defined with respect to the normal to the silvered surfaces (the center of the mercury rings). This set of measurements will give the wave-lengths at the several points and consequently also the distribution of the radial velocities.

Let us examine first the case where there is only a motion of the whole. One of the mercury photographs is placed on a comparator, and the diameters of the successive rings are measured, for example the first five. Let N be the number of the order of the smallest ring measured; it is a whole number which is always known from the observation of coincidences made before the photographic exposures. The problem is to find the order of interference at the center, which can be formulated as $N+\epsilon$. The semidiameters ρ measured from the center obey the law $K\rho^2=N+\epsilon$.

K is a constant and takes successively the values 0, 1, 2, 3, 4, beginning with the central ring. Combining the five equations which result from the measurement of the five rings, ϵ is calculated together with the constant K. This last quantity can be deduced from the data furnished by the apparatus, but it is more correct to obtain it directly on the plate. The order of interference $N+\epsilon$ is thus determined within a few thousandths of a fringe.

We operate in the same manner on the second mercury plate. We should find precisely the same value for the order of interference if there were no change in the thickness of the étalon or in the index of refraction of the air. In point of fact, the two values obtained differ very little indeed when an étalon with invar blocks has been used. The difference is often less than one-hundredth of a ring and never surpasses 0.03, although the two plates may have been obtained at an interval of two hours and no precaution was taken to eliminate variations of temperature. The average of the two values found is adopted as the value of the order of interference of the violet mercury light.

The plate of the nebula is measured in the same manner. The whole number of the order of interference results from an approximate knowledge of the wave-length of the radiation under observation. The fractional figure is obtained as before by the

measurement of the ring diameters; the corresponding constant K may be deduced from that of the mercury by the observation that it varies in inverse ratio with the wave-lengths. Knowledge of the order of interference in mercury light and of the nebula yields immediately the ratio of the two wave-lengths and consequently that of the radiation of the nebula.

In reality there are differences of radial velocity from point to point and the measurements can only be made point by point. The first thing to be determined in the photograph of the nebula is the foot of the normal to the silvered surfaces whose position on the mercury photographs is defined by the center of the rings. It is in order to correlate the position of this point on our two classes of photographs that we have provided the reticle whose image is reproduced on the photographic plate.

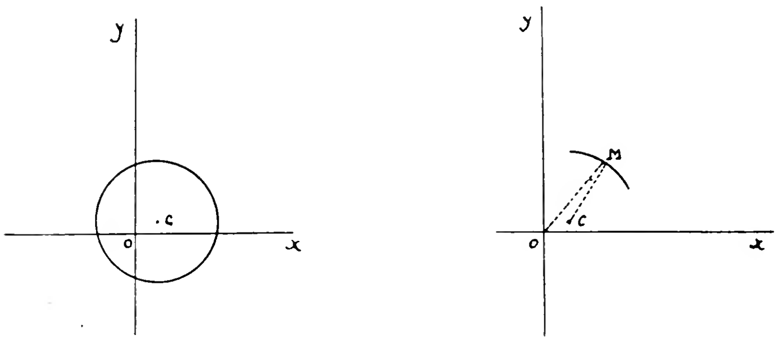


FIG. 2

On the mercury prints the center is determined in the following manner. Let Ox and Oy be the images of the threads of the reticle, and C the center of the rings (Fig. 2). We first orient the print on the comparator in such fashion that the displacement shall be parallel with Ox. With a thread parallel to Oy we then successively set on the two edges of the ring and Oy; from these measurements we deduce the distance from the point C to the axis Oy, that is, the abscissa of the point C with respect to the system of axes xOy. To increase precision, measurements are made on the first five rings. Operating in the same fashion after turning the plate 90°, we obtain the ordinate of the point C.

These co-ordinates, now known, are registered without alteration upon the photograph of the nebula where they determine the foot of the normal to the silvered surfaces.

Passing to the photograph of the nebula we measure, in a known direction from the point O, the distance OM from the origin of the co-ordinates to a point M of one ring. We shall have to deduce the wave-length λ' of the radiation which produced the ring at M. For this purpose we calculate the distance ρ' from the point M to the point C. This is easy because the co-ordinates of these two points are known. The calculation is simplified by the fact that points O and C are very near to each other and hence ρ' differs from OM only by a small amount. Let N' be the known order of interference of the ring which passes through M. $N'+\epsilon'=\mathrm{P}'$ is the order of interference which we should have at the point C if the wave-length everywhere had the same value λ' which it had at M. We then have

$$\epsilon'=K'\rho'^2.$$

K' has the same significance as before and is related to K by the formula

$$K'=K\frac{\lambda}{\lambda'}.$$

On the other hand, we know the order of interference $P=N+\epsilon$ which the mercury line λ yields at the center, and we thus have for the desired wave-length the value

$$\lambda'=\lambda\frac{P}{P'}.$$

This measurement can be made point by point on every ring, and from it we deduce the chart of the radial velocities in the nebula, as we shall see.

For these measures we used a comparator constructed by Gaertner of Chicago, after a slight modification of it. On the carriage moved by the screw, a vertical plate-holder is placed, which can turn in its own plane and which carries a divided circle. The reticle of the microscope which serves to make the settings

has but a single wire for the measurement of the mercury rings; whereas for the rings of the nebula, where we must take not a tangent but a definite point, the intersection of two cross-hairs is used.

THE STUDY OF THE RADIAL VELOCITIES

Absolute measurements of radial velocities can be made only by means of radiations capable of emission from terrestrial sources. With radiations of unknown origin nothing but a differential study of the velocities of different points can be accomplished.

For a known radiation we have employed the line H_γ of hydrogen. The measurements were made on photographs that were obtained with an étalon 1 mm thick, that is to say, with rings of the approximate order of 4600. They have been conducted in such a way as to eliminate the wave-length of the mercury line and to compare the radiation of the nebula with that of a hydrogen tube. For this purpose with the same étalon and with the same silvered surfaces, a comparison is made between the mercury line and the artificial line of hydrogen. The whole arrangement represented in Fig. 1 is transported to the laboratory. Here one photograph is taken with the mercury line and one with the line H_γ separated from the other hydrogen lines by a dispersive apparatus. This group of photographs gives the wave-length of H_γ, the value 4338.341 for the mercury line being used. The photograph of the nebula is calculated as we indicated above, the same value for the wave-length of mercury being used. The radial velocity at each point of the nebula results from a comparison between the wave-length at this point and the wave-length which has been found in the laboratory for H_γ. The result is independent of the value adopted for the mercury line, since this has served only as a convenient intermediary, on account of its great intensity.

With rings of the order 4600 a radial velocity of 1 km per second produces a change in the order of interference equal to 0.015, that is, a change of the same order of magnitude as the probable error of a single measurement. To give an idea of the effect of radial velocities on interference rings, let us consider the motion of the earth in its orbit, which produces the maximum radial velocity when the nebula is at 90° from the sun, i.e., in quadrature;

between one quadrature and the next, the difference of radial velocity is 60 km per second. This is equivalent to a change of one whole ring in the interference rings. If it were possible to photograph the rings every day between the two quadratures we should see them contract until every ring took the place of the next preceding one.

In performing the calculations as we have indicated, the absolute radial velocity with respect to the observer is obtained for every point which has been measured on the nebula. An average of these velocities can be taken for a definite surface and can then be corrected for the velocity of the earth in order to obtain the average velocity of the nebula with respect to the sun. On the other hand, the internal movements of the nebula will be represented by calculating the velocities of its several points with respect to this mean velocity.

The differential study of velocities can also be made on photographs obtained with the line of unknown origin λ 3728. This would not permit one to obtain absolute values. This line is finer than H_γ; it therefore permits the employment of fringes of a higher order, upon which the effect of radial velocities is even greater. Since the line is double, we shall choose the thickness of the étalon in such a manner as to have the two systems of rings coincide.

In this kind of study the interference method presents the advantage of yielding the radial velocities of the whole surface of the nebula simultaneously, whereas an ordinary spectroscope permits us to study only those points which are projected on the slit.

Results.—All our prints of the nebula were made with the rings centered upon the region of the trapezium; we utilized the first seven or eight rings. This permits us to study the radial velocity within a circle of about 4′ diameter. The luminous intensity is nevertheless sufficient to permit more extensive measurements, but for this it would be necessary to place the center of the rings in other regions of the nebula, because the measurements lose their precision when rings too remote from the center are utilized.

In the region surrounding the trapezium, the mean radial velocity with respect to the sun is $+15.8$ km per second; i.e., the

nebula is receding from the sun. This number is the average for values found for 58 points distributed in 12 directions about the trapezium in a radius of about $2'$.[1]

Again, the measures show variations of radial velocities from one point to another; this enormous gaseous mass is not at rest relatively. The rings show local deformations in certain regions, indicating, in certain portions of the nebula covering very small areas, irregularities of speed which may amount to about 10 km per second. Movements of this sort are manifested in the region to the southeast of the trapezium in the direction of the star Bond 685. Moreover, there are great collective movements; with respect to the mean velocity, the northeast region is withdrawing at a speed of something like 5 km per second, while the southwest region is approaching at pretty nearly the same speed. In general, the part of the nebula which we have studied has a sort of rotary movement about the line southeast-northwest, but with numerous irregularities.

WAVE-LENGTHS OF THE LINES OF NEBULIUM

We have measured wave-lengths of the ultra-violet group, which with a reflecting telescope is, for photography, the most intense of the whole spectrum. According to Wright[2] this group is composed of two lines, whose wave-lengths he was able to measure on only one plate. The precise determination of the wave-lengths is important because it furnishes a sure basis for attempting the identification of these lines with those of terrestrial elements.

Since it is out of the question to separate the two lines by an absorbing filter, we measure both of them on the same plate,

[1] The velocities found up to the present are:

Keeler (1891)	+17.7
Wright (1901)	+16.2
Vogel (1902)	+17.4
Frost and Adams (1904)	+18.5

The agreement of these numbers with each other and with ours may be considered as satisfactory, especially if one remembers that the velocity is not the same for all points and that the various observations probably do not apply to the same region.

[2] *Astrophysical Journal*, **16**, 53, 1902.

obtained with an étalon of thickness so chosen that the systems of rings of the two radiations are entirely separated. A preliminary measurement made with a difference of path of 250 microns gave a first approximation; the definite measurement was made with a difference of path of 1.3 mm.

To eliminate the effect of radial velocities in the result, the measurement was made according to the method set forth above, by determining the radii of the first five rings in the northwest direction from the trapezium. It is there that the study of radial velocities had shown the fewest inequalities of speed from point to point. The value +17.6 was adopted for the radial velocity of this region.

When the measured lines are compared with the violet radiation of mercury, which is rather far from them in the spectrum, it is necessary to make a small correction to take account of the dispersion of the change of phase by reflection from silver.[1] This correction has been determined by a study in the laboratory. For that purpose we had an étalon of small thickness constructed (130 microns) with the same silvered surfaces, and the resultant rings were measured, radiations of known wave-length being used. The difference in optical thickness in passing from λ 4358 to λ 3728 is only 0.0025 μ; the correction of the wave-length is 0.014 A.

Reduced to the international system the wave-lengths for a source at rest with respect to the observer are:

3726.100
3728.838.

The first of these two lines is the more intense.

The values we have given are exact to a hundredth of an angstrom.

The values given by Wright reduced to the international system are:

3726.25
3728.85 with an uncertainty of ±0.2.

In the list of the lines of known elements, none is found that can be identified with either of these two rays. Before precise measurements had been taken and before we knew that this line

[1] *Ibid.*, **28**, 169, 1908.

was double, the idea had been expressed that it could be attributed to oxygen, which has a rather strong line in this region. Now the wave-length of this line of oxygen in the international system is 3727.35. It falls at about an equal distance from the two lines of the nebula and the interval between both of them and it is much too large for identification to be even contemplated.

ATOMIC WEIGHT OF NEBULIUM. TEMPERATURE OF THE NEBULA

The kinetic theory of gases establishes a correlation between the velocity of agitation of the luminous particles, and thus the width of the lines, and the atomic weight and the temperature of the luminous gas. Now the study of interference rings permits us to obtain the width of the lines by increasing the difference of path and finding to what limit the interference rings are visible. All calculations lead to the following formula: Let T be the temperature of the gas, m the atomic weight of the luminous particles, referred to the ordinary system of atomic weights ($O=16$), and N the order of interference from which point the rings cease to be visible, and we have

$$N=1.22\times10^6\sqrt{\frac{m}{T}}.$$

Experiment has, in every case studied up to date, verified this formula when for m is substituted the atomic mass of the luminous gas, for the reason that the luminous particles have the same mass as the atom.[1]

The experimental determination of N therefore reveals a relation between m and T. In utilizing the radiations of a known gas, we have a measurement of the temperature of the source. Inversely, if the temperature is known, one can determine the atomic weight of a gas, known to us only by its spectrum. More simply, if the source gives at the same time known and unknown lines, the temperature is eliminated, and the relations of the atomic weights is given by the square of the ratio of the limits of interference.

We looked for the limits of interference for the hydrogen and for the lines of unknown origin, in particular the double ultra-

[1] H. Buisson and Ch. Fabry, *Journal de physique* (5), 2, 442, 1908.

violet line. Étalons are used of gradually increasing thickness, until the interference rings cease to appear.

Hydrogen.—We used only the ray H_γ, operating by photography. Interference rings are still visible with a difference of path of 4 mm (order of interference 9200). The limit of interference is a little above this number, probably very close to 10,000.

Nebulium.—We studied by photography the double line $\lambda\lambda$ 3726–3729. To obtain the limit of interference, we worked with increasingly large thicknesses selected in such a manner that the systems of fringes given by the two lines coincide. Exact knowledge of the wave-lengths permits a simple calculation of the differences of path for which these coincidences take place, and we found that it occurred for multiples of 0.5074 mm.

The interference rings still exist for a difference of path of 5.6 mm, that is, for a number of the order of 15,000. The limit is a little higher, and probably approaches 16,500.

This result shows that the unknown gas which emits the double ultra-violet line has an atomic weight higher than that of hydrogen. The ratio of the two atomic weights is $\left(\frac{16{,}500}{10{,}000}\right)^2 = 2.74$. A figure in the neighborhood of 3 is therefore the probable value of the atomic weight of this gas.

A strong green line $\lambda = 5006$ is also due to an unknown gas. We have made, thus far, only visual observations on this radiation, less exact than photographic observations. In spite of the feeble intrinsic brightness of the nebula, rings of which the order of interference reaches 11,000 were distinctly seen. The green line is therefore also emitted by a gas of greater atomic weight than hydrogen. It is not easy to obtain an exact value of the limit, but we consider as probable that this limit is less than 16,500, and, consequently, the green ray is emitted by a gas of lower atomic weight than the body which emits the ultra-violet group.

It is curious to note that the classification of elements recently given by Rydberg leads to the admission, between hydrogen and helium, of two unknown elements having respectively the atomic weights 2 and 3.

Temperature.—The limit of interference of the hydrogen line permits, by means of the formula given above, the calculation of

the temperature of the luminous gas. Assuming 10,000 for this limit, we find a temperature of 15,000 degrees. This number is a maximum; every accessory cause tending to diminish the clearness of the fringes will cause us to find a temperature that is too high, e.g., the differences in radial velocities of gaseous masses radiating to the same point.

CONCLUSIONS

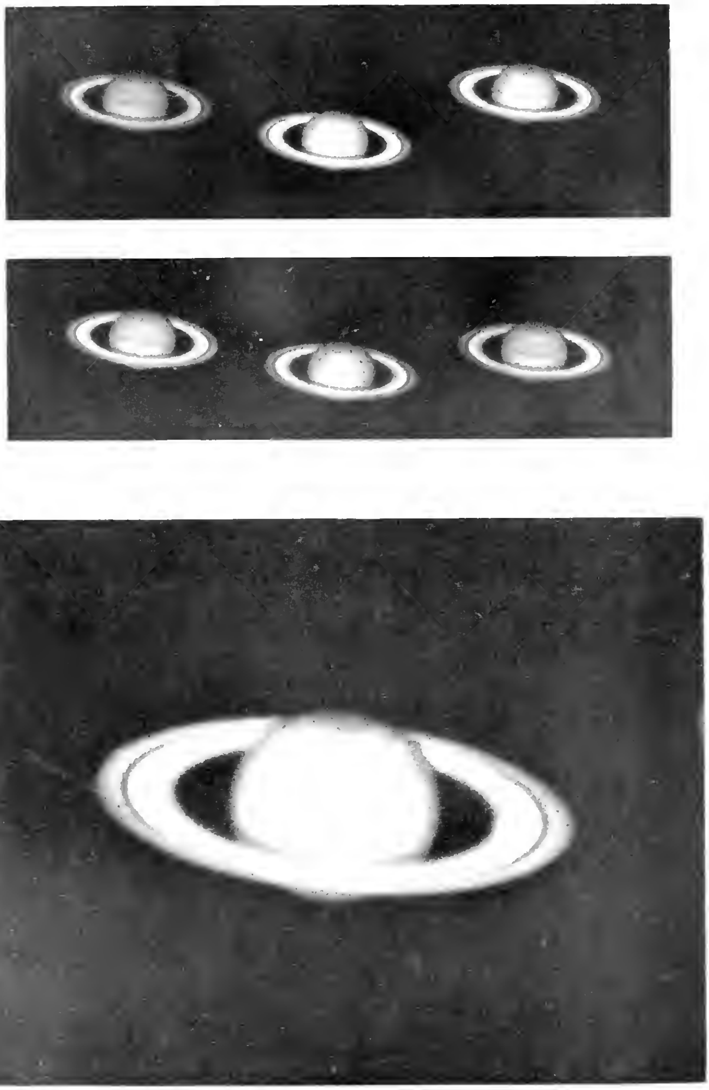
It is to be hoped that further results may be obtained by following the method which we have indicated. There are in particular still to be made the more detailed study of the velocities, on a greater scale, and the study of the green line by photography. The use of absorbents to isolate a radiation is never completely satisfactory. It might be possible to separate the images produced by the various lines by means of dispersive apparatus which would give them all on a single plate. There would be opportunity further to apply the interferential method to other nebulae, in particular to the planetary nebulae.

We must emphasize the simplicity of the apparatus used and the ease with which it can be mounted on the telescope. When the silverings have been carefully selected, the interferential apparatus does not cause the loss of much light and permits the study of objects of very feeble intrinsic brightness.

University of Marseilles
June 1914

PLATE VIII

South



SATURN, 1911, NOVEMBER 19 (E. E. Barnard

60-inch reflector, Mount Wilson

PHOTOGRAPHIC MEASURES OF SATURN AND ITS RINGS

By E. E. BARNARD

During part of November 1911, the writer, through the courtesy of Professor Hale, made some observations and photographs of the planet Saturn with the 60-inch reflector of the Solar Observatory at Mount Wilson. See *Year Book No. 11* of the Carnegie Institution for 1912, where a plate of Saturn on 1911, November 19, is given. This plate was also reproduced in the January, 1914, number of *L'Astronomie*. The photographs were made with a Brashear enlarging lens on the 60-inch reflector with the long focus mirror—the exposures being made near the great mirror just outside the tube. To secure sharper contrast a yellow color-filter and a Cramer Instantaneous Iso plate were used. The exposures, with one exception, were ten seconds each.

On the original negative the extreme diameter of the ring of Saturn is 19.01 mm, which is equivalent to the use of a telescope 97.5 meters (320 feet) in focal length.

Twelve images, one of which was purposely overexposed to show the crape ring, were made on the original plate. The crape ring, which extends half-way to the ball, is shown distinctly on the overexposed image (40 seconds' exposure). Several of the other exposures on this plate were injured by bad seeing, but most of them are very perfect and can be measured quite accurately. With the exception of the crape ring they show everything that can be seen with the eye. Indeed, any object as large as one-tenth of a second of arc would readily be visible on this plate—especially if it were an elongated marking. The Cassini division is so clearly shown that the micrometer wires can readily be placed on its edges. It can easily be traced all around the ring. Where it passes in front of the north part of the planet it is very thin but less distinct, and lighter, as if the ball were seen through it. The outer ring where it crosses the ball is lighter than elsewhere.

The relative brightness of the ball and different parts of the ring is well shown. The outer ring and the belts on the globe are about equal in brightness. The outer or brightest part of the inner bright ring is brighter than any portion of the globe. The polar regions are quite dark—much darker than the outer ring. The shadow of the planet on the rings is visible on both sides of the ball, perhaps a little stronger on the following side. The shadow seems to bend out where it reaches the Cassini division, just as it appears in visual observations near opposition.

Herr K. Graff of the Bergedorf (Hamburg) Observatory has recently made a set of measures of the images on the collotype plate as it appeared in *L'Astronomie*. These measures are given in the same publication for March 1914. Graff's results from the paper prints have led me to make a series of measures of the original negative, which, of course, is much superior to the reproduction.

In the work that follows I have measured the original negative and also two glass positives from it, one slightly enlarged and the other very much reduced in size. The original negative and the larger glass positive were measured on a Repsold machine belonging to the Columbia University, for the use of which I am indebted to both Dr. Harold Jacoby and Dr. S. A. Mitchell. The smaller glass positive (a lantern slide) was measured on a Gaertner machine, using a low magnifying power (5 diameters) to produce as sharp an image as possible. The original negative and the larger glass positive (which contained only six of the original images) were measured to see what difference would result from the negative and positive form of image.

As the actual scale of the photographs in seconds of arc is not known, Herr Graff gave his measures in terms of the equatorial diameter of Saturn. For this purpose I have adopted the diameter of the center of the Cassini division, which will give a better representation, as it is very definite and is less liable to halation effect. At the same time it is possible that through some eccentricity of the rings this may not be a constant quantity. It is easy, however, to transfer the measures to the equatorial diameter if any suspicion of change in the Cassini division should occur. The measures are, therefore, independent of scale, and though one cannot give

the quantities accurately in seconds of arc, they are perfectly accurate for the detection of any change that may hereafter occur in the Saturnian ball and ring system. These photographs were made within a few days of opposition and the phase effect would be insensible.

For comparison I give my visual measures of these same quantities, also referred to the Cassini division. A fairly close agreement between the visual and photographic quantities is shown. The most striking difference is in the polar dimensions of the ball—the polar diameter coming out decidedly less in the photographs, thus giving a greater polar compression.

In the original negative the overexposed image shows the crape ring fairly well, though it is faint. To make its edges more definite a greatly underexposed glass positive was made, containing the overexposed image. The measures of the crape ring on this positive are more satisfactory than on the original because of the density of the image of the other rings in the original. The Cassini division does not show, however (though visible on the original). I have, therefore, measured it on several of the other images on the same positive to get the relation of the diameter of the crape ring to that of the Cassini division.

These measures give the ratio of the diameter of the crape ring to that of the Cassini division:

Negative 0.620 Positive 0.627

or, mean ratio 0.623.

The measures of both the negative and the positive make the crape ring extend 0.51 of the space from the inner bright ring to the ball. Visual measures make this ratio a little greater.

By comparing the measures of the same quantities made on both the preceding and following parts of the planet, it was hoped to detect some eccentricity in the rings. There is, however, a very close agreement in these quantities, showing that no measurable eccentricity was visible in any of the elements of the rings at the time of the photographs.

In my measures of the ball and ring system made with the 36-inch refractor of the Lick Observatory in 1894 and 1895, no

eccentricity was apparent in the rings, though special observations were made at the time to detect any displacement of the ball. It would seem, therefore, that if any eccentricity exists in the rings it must be very small.

To show this apparent absence of eccentricity the following results (both for the visual measures with the 36-inch and from the present photographs) are given, where

α is the distance from outer edge of outer ring to center of ball.
β is the distance from center of Cassini division to center of ball.
γ is the distance from inner bright ring to center of ball.

		Visual		From Negative		From Enlarged Positive		From Reduced Positive	
		″	Ratio	mm	Ratio	mm	Ratio	mm	Ratio
α	Prec.	20.102	0.582	9.510	0.571	10.179	0.569	4.953	0.568
	Fol.	20.174	0.584	9.502	0.570	10.214	0.571	4.971	0.570
β	Prec.	17.100	0.495	8.336	0.500	8.952	0.500	4.351	0.499
	Fol.	17.206	0.498	8.326	0.500	8.958	0.500	4.367	0.501
γ	Prec.	12.744	0.369	6.335	0.380	6.778	0.379	3.315	0.380
	Fol.	12.714	0.368	6.343	0.381	6.792	0.379	3.331	0.382

For comparison with my measures, I have referred Graff's measures of the quantities, A, B, D, E, F, G, and (H, I) to his value (C) of the mean diameter of the Cassini division as unity. With the exception of F and G they are identical with my results from the original negative. These last quantities are:

$$F=0.497$$
$$G=0.442$$

These make the equatorial and polar diameters a little larger than my values. The exact agreement of the other values is remarkable.

The measures of the various quantities on the original negative and on the two glass positives follow.

TABLE I

ORIGINAL NEGATIVE

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
18.996	16.955	16.650	16.346	12.694	8.323	7.546	0.308	0.301	4.164	4.163	8.326	8.324
19.034	16.988	16.675	16.362	12.706	8.318	7.654	0.302	0.324	4.193	4.164	8.352	8.323
18.984	16.952	16.659	16.366	12.642	8.339	7.601	0.320	0.266	4.159	4.161	8.328	8.331
18.970	16.991	16.666	16.343	12.705	8.316	7.574	0.319	0.329	4.166	4.184	8.324	8.342
19.026	16.962	16.647	16.331	12.677	8.268	7.565	0.301	0.330	4.200	4.179	8.334	8.313
19.034	16.999	16.678	16.358	12.679	8.364	7.608	0.308	0.333	4.170	4.144	8.352	8.326
19.041	16.958	16.658	16.359	12.644	8.342	7.552	0.275	0.324	4.164	4.152	8.335	8.323
(19.200)				(12.412)	(8.567)	7.584						
						(7.692)						
19.012	16.972	16.662	16.352	12.678	8.324	7.586	0.305	0.315	4.174	4.164	8.336	8.326

TABLE II

ENLARGED GLASS POSITIVE

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
20.398	18.222	17.902	17.581	13.580	8.992	8.117	0.329	0.312	4.430	4.480	8.926	8.976
20.415	18.239	17.919	17.599	13.584	8.970	8.079	0.328	0.312	4.476	4.473	8.961	8.958
20.447	18.264	17.916	17.568	13.615	8.946	8.146	0.366	0.330	4.471	4.499	8.944	8.972
20.330	18.233	17.878	17.522	13.548	8.927	8.086	0.364	0.347	4.461	4.490	8.974	8.954
20.390	18.197	17.893	17.590	13.520	8.988	8.080	0.314	0.293	4.457	4.448	8.951	8.937
20.380	18.239	17.908	17.578	13.574	8.979	8.072	0.337	0.324	4.464	4.465	8.954	8.954
20.393	18.232	17.903	17.573	13.570	8.967	8.097	0.340	0.320	4.460	4.476	8.952	8.958

TABLE III

Reduced Glass Positive

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
9.944	8.860	8.719	8.579	6.656	4.285	3.995	0.128	0.153	2.213	2.221	4.355	4.364
9.898	8.827	8.701	8.575	6.631	4.270	3.971	0.138	0.114	2.216	2.215	4.351	4.350
9.931	8.875	8.733	8.591	6.642	4.298	3.961	0.118	0.166	2.212	2.223	4.361	4.372
9.917	8.868	8.714	8.560	6.645	4.282	3.951	0.158	0.150	2.202	2.230	4.343	4.371
9.935	8.877	8.727	8.576	6.651	4.283	3.938	0.147	0.154	2.225	2.219	4.366	4.361
9.925	8.858	8.710	8.563	6.659	4.308	3.958	0.131	0.164	2.190	2.212	4.344	4.366
9.917	8.860	8.724	8.588	6.635	4.295	3.971	0.121	0.151	2.206	2.223	4.354	4.370
9.920	8.856	8.720	8.582	6.668	4.286	3.931	0.143	0.131	2.194	2.240	4.337	4.383
9.930	8.886	8.720	8.554	6.628	4.302	3.981	0.150	0.182	2.198	2.220	4.349	4.371
(9.952)		8.715		(6.520)	(4.392)	3.992			(2.153)	(2.170)	4.349	4.366
						3.938						
						3.979						
9.924	8.863	8.718	8.574	6.646	4.290	3.964	0.137	0.152	2.206	2.223	4.351	4.367

The approximate values of the scale in the above measures, for the three sets, are:

Original negative, 1 mm = 2″.09
Enlarged positive, 1 mm = 1.95
Reduced positive, 1 mm = 4.00

The means of these various measures form Table IV, but the measures of the polar diameter in Table IV have been corrected for the tilt of the planet toward us. In Tables I, II, and III the apparent polar diameter is given.

TABLE IV

	Visual		From Negative		From Enlarged Positive		From Reduced Positive	
	″	Ratio	mm	Ratio	mm	Ratio	mm	Ratio
A.....	40.186	1.164	19.012	1.141	20.393	1.139	9.924	1.138
B......	35.034	1.015	16.972	1.019	18.232	1.018	8.863	1.017
C......	34.517	1.000	16.662	1.000	17.903	1.000	8.718	1.000
D.....	34.000	0.985	16.352	0.981	17.573	0.982	8.574	0.983
E.....	25.626	0.742	12.678	0.761	13.570	0.758	6.646	0.762
F......	17.798	0.516	8.324	0.500	8.967	0.501	4.290	0.492
G......	16.246	0.471	7.512	0.451	8.006	0.447	3.919	0.450
(H, I)..	0.517	0.015	0.310	0.019	0.330	0.018	0.145	0.017
N.....	20.528	0.595	10.340	0.620	....	....	....	....

The only complete measures of the planet and rings made with a large telescope by other observers, with which I am familiar, are those by Professor A. Hall[1] (1884–1887) and by Dr. T. J. J. See[2] (1901), both using the 26-inch Washington refractor, and by Messrs. F. W. Dyson and T. Lewis[3] (1895) with the 28-inch Greenwich refractor. Professor Hall and Professor See did not measure the polar diameter. From their measures I have deduced for comparison (Table V, p. 266) the following ratios with respect to the Cassini division.

It will be noticed that the different sets of visual ratios generally confirm each other, but they differ slightly from the photographic ratios, which are also consistent among themselves.

[1] *Washington Observations*, 1885, Appendix II.

[2] *Astronomische Nachrichten*, **157**, 389, 1902.

[3] *Monthly Notices, R.A.S.*, **56**, 14, 1895.

TABLE V

	Hall		Dyson and Lewis		See	
	"	Ratio	"	Ratio	"	Ratio
A	40.45	1.171	40.590	1.182	39.971	1.171
B	34.95	1.012	34.870	1.015	34.605	1.014
C	34.53	1.000	34.349	1.000	34.138	1.000
D	34.11	0.988	33.828	0.985	33.617	0.986
E	25.75	0.746	25.647	0.747	25.932	0.760
F	17.72	0.513	17.754	0.517	17.240	0.505
G	....	....	16.793	0.489	....	....
(*H*, *I*)	0.42	0.012	0.521	0.015	0.818	0.024
N	20.52	0.594	20.765	0.604	20.434	0.599

The following letter scheme will explain the quantities given in the tables. The measures in Tables I, II, and III are designated by small letters, and the means of the measures, given in Table IV, by capitals.

a=Outer diameter of outer ring.
b=Inner diameter of outer ring.
c=Diameter of middle of Cassini division.
d=Outer diameter of inner ring.
e=Inner diameter of inner ring.
f=Equatorial diameter.
g=Polar diameter.
h=Width of Cassini division on preceding side.
i=Width of Cassini division on following side.
(h, i)=Width of Cassini division—mean of h and i.
j=Middle of Cassini division to preceding limb.
k=Middle of Cassini division to following limb.
l=Middle of Cassini division (pr.) to center of planet.
m=Middle of Cassini division (fol.) to center of planet.
n=Inner diameter of crape ring.

Yerkes Observatory
Williams Bay, Wisconsin
June 1, 1914

NOTE ON THE TRANSPARENCY OF THE OUTER RING

In *Monthly Notices* of the Royal Astronomical Society for June 1914, Mr. Patrick H. Hepburn calls attention to the fact that these same photographs (glass positives of which I had sent to the R.A.S.)

show that the outer ring of Saturn is transparent. Though I had noticed that this ring was brighter where it crossed the ball (see the present paper), I did not attribute the effect to the ball being seen through the rings. It is a fact, however, that the photographs clearly show this ring to be transparent.

In Mr. Hepburn's paper he states that his measures of one of these glass positives show that the inner bright ring is wider on the preceding side than on the following, and he attributes this to some eccentricity in the ring (*M.N.*, **74**, 724). My measures do not show any difference in these quantities, as will be seen from the following, which are the means of the ratios with respect to the Cassini division from the three different plates.

		Ratio
Width of Ring *B*	preceding	= 0.111
	following	= 0.111

E. E. B.

September 1, 1914

STELLAR WAVE-LENGTH OF λ 4686 AND OTHER LINES IN THE SPECTRUM OF 10 LACERTAE

By EDWIN B. FROST and FRANCES LOWATER

The need has been felt for some years for the accurate determination of the stellar wave-length of the important line which has long been known as the first line of the principal series of hydrogen, but which recent theoretical studies have assigned with some degree of plausibility to helium. The line and its congeners at λ 4542, λ 4200, λ 4026 play an important rôle in the spectra of stars at that end of the spectroscopic sequence which we are accustomed (probably correctly) to call the beginning.

PREVIOUS DETERMINATIONS

The wave-length was originally inferred from objective-prism plates at Harvard as 4688, but later given more precisely as 4685.4. Rydberg derived a wave-length of 4687.88 from his formula,[1] based upon the values he had available for the so-called "sharp" series.

A note on the wave-length of this line in stellar spectra, chiefly in the star 10 Lacertae, with which we are concerned in the present paper, was presented by Frost and Adams at the Washington meeting of the Astronomical Society in December 1902.[2] The value obtained was closely in accord with the value now given in this paper, which is based upon a much greater amount of material.

Meanwhile Professor A. Fowler has been successful in producing the line λ 4686 and others in the laboratory[3] and from accurate measures on plates taken with a concave grating he derives the wave-length 4685.90 (on Rowland's scale, to which are referred all data in this paper). The scale of his spectrogram was 5.5 A per mm, insuring a high degree of accuracy in the result.

[1] *Harvard Annals*, **56**, 103; *ibid.*, **28**, 150; *Astrophysical Journal*, **6**, 236, 1897.

[2] *Publications of the Astronomical and Astrophysical Society of America*, **1**, 207.

[3] *Monthly Notices, R.A.S.*, **73**, 62, 1912.

The line has often been photographed in the flash spectrum at solar eclipses, e.g., by one of us[1] in 1900 at λ 4685.7; and by H. C. Lord at the same eclipse.[2] More accurate wave-lengths were later given by Lockyer, λ 4685.90 (in exact agreement with our stellar wave-length); and by Dyson, 4685.86; and most recently by S. A. Mitchell, λ 4686.00.

CLASSIFICATION OF STAR

This line is sharper in the spectrum of 10 Lacertae ($\alpha=22^h35^m$; $\delta=+38°32'$; Mag. $=5.0$) than in other stars in the collection made with the Bruce spectrograph, as was recognized when the first plates were taken. The other lines of hydrogen and helium are, in this spectrum, also sharper than usual, so that the radial velocity can be pretty well determined from those well known lines. The lines will not bear too high a dispersion, however, and we have found the use of one prism most suitable, as for most of the helium stars. With one prism the scale-value on the spectrogram is 35 A = 1 mm at λ 4686; or 20 A = 1 mm at λ 4089.

The star is placed by Miss Cannon in class Oe5, resembling closely the prototype 30 τ Canis Majoris.[3] It is unfortunate, but hardly avoidable, that so many of the typical stars of this classification have proved to be spectroscopic binaries, for where lines of the second component are visible, the spectrum is likely to be unduly complicated—two spectra instead of one. For instance, the three typical stars of the adjacent classes Oe, Oe5, and B have been found by one of us to be spectroscopic binaries. 10 Lacertae itself, unfortunately enough, proves to vary in its radial velocity, but this does not affect the wave-lengths, as the values from each plate are corrected for the velocity found on that plate from the well known lines of hydrogen and helium. But on a few plates some of the lines could be seen to be double, complicating or frustrating the measurement of radial velocity, so that these plates had to be excluded when wave-lengths were determined.

The general resemblance to 30 τ Canis Majoris as noted at Harvard is confirmed on the Yerkes plates, but the lines λ 4089,

[1] *Astrophysical Journal*, **12**, 344, 1900. [2] *Ibid.*, **13**, 159, 1901.

[3] *Harvard Annals*, **56**, 102.

4097, 4116, 4686 are very much better defined in 10 Lacertae. The resemblance on our plates is much closer in case of 46 ϵ Orionis, the typical star of class B0, amounting to a practical identity except for the superior sharpness of λ 4686 in 10 Lacertae. We should therefore be disposed to classify 10 Lacertae under group B0.

The lines above named may be properly designated as sharp with our dispersion. Other conspicuous lines are the helium lines λ 4472 and λ 4026, and Ca K λ 3933, all quite sharp. Calcium H λ 3968 is much sharper than H_ϵ. H_δ is fairly measurable, and has beside it, tolerably distinct, the silicon (?) line λ 4103. H_β and H_γ are measurable with less difficulty than in most B stars.

Of the other helium lines, λ 3964 is fairly clear, λ 4009 is faint on our plates. It is a singular fact that λ 4026.3 of helium seems to be single and not confused by the line of the "sharp" series at λ 4026, which Pickering designated as ϵ'. λ 4024 of helium was not measured on any of the plates, but seems to be faintly present.

On plates 89, 419, 431, 439, 3481, several lines were double or complex, showing the presence of the fainter component; the principal ones were H_β, H_γ, H_δ, and 4388.

MEASUREMENTS AND RESULTS

All the plates were measured by each of us, and from the well known hydrogen and helium lines a first approximation to the radial velocity on the plates was made. Titanium and iron furnished the comparison spectrum. The wave-lengths, to a first approximation, of lines required were then derived after applying the proper correction for the radial velocity, the results for each observer being left separate. A second approximation for velocity and wave-lengths was then made, of course differing only slightly from the first. The results from the two observers were combined, and then, utilizing these wave-lengths of the lines λ 4089, 4097, 4116, and 4686, a final value of the radial velocity for each observer was obtained. These wave-lengths are given in Table I.

It will not be necessary to tabulate the separate determinations of wave-length for the four lines, which were in excellent accord among themselves and between the two observers. The condensed results are as tabulated.

The last three lines are of considerable importance in stellar spectra of classes Oe to B2. Approximate wave-lengths were given by Miss Cannon from objective-prism plates, and the chemical

TABLE I

Frost :4685.897±0.016 (15 plates) Lowater: .903±0.018 (15 plates)	4116.338±0.015 (12 plates) .328±0.018 (14 plates)
Means: 4685.90	4116.33
Frost :4097.536±0.011 (12 plates) Lowater: .564±0.014 (14 plates)	4089.127±0.012 (13 plates) .115±0.011 (14 plates)
Means: 4097.55	4089.12

origin and position of these lines have been studied in the laboratory by others, most recently by Lunt, by Lockyer, Baxandall, and Butler, and by Sir William Crookes. The values obtained in the spark spectrum of silicon are:

Lunt[1]	4089.00	....	4116.35
Lockyer, Baxandall, Butler[2]	.04	4097.49	.29
Crookes[3]	.02	....	....

Crookes does not find λ 4116 in the spectrum of pure elementary silicon, and he gets the wave-length 4097.02 for a silicon line. This cannot be the stellar line above measured by us at λ 4097.55, which has been attributed by Lunt to some impurity in his silicon, and by Lockyer and his collaborators to nitrogen. For this, laboratory values have been published as follows:

Neovius	4097.4
Hemsalech	4097.3
Exner and Haschek	4097.43

From plates of ϵ Orionis taken with an objective-prism, Lockyer and his colleagues find for these lines wave-lengths as follows:

4089.14 4097.59 4116.54

The other lines of the so-called "sharp" series of hydrogen, at λ 4542 and 4200, were vague and broad on these spectrograms, so

[1] *Annals of the Cape Observatory*, 10, Part 2, 1906.

[2] *Proc. Roy. Soc.*, **82** A, 532, 1909.

[3] *Ibid.*, **90**, 512, 1914.

that their wave-lengths could be determined only roughly. We are not familiar with any stellar spectrum in which they are well defined. The results are:

Frost	λ 4541.8	(10 plates)	4200.3	(6 plates)
Lowater	4542.0	(10 ")	4200.3	(5 ")
Mean	4541.9		4200.3	

We have mentioned above that the next line of this series, at about λ 4026, is not measurable, either because of its faintness or because it is masked by the excellent helium line at λ 4026.3. The

TABLE II

RADIAL VELOCITY OF 10 LACERTAE

Plate	Date	G.M.T	Frost		Lowater		Mean Km	Quality	Taken By
			No. of Lines	Km	No. of Lines	Km			
89	1903, Sept. 25	$14^h 44^m$	......	......	......	......	......	......	F
128	Oct. 17	18 34	8	−11.6	9	−10.1	−10.9	g.	F
150	Oct. 24	14 50	9	13.6	8	16.3	14.9	v. g.	A
409	1904, Nov. 1	15 14	11	14.7	10	15.5	15.1	v. g.	B
419	Nov. 11	14 31	9	9.6	9	10.5	10.0	g.	F
431	Nov. 15	14 37	10	9.9	10	7.8	8.9	g.	B
439	Nov. 18	14 34	11	12.0	8	8.9	10.4	v. g.	F
556	1905, July 14	19 07	5	5.4	4	4.6	5.0	f.	B
852	1906, Sept. 17	15 45	11	15.6	10	19.9	17.8	g.	F
1080	1907, June 10	20 46	11	18.4	10	15.6	17.0	v. g.	F
1118	July 26	19 38	10	19.2	8	16.0	17.7	v. g.	F
1239	Nov. 22	14 21	9	10.5	9	8.7	9.6	v. g.	B
1662	1908, July 31	20 49	10	15.8	9	12.5	14.2	v. g.	B
2165	1909, Oct. 29	15 10	9	15.8	8	18.4	17.1	g.	L B
3473	1913, Aug. 29	20 09	7	8.5	7	6.9	7.7	p.	F
3481	Sept. 4	19 17	8	10.8	9	11.5	11.2	g.	F
3483	Sept. 5	15 47	6	9.2	8	9.0	9.1	g.	L
3490	Sept. 8	18 22	9	8.6	8	9.4	9.0	g.	B
3496	Sept. 10	19 55	10	14.2	10	12.2	13.2	g.	L

In the last column A=Adams, B=Barrett, F=Frost, L=Lee. Mr. Sullivan assisted as usual. g.=good; f.=fair; p.=poor; v.=very.

next line of this series beyond this, at λ 3924, is too far from the center of good focus for measurement, and it was not measured on any plate; but on strongly exposed plates a line is seen near this point.

For plate 89 the results must be given separately for the two components (Table III).

TABLE III

	Single Lines		Double Lines		
Frost.........	5	−14 km	3	−57 km	+22 km
Lowater.......	6	−18	3	−42	+26
Mean.........		−16 km		−50 km	+24 km

The above data are insufficient for indicating the period of this star, but there seems to be little to suggest a period of less than several days. The range on the best of the above plates is about 8 km, extreme range for the different plates, 13 km. The motion of the system is probably about −12 km, almost wholly due to the solar motion.

It might be thought necessary to correct the wave-lengths given above for the discrepancy of about +5 km found to be inherent at present in our determinations of radial velocity of B stars, but a moment's reflection will show that these wave-lengths depend upon the laboratory values of the hydrogen and helium lines, and they should therefore be unaffected by this systematic error.

YERKES OBSERVATORY
September 3, 1914

THE ÅNGSTRÖM COMPENSATION-PYRHELIOMETER AND THE PYRHELIOMETRIC SCALE[1]

By A. K. ÅNGSTRÖM

I propose to discuss here a small source of error which has appeared in the construction of the Ångström compensation-pyrheliometer. The correction involved is shown not to exceed 2 per cent, but since the instrument has been accepted as a standard instrument by the International Congress at Oxford, and since this correction is of particular interest in the construction of instruments of this type, a somewhat detailed discussion may be desirable.

In this paper, therefore, I intend (1) to deduce the magnitude of the correction theoretically, and (2) to give an experimental determination of this correction.

As is well known, the principal part of the Ångström pyrheliometer consists of two equally blackened manganin strips, to the middle points of which are connected the two junctions, m and n, respectively, of a thermo-element. The two metal strips are soldered at both ends to the metal supports O and P, O' and P'.

The measurement of the radiation is made as follows: One of the strips is exposed to the radiation, the other screened from it. We then measure the current which must flow through the screened strip in order to keep the attached junction of the thermo-element in the middle of the strip at the same temperature as the junction at the middle of the illuminated strip. The two strips are then interchanged so that the strip previously covered is illuminated, and vice versa. The intensity of the radiation is computed from the mean value obtained from three such measurements of the current.

If the distribution of temperature in the two strips is exactly the same, this procedure will naturally give entirely correct results. But this will not be the case if the distribution of temperature is not the same. Such a difference in distribution is due principally to two causes.

[1] This paper will appear also in the *Meteorologische Zeitschrift.*

1. The illuminated strip is warmed from the surface, whereas the shaded strip is heated throughout its entire volume by the electric current. The consequence is that the temperature-gradient, taken from the surface inward, is not quite the same in the two cases. It is easy to show that in practice (length of the manganin strips 2 cm, thickness 20 μ, thickness of the coating of soot 10 μ) the error resulting from this cannot exceed one-half of 1 per cent.[1] This is on the assumption that the conductivity of

[1] We consider first the problem of ascertaining the illumination necessary to impart a temperature t_2 to the back surface of a thin strip consisting of two portions laid face to face. We obtain easily

$$I = t_2(\alpha+\beta+\alpha\beta\mu)$$

where

$$\mu = \frac{c_2 d_1 + c_1 d_2}{c_1 c_2}$$

and α is the rate of efflux of heat per element of area of the illuminated face and β is the rate of efflux of heat per element of area of the back surface. d_1 and d_2 are the thicknesses, c_1 and c_2 the thermal conductivities, of the two substances 1 and 2.

In the case where the second portion is heated by an electric current, producing a quantity of heat W in an element of volume determined by the element of area and the thickness of the strip, we can deduce

$$W = t_2 \frac{\alpha+\beta+\alpha\beta\mu}{1+\alpha\mu-(\alpha d_2/2c_2)},$$

whence

$$\left(\frac{I}{W} = 1+\alpha\mu-\frac{\alpha d_2}{2c_2}\right)$$

If now, as in the compensation-pyrheliometer, the first substance is soot, the second manganin, c_1 may be neglected relatively to c_2, and we get

$$\left(\frac{I}{W} = 1+\frac{\alpha d_1}{c_1}\right)$$

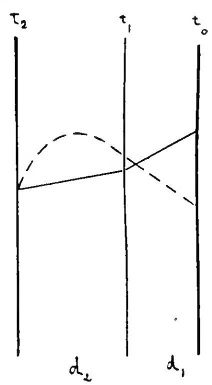


FIG. 1

If then we set: $\alpha = 0.00026$ (MacFarlane); $d_1 = 0.001$; $d_2 = 0.002$, and $c_1 = 0.57 \times 10^{-4}$ (conductivity of air), we obtain

$$\left(\frac{I}{W} = 1.0046\right)$$

wherein we have taken the conductivity of soot as equal to that of air, which probably gives much too small a value for c_1, and hence too great a value for $\alpha d_1/c_1$. We therefore conclude that the correction to W required by (1) alone cannot exceed one-half of 1 per cent.

the soot is not less than that of the air. Such a result can also be deduced from the empirical investigations of Kurlbaum.

2. A second cause of the difference in temperature-distribution between the strips arises if the exposed strip is illuminated along a part of its length only. This in fact is the case. In a pyrheliometer (No. 158) used by me in expeditions to Algiers and to Mount Whitney, it was found that about one millimeter at the edges of the illuminated strip (the length of which was 2 cm) was in the shadow of the diaphragm.

An idea of the magnitude of this border-effect, which is an important source of error in all instruments of similar construction, will be gained from what follows. I will assume the following simplifications: that the temperature is the same throughout each cross-section, and that the temperature of the supports O, P, and O', P' is the same as that of the surroundings. If, further, we assume that the rate at which each surface-element loses heat is proportional to its difference of temperature from the surroundings $(-kt)$, we obtain for the determination of the thermal condition of the electrically heated strip

$$Wadx = ktadx - abc\frac{d^2t}{dx^2}dx \tag{1}$$

where W is the heat evolved per element of area of the strip, a the breadth of the strip, b its density, and c its internal thermal conductivity.

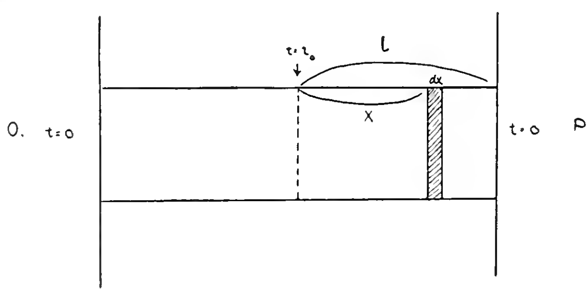


FIG. 2

From (1) we obtain

$$\frac{d^2t}{dx^2} - \frac{kt}{bc} + \frac{I}{bc} = 0 \tag{2}$$

The general solution is of the form:

$$t = Ae^{-\mu x} + Be^{\mu x} + S$$

where

$$\mu = \sqrt{k/bc} \qquad S = W/k$$

If the constants A and B are determined from the conditions $t=0$ when $x=l$ and $dt/dx=0$ when $x=0$ (the latter follows from symmetry), we obtain

$$A = B = -\frac{S}{e^{-\mu l} + e^{\mu l}}$$

For the temperature t_0 at $x=0$, i.e., the temperature at the junction, we have

$$t_0 = S\left[1 - \frac{2}{e^{-\mu l} + e^{\mu l}}\right] \tag{3}$$

This temperature t_0 must be equal to the temperature of the middle point of the illuminated strip. For the illuminated part of this strip there holds, in analogy with (1), the equation:

$$t = A_1(e^{-\mu x} + e^{\mu x}) + S_1 \tag{4}$$

where

$$S_1 = I/k,$$

I being the intensity of radiation per element of area. For the part not illuminated we have

$$\frac{d^2t'}{dx_1^2} - \frac{k}{bc}t' = 0 \tag{5}$$

which gives the solution:

$$t' = Me^{-\mu x'} + Ne^{\mu x'}$$

where A_1, M, and N are determined by the conditions that at $x=d$, $t_1=0$, and at $x=0$, $t_1=t$, and the third condition,

$$\left(\frac{dt}{dx}\right)_{x=l-d} = \left(\frac{dt'}{dx_1}\right)_{x_1=0}$$

whence we obtain

$$\left.\begin{aligned} Me^{-\mu d} + Ne^{\mu d} &= 0 \\ -M + N &= A_1[e^{\mu(l-d)} - e^{-\mu(l-d)}] \\ M + N &= A_1[e^{\mu(l-d)} + e^{-\mu(l-d)}] + S_1 \end{aligned}\right\} \tag{6}$$

and from (6)

$$A_1=\frac{S_1}{2}\left[\frac{e^{-\mu d}+e^{\mu d}}{e^{-\mu l}+e^{\mu l}}\right],$$

which together with (4) gives

$$t_0=S_1\left[1-\frac{e^{-\mu d}+e^{\mu d}}{e^{-\mu l}+e^{\mu l}}\right]. \tag{7}$$

From (3) and (7) follows the relation desired:

$$\frac{I}{W}=\frac{\psi(l)-1}{\psi(l)-\psi(d)}, \tag{8}$$

where

$$\psi(l)=\frac{e^{-\mu l}+e^{\mu l}}{2}.$$

Knowing l, d, and μ, we can now determine from (8) the relation between the quantity of heat generated by the current per element of area, and that imparted by the radiation. We thus obtain a value $(1+\delta)$, where δ is the correction which must be made upon the value for the quantity of heat computed from the strength of the current. If c is set$=0.52$ (from Jaeger and Diesselhorst), and k is set$=0.00018+0.00026=0.00044$ (from MacFarlane), the computed value for μ is 0.65, since the thickness of the strip is 20 μ.

The length of the strip is 2 cm; hence $l=1$ cm. Hence we have

$$\frac{I}{W}=\frac{0.2188}{1.2188-\psi(d)}$$

The following table is instructive:

d	I/W
0.05	1.0025
0.1	1.011
0.15	1.023

The correction increases rapidly as d increases. In our special case d was estimated at about 1 mm. The corresponding correction, according to the theoretical considerations, is 1.1 per cent.

Experimentally the correction was determined by replacing the diaphragm belonging to the instrument by another, which permitted the illumination of the whole strip. In both cases the

readings were compared with those of a pyrheliometer belonging to the Smithsonian Institution (A.P.O. 9). The mean value of five series gave in the first case the relation

$$\frac{\text{A.P.O. 9}}{\text{Ångström}^{1}} = 1.0430$$

and in the second case

$$= 1.0299$$

The difference of 1.3 per cent is, therefore, to be ascribed to a border-effect, the sign of which is such that the readings give values too small. In my opinion, therefore, all measurements made hitherto with the Ångström pyrheliometer should be corrected by +1.3 per cent. As I have since found that my pyrheliometer No. 158 gives values about 1 per cent lower than most Ångström pyrheliometers, the difference between the International scale and that of the Smithsonian Institution becomes after this correction about 2.5 per cent (±1 per cent).[1]

As regards this difference, it is perhaps too early to make a definite statement. It must be conceded, however, that Abbot's method (the water-flow pyrheliometer) has a distinct tendency to give values too high. Thus in 1908 the difference between the two scales was 9.2 per cent,[2] in 1911 it was 5.5 per cent,[3] and in 1913 it was 3.9 per cent,[4] whence it is evident that the Abbot scale after revised measurements has gone considerably downward. On the other side, all the sources of error in the Ångström method tend to result in too small values, and the possibility of a necessary extra per cent of correction is not excluded. It is, therefore, probable that the true pyrheliometric scale lies between limits which do not differ by more than 2.5 per cent of the quantity of heat measured.

In this connection it may be of interest to mention briefly an experimental result which my friend Professor C. G. Abbot has

[1] Vol. 3 of the *Annals* of the Astrophysical Observatory of the Smithsonian Institution gives the difference as 3.9 per cent.

[2] *Ibid.*, 2, 47.

[3] Abbot and Aldrich, *Astrophysical Journal*, 33, 125, 1911.

[4] *Annals*, 3, 72.

kindly placed at my disposal. It follows from this that the measurements with the compensation-pyrheliometer are practically independent of pressure. Abbot inclosed the Ångström pyrheliometer in a glass tube which could be evacuated, and compared the readings at different pressures with those of a pyrheliometer outside. He found that at very low pressures (0.1 mm) the instruments gave values differing by 4.9 per cent from those obtained at atmospheric pressure. At a pressure of 34 cm nothing remained of this effect. When the ratio of the readings of the two pyrheliometers is set $=1$ at atmospheric pressure, the value of the ratio at 34 cm comes out 1.0001 (eight measurements).

The erroneous result at very low pressures is probably to be ascribed to a magnification of the above-mentioned border-effect.

SUMMARY

Theory and experiment show that the readings of the Ångström pyrheliometer must be corrected by about $+1.3$ per cent. The difference between this scale and that employed by the Smithsonian Institution appears, therefore, to be 2.6 per cent.

Experiments show that the constant of the instrument is independent of pressure within the limits of practical work.

I am much indebted to Professor C. G. Abbot for valuable discussions with him.

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UPSALA
May 1914

[1] *Ibid.*

A SHORT METHOD FOR DETERMINING THE ORBIT OF A SPECTROSCOPIC BINARY

By HENRY NORRIS RUSSELL

So many ways of determining the orbit of a spectroscopic binary have been published that an additional one would appear to demand some justification. If the problem were only to determine the orbital elements from a given velocity-curve, of the exact theoretical form consistent with elliptic motion, any one of several existing methods (some of them of great mathematical elegance) would afford a satisfactory solution. But the computer has actually to fit such a curve to a series of points, derived from observations which are often affected by considerable errors; and this makes the problem much more difficult, so that it is not unusual to read that several trials were necessary to reach preliminary elements which appeared near enough to the truth to be used as a basis for a correction by least squares, while sometimes a second approximation by this laborious method has been necessary.

For nearly circular orbits, the analytical method of Wilsing, as extended by the writer,[1] affords a satisfactory solution; but it is convenient only for eccentricities less than 0.3. King's graphical method[2] also makes it possible to use the whole course of the velocity-curve in finding the preliminary elements, and is applicable to orbits of any eccentricity. Both these methods depend upon the velocities at equal intervals of time, as read from a freehand curve drawn to represent the observations. The method here proposed is free from this limitation, and enables the computer to fit his elements directly to the observations themselves, though it may be applied also to a freehand curve. It is equally applicable to orbits of all eccentricities.

Let it be supposed that a period, satisfactory at least as a first approximation, has been found, and that the observations have been assembled on this period, and combined into normal places,

[1] *Astrophysical Journal*, **15**, 252, 1902.

[2] *Ibid.*, **27**, 125, 1908.

in the usual manner. We know for each normal place the radial velocity ρ and the time t referred to some arbitrary initial epoch. If M is the mean anomaly in the orbit, T the time of periastron passage, and n the mean motion, we have

$$M=n(t-T).$$

Setting $M_0=nT$, we may write

$$nt=M+M_0 \tag{1}$$

so that the value of $M+M_0$ is known for each observation.

The fundamental equation for the radial velocity may be written, with the usual notation,

$$\rho=\gamma+Ke\cos\omega+K\cos(v+\omega)=G+K\cos(v+\omega) \tag{2}$$

where v is the true anomaly in the orbit. The maximum velocity is $G+K$, and the minimum $G-K$, so that G and K may be estimated at once from a freehand curve. We may then write (2) in the form:

$$\cos(v+\omega)=\frac{\rho-G}{K} \tag{3}$$

and compute $v+\omega$ from each observed value of ρ. If we subtract the corresponding values of $M+M_0$ from each of these, we shall have values of $(v-M)+(\omega-M_0)$. The second part of this expression is constant, while the first is the equation of the center in the elliptic motion. During a revolution this varies between equal positive and negative limits which depend only on the eccentricity, and are nearly proportional to it, as is shown in the following table.

TABLE I

Eccentricity	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
Max. equation of center	11°5	23°0	34°8	46°8	59°2	72°3	86°4	102°3	122°2

If the values of $v-M+\omega-M_0$ are plotted against those of $M+M_0$, we obtain a diagram which, since it represents the relations between the mean and the true anomalies, we may call the anomaly diagram. If on this diagram a curve is drawn to represent

the plotted points, half the difference between its maximum and minimum ordinates will be the greatest value of the equation of the center, from which e may be found at once by means of Table I.[1] The mean of the maximum and minimum ordinates will be the value of $\omega - M_0$. The instants when $v - M + \omega - M_0$ has this value are those of periastron and apastron passage, the former corresponding to the ascending branch of the curve, which is always the steeper. The abscissae of the corresponding points of the curve are M_0 and $M_0 + 180°$. The values of e, M_0, and ω are now known, and the remaining elements may be found at once from K and G.

The principal advantage of this method is that the form of the curves which give $v - M$ as a function of M depends upon e alone. For orbits of the same eccentricity, a change in M_0 (that is, in T) shifts the curve on the anomaly diagram horizontally, and one in $\omega - M_0$ shifts it vertically, without otherwise altering it. It is therefore possible to draw upon tracing-cloth, once for all, a set of curves with $v - M$ as ordinates and M as abscissae, for every 0.05 or 0.10 of e. Such a diagram may very easily be prepared with the aid of the tables for the true anomaly in elliptic orbits given in Vol. 2 of the *Publications of the Allegheny Observatory* (pp. 158–190). By superposing this sheet upon the anomaly diagram for any star (plotted to the same scale), and shifting it in both co-ordinates (keeping the co-ordinate axes on the two sheets parallel), it is easy to determine what value of e, and what position of the periastron in time and space, will give the best representation of the observations.

In this connection it must be remembered that the values of $v + \omega$ determined from values of its cosine near ± 1—that is, from radial velocities near the maximum or minimum—are subject to considerable uncertainty. The corresponding points should be distinguished from the others in the plot, and attention directed mainly to obtaining a good fit for the well-determined points. It

[1] If ϵ is the maximum equation of the center, in degrees, the approximate formula

$$e = \tfrac{7}{8} \left\{ \frac{\epsilon}{100} - 10 \left(\frac{\epsilon}{100} \right)^3 \right\}$$

gives results which are correct within 0.001 for values of e less than 0.80, and only 0.005 too great when $e = 0.90$.

is best to compute $v+\omega$ directly from the observed velocity, except when this is near the maximum or minimum, in which case the reading of a smooth curve drawn to represent the course of the observations is usually preferable. It will usually be found that e cannot be varied by more than 0.02 on either side of the best value, or $\omega-M_0$ by more than a degree or two, without introducing inadmissible discordances with the observations. The precision with which M_0 is defined depends upon the eccentricity. When this is large, it may be uncertain by less than a degree; when it is small, by many degrees.

When a satisfactory fit has been obtained, the theoretical velocity-curve, corresponding to the assumed elements, may be computed at once. Keeping the tracing-cloth fixed, relative to the anomaly diagram below it, the values of $v-M+\omega-M_0$ corresponding to any desired values of $M+M_0$ may be read off, by interpolation between the curves for the adjacent values of e. Adding, we have the values of $v+\omega$, and the velocity is given by (2). Near periastron, in very eccentric orbits, the curves run so steeply that the values of $v-M+\omega-M_0$ cannot be read from them with precision, and it is well to use the Allegheny tables to find v.

If the points on the anomaly diagram cannot be satisfactorily represented by means of a curve of the theoretical form, it is probable that the assumed values of K and G are in error. Even in such a case, good approximations to e, ω, and M_0 may usually be obtained from the diagram. If a velocity-curve is computed from these values, and the assumed K and G, and plotted along with the observations, it is easy to see what changes in the maximum and minimum, and hence in K and G, are needed. It is still better to plot the residuals for the individual observations from this first approximation against the computed velocities, and draw a straight line to represent them as well as may be. The readings of this line opposite the originally assumed maximum and minimum velocities give the corrections which must be applied to them in order to get the best representation of the observations with the values of e, ω, and M_0 derived in the first solution. From these, new values of K and G may be found, and a second anomaly diagram prepared, from which new values of e, ω, and M_0 are found as

above. Such a second approximation is necessary only in difficult cases, and a third is seldom if ever requisite.

The method here developed is closely analogous in principle to the graphical method of Dr. King, but has the advantage that a single diagram takes the place of the numerous "protractors" which must be constructed for each separate value of the eccentricity. It is also probable that the graphical processes, which demand the exact drawing of lines and measurement of angles will consume more time than the methods here suggested. After the observations are plotted, and the values of K and G estimated (which is necessary in both methods), the individual values of $v+\omega$ can be read from a slide-rule as fast as they can be written down. It takes then but a few minutes to plot the anomaly diagram, and a few more to adjust the tracing-cloth over it, except in cases of unusual difficulty. The preparation of this latter diagram, which is done once for all, with the aid of the Allegheny tables, takes two or three hours. The complete solution for an ordinary star, from the beginning of plotting the observations to the determining of the elements and the residuals from the theoretical curve, may be made in less than an hour.

As an example of the practical application of this method the discussion of α Draconis may be given in detail. An orbit of this system, determined by Harper[1] from 59 plates taken at the Dominion Observatory, Ottawa, after a thorough discussion by least squares, may be taken as a standard of comparison.

The period, 51.38 days, was found by comparison with observations made several years earlier, and no correction to it was attempted in the final adjustment. Harper's normal places, with phases counted from a preliminary determination of the periastron, are given in the first two columns of Table II, and the weights assigned by him follow. The remaining portion of the table gives the solution by the method of this paper.

The values of $M+M_0$ for each normal place are computed by (1), and the observations plotted. From a freehand curve drawn to represent them, the maximum and minimum velocities are

[1] *Report of the Chief Astronomer* for 1910, Appendix No. 2, pp. 147–149 (Ottawa: Government Printing Office, 1912).

read off to be $K+G=+48.0$, $G-K=-45.0$, whence $K=46.5$, $G=+1.5$. We then tabulate $\rho-G$, and compute $v+\omega$ by (3), and then $v-M+\omega-M_0$. The values of $v+\omega$ near 0° or 180° are uncertain, and are indicated by colons in the usual manner.

Plotting $v-M+\omega-M_0$ against $M+M_0$, we obtain the anomaly diagram shown in Fig. 2, in which the uncertain values are represented by the open circles. Superposing upon this the tracing-cloth on which are drawn the theoretical curves (shown in Fig. 1),

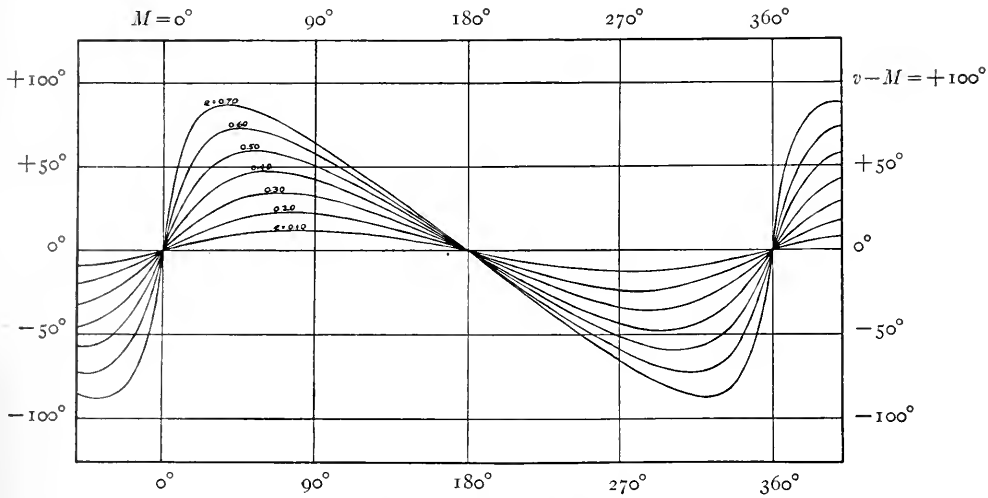


FIG. 1.—Curves giving $v-M$ as a function of M (tracing-cloth diagram)

we find that the curve for $e=0.40$ fits the observations very closely if it is shifted so that the periastron point upon it (the intersection of the steeper branches of the various curves) has the horizontal co-ordinate $-4.^\circ5$ and the vertical co-ordinate $+18°$. But these quantities are respectively the values of M_0 and $\omega-M_0$; hence $M_0=-4.^\circ5$ and $\omega=13.^\circ5$. To find the remaining elements we then have $T=P\times M_0/360°$ and $\gamma=G-Ke \cos \omega$, which give $T=-0.^d32$ and $\gamma=-17.1$ km.

Even before solving these equations, we may compute the velocity-curve corresponding to these elements. Keeping the tracing-cloth in position above the anomaly diagram, we read off

from the latter the values of $v-M+\omega-M_0$ corresponding to $e=0.40$ and the tabular values of $M+M_0$. These are entered in the latter part of the table. From them the computed values of $v+\omega$ may be written down at once, and the corresponding values of $\rho-G$ read from a slide-rule. The residuals $O-C$ are given in the last column of the table.

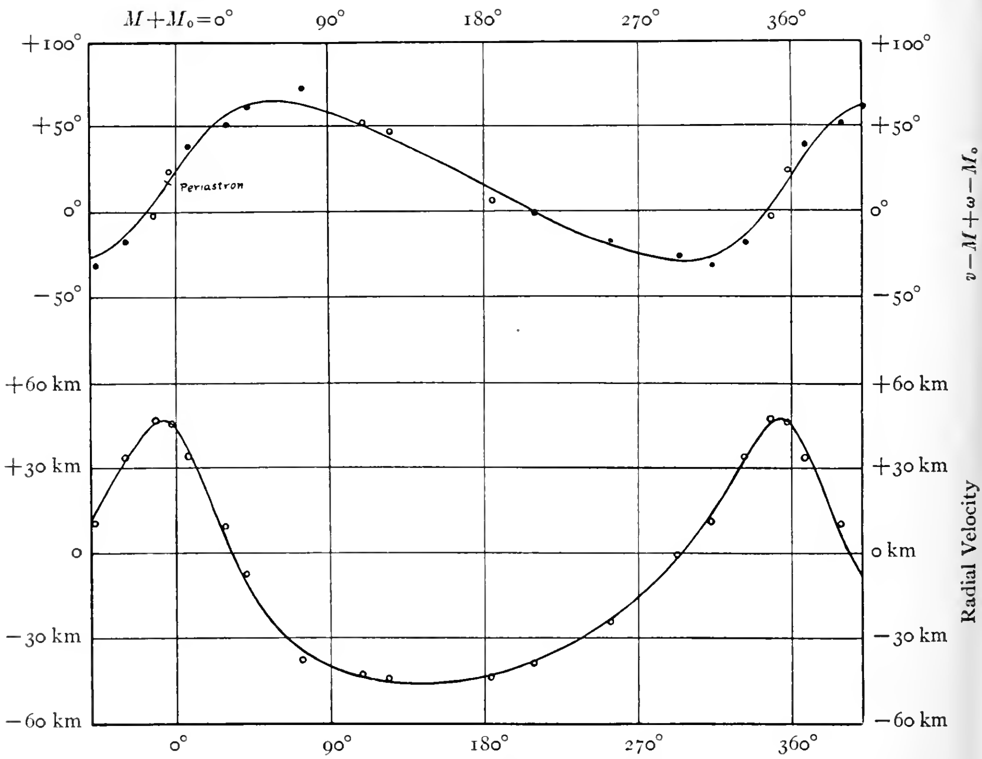


Fig. 2.—Anomaly diagram and velocity curve of α Draconis

From these residuals it appears that the representation of the observations by these elements, though already good, is capable of decided improvement. The residuals are negative near the maximum and minimum of the curve, and positive on the ascending and descending branches. Now the computed values at maximum and minimum can be altered only by changing G and

K, while it appears from the anomaly diagram that the points on the ascending and descending branches correspond to values of the equation of the center near its maxima, and are very sensitive to changes of e. A diminution of e, and therefore of the equation of the center, will diminish the values of v for points on the descending branch, and increase them for points on the ascending branch, and in both cases increase the computed velocity and raise the curve.

TABLE II

DATA		SOLUTION					FROM CURVES		
Phase	Wt.	ρ	$M+M_0$	$\rho-G$	$v+\omega$	$v-M$ $+\omega-M_0$	$v-M$ $+\omega-M_0$	Comp. $\rho-G$	O−C
4.18....	1.5	+ 9.26	29°.3	+ 7.8	80°.3	+51.0	+57°.0	+ 3.0	+4.8
5.92....	5	− 8.61	41.5	−10.1	102.6	+61.1	+63.0	−11.6	+1.5
10.52....	4	−37.60	73.7	−39.1	147.2	+73.5	+62.4	−33.5	−5.6
15.65....	4	−43.00	109.7	−44.5	162.:	+52.:	+50.4	−43.8	−0.7
17.76....	4	−44.48	124.4	−46.0	172.:	+48.:	+44.	−45.6	−0.4
26.29....	4	−44.30	184.2	−45.8	190.:	+ 6.:	+13.6	−44.3	−1.5
29.89....	2	−39.25	209.4	−40.7	208.9	− 0.5	+ 0.2	−40.5	−0.2
36.17....	2	−24.33	253.4	−25.8	236.3	−17.1	−20.0	−27.8	+2.0
41.87....	1.5	− 0.21	293.6	− 1.7	267.8	−25.8	−28.3	− 3.8	+2.1
44.56....	1.5	+10.62	312.1	+ 9.1	280.3	−31.8	−25.5	+13.3	−4.2
47.21....	4	+33.21	330.8	+31.7	313.0	−17.1	−13.8	+34.0	−2.3
49.53....	3	+46.20	347.0	+44.7	344.:	− 3.:	+ 4.8	+46.0	−1.3
50.86....	3	+45.30	356.3	+43.8	20.:	+24.:	+19.2	+44.9	−1.3
1.02....	1.5	+33.60	7.1	+32.1	46.2	+39.1	+33.8	+33.6	−1.5

The mean of the four residuals nearest the maximum velocity is -1.6, and that of the five residuals nearest the minimum is -1.7. We therefore, in beginning a second approximation, correct the previously assumed maximum and minimum by these amounts, and so find $G=-0.1$, $K=46.5$. We then compute new values of $v+\omega$, and construct a second anomaly diagram, in precisely the same way as before. The numerical data are given in Table III. When we superpose the tracing-cloth on this diagram, it appears at once that we must have approximately $M_0=0°$, $\omega-M_0=20°$, with e somewhat less than 0.40. A better determination of e may be made as follows. Keeping the tracing-cloth in such a position that $M=0°$, we shift it vertically until the curve for $e=0.40$ gives as good a representation as possible of the points near the top of the anomaly diagram. We then read from this

curve the maximum value of $v-M+\omega-M_0$, which is found to be $+64^\circ.8$. Displacing the tracing-cloth vertically until the best representation of the points near the bottom of the diagram is obtained, we read from the curve in its new position a minimum of $-26^\circ.2$. The mean of these, $+19^\circ.3$, gives an improved value of $\omega-M_0$, while half their difference is the maximum equation of the center, $45^\circ.5$, to which, by the table given earlier, corresponds an eccentricity of 0.390. If now we place the tracing-cloth so that the vertical co-ordinate of the periastron point corresponds to this value of $\omega-M_0$, we find easily that the best fit is obtained, on moving it horizontally, for $M_0=+1^\circ$.

TABLE III

p Sin² $(v+\omega)$	$\rho-G$	$v+\omega$	$v-M$ $+\omega-M_0$	M	v	Comp. $\rho-G$	O−C
1.4.....	+ 9.36	78°.4	+49°.1	+ 28°.3	+ 62°.4	+ 5.9	+3.5
4.8.....	− 8.51	100.6	+59.1	+ 40.5	+ 81.9	− 9.8	+1.3
1.4.....	−37.50	143.8	+70.1	+ 72.7	+117.4	−34.3	−3.2
0.6.....	−42.90	157.3	+45.6	+108.7	+143.1	−44.5	+1.6
0.3.....	−44.38	162.5:	+38.1:	+123.4	+151.7	−46.0	+1.6
0.3.....	−44.20	198.0:	+13.8:	−176.8	−178.5	−43.1	−1.1
0.5.....	−39.15	212.6	+ 3.2	−151.6	−166.3	−38.5	−0.7
1.5.....	−24.23	238.6	−14.8	−107.6	−142.5	−24.1	−0.1
1.5.....	− 0.11	269.9	−23.7	− 67.4	−112.7	− 1.9	+1.8
1.5.....	+10.72	283.3	−28.8	− 48.9	− 92.9	+13.9	−3.2
2.0.....	+33.31	315.8	−15.0	− 30.2	− 65.8	+32.6	+0.7
0.0.....	+46.30	357.::	+10.::	− 14.0	− 33.5	+45.3	+1.0
0.1.....	+45.40	12.5:	+16.3:	− 4.7	− 11.6	+46.0	−0.6
0.7.....	+33.70	43.5	+38.4	+ 6.1	+ 14.9	+38.0	−4.3

In making the compromises between the representation of different points, which are always necessary, it must be borne in mind that the relative weights of these points, *on the anomaly diagram*—that is, of the values of $v+\omega$ computed from the individual observations—are proportional to $p\ \sin^2(v+\omega)$ where p is the weight of the observed radial velocity. These need of course be computed only very roughly, and may usually be estimated mentally. Table III may be regarded as a continuation of Table II, and no unnecessary repetitions have been made in it. The latter part of this table gives the computation of the final velocity-curve, which has been made with the Allegheny tables. The values of M are found from those of $M+M_0$ given in Table II, and those of v taken

directly from the tables. The representation of the observations now appears to be satisfactory, and we proceed to calculate the final elements, in the same manner as in the first approximation.

The two sets of elements determined by the present method compare as shown in Table IV with the preliminary elements used by Harper, and the results of his least-squares solution. The first approximation by the present method is not so good as Harper's preliminary elements, which were obtained by King's graphical method (whether at the first trial or after more numerous attempts is not stated); but the second approximation gives results practically identical with those of the least-squares solution. Only one of the five quantities determined differs from the least-squares values by more than the probable error of the latter, and the weighted sum of the squares of the residuals is the same for both (within the accuracy of the reckoning).

TABLE IV

	HARPER		PRESENT METHOD	
	Preliminary	Final	1st Approx.	2d Approx.
P..........	51.38 d. (assumed throughout)	....	....	....
K..........	47.0 km	46.25 ±0.24	46.5	46.5
e..........	0.41	0.384±0.003	0.40	0.390
ω..........	15°	19°.07 ±2°.21	13°.5	20°.3
T..........	0.00 d.	+ 0.284±0.230	− 0.32	+ 0.14
γ..........	−17.61 km	−17.03 ±0.15	−17.1	−17.1
$a \sin i$.......	....	30,173,000 km	30,000,000	30,150,000
pvv..........	213.5	148.6	259.4	148.2

It would therefore appear that the new method is of very considerable precision. As regards its rapidity, an account of the time consumed was kept, showing that the first approximation, from the time of beginning to tabulate and plot the observations to the computation of the elements and residuals, required 78 minutes, and the second approximation 42 minutes more (not including the calculation of the sum of the squares of the residuals).

That these conclusions regarding the accuracy and rapidity of the method are generally applicable is shown by the results of several other test computations, for which an account of time was

kept. Some of the most difficult cases in the recent literature of the subject were purposely chosen. The results of previous computers have been derived by least squares in all cases except β Arietis, but in some instances the probable errors are not given. The normal places of the previous computers have been discussed independently by the present method, with the results given in Table V.

TABLE V

β ARIETIS

$P = 107.0$ Days

	Present Method	Ludendorff*
K	33.0 km	32.6
γ	− 0.5 km	− 0.6
e	0.88	0.88
ω	18°	19°.7
T	0.0 d.	+ 0.1
Time	63 min.	....

* *Astrophysical Journal*, **25**, 324, 1907.

ϵ HERCULIS

$P = 4.0235$ Days

	Present Method	Baker†
K	71.25 km	70.39 ±1.08
γ	−24.0 km	−24.03 ±0.83
e	0.033	0.023±0.016
ω	180°	180° (assumed)
T	0.00 d.	−0.003±0.011
Time	51 min.	

† *Publications of the Allegheny Observatory*, **2**, 20, 1910.

θ TAURI

$P = 140.50$ Days

	1st Approx.	2d Approx.	Plaskett‡
K	28.5 km	29.5	29.1
γ	+43.7 km	+42.2	+42.9
e	0.67	0.70	0.694
ω	48°	50°	48.6
T	56.2 d.	56.6	56.12
Time	72 min.	59 min.	

‡ *Journal of the Royal Astronomical Society of Canada*, **6**, 231–239, 1912.

ι Orionis

$P = 29.136$ Days

	1st Approx.	2d Approx.	3d Approx.	Plaskett§
K	116 km	111.5	109.5	109.9 ±1.1
γ	+ 18.6 km	+ 23.0	+ 20.6	+ 21.3 ±0.86
e	0.77	0.75	0.75	0.754±0.005
ω	106°	110°	110°	113.3 ±1.0
T	+ 1.86 d.	+ 1.95	+ 1.91	+ 1.99 ±0.022
Time	2 hrs. 12 min.	Not noted, owing to interruptions		

§ *Astrophysical Journal*, **27**, 275, and **28**, 274, 1908.

B.D. −1°1004

(Including secondary oscillation of same period as primary)

$P = 27.160$ Days

	1st Approx.	2d Approx.	Harper¶
K	94.5 km	95.5	93.04 ±2.2
γ	+26.5 km	+27.8	+26.12 ±0.7
e	0.80	0.77	0.765±0.007
ω	87°	87°	87°.03 ±2°.5
T	0.0 d.	+ 0.015	− 0.016±0.015
K'	....	10.0 km	10.15 ±1.4
T'	....	+13.58 d.	+12.32 ±1.3
Time	78 min.	68 min.	

¶ *Report of the Chief Astronomer*, 1910, Appendix 2, pp. 137–143.

The last three systems present problems of exceptional difficulty, and in the original discussions several successive least-squares solutions were necessary. Even in these cases, the present method has been found capable of giving solutions which agree very closely with the final values, with only an afternoon's work—probably less labor than was involved in finding the period and forming the normal places.

It would therefore appear probable that the new method may be of some utility to the practical computer.

Princeton University Observatory
June 30, 1914

THE SPECTRA OF FOUR OF THE TEMPORARY STARS[1]

By W. S. ADAMS and F. G. PEASE

An important observation by Hartmann[2] in 1907 upon the spectrum of Nova Persei of 1901 showed that the principal nebular lines at λ 4960 and λ 5007, which were very prominent in the spectrum of the star when last observed during its brighter stages, had disappeared, and that the spectrum was essentially identical with that of the Wolf-Rayet star B.D.+35°4001. The interesting question thus raised, whether the disappearance of the principal nebular lines is characteristic of temporary stars in their later history, makes it desirable to secure spectra of such other Novae as can be obtained. Four such stars have been shown by the observations of Professor Barnard[3] to be of the fourteenth magnitude or brighter. With the magnitudes which he gives they are as follows:

	Magnitude
Nova Aurigae of 1891	14
Nova Persei of 1901	12.4
Nova Lacertae of 1910	12.5
Nova Geminorum No. 2 of 1912	10

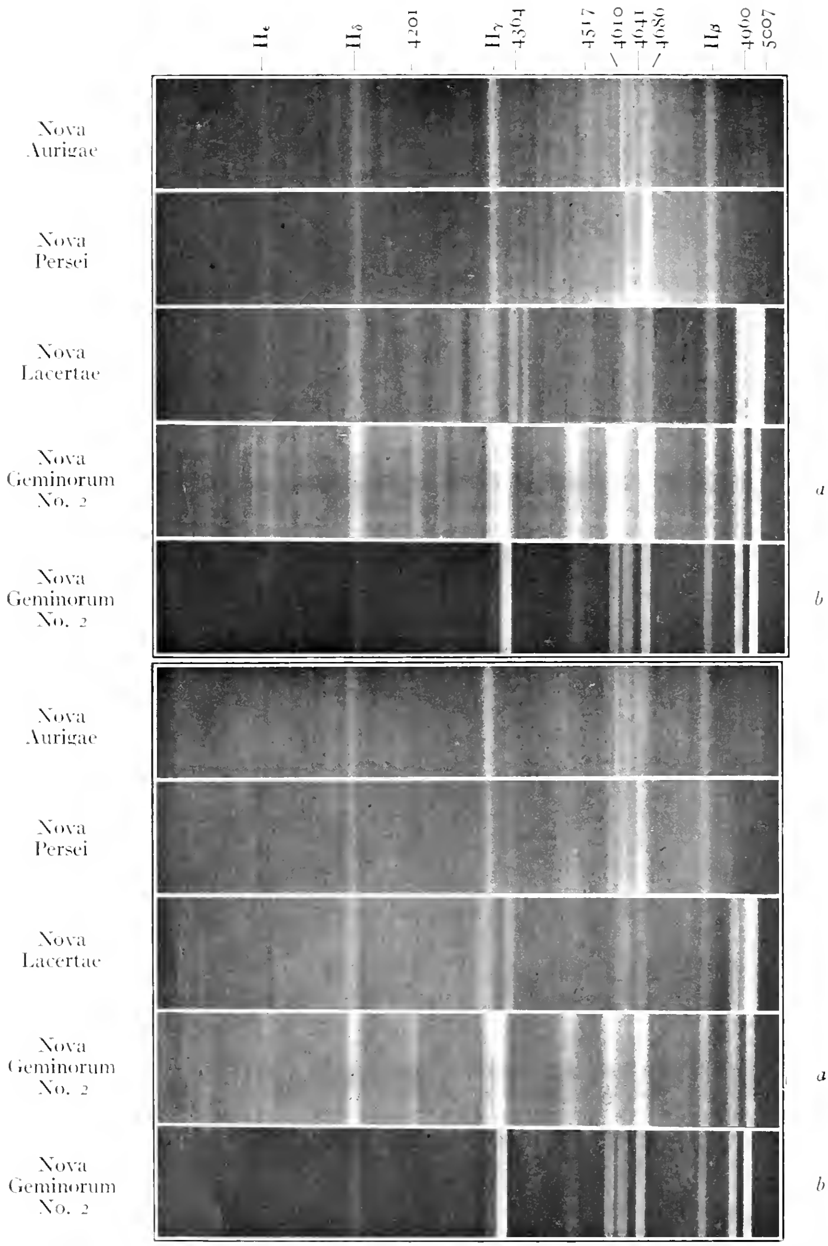
We have succeeded in obtaining photographs of the spectra of all of these stars with a small slit spectrograph at the primary focus of the 60-inch reflector. The spectrograph consists of a 60° prism of ultra-violet flint glass used with collimating and camera lenses of about 16 cm focal length by Steinheil. The instrument is mounted in the opening of the double-slide plate-holder, and after the star has been brought upon the slit the guiding is carried on as in the case of direct photography by keeping auxiliary stars upon the cross-wires of two eyepieces, one at either side of the field, the spectrograph being moved bodily by the guiding screws. A diagonal prism mounted in a tube with an eyepiece may be

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 87.

[2] *Astronomische Nachrichten*, **177**, 113, 1908.

[3] *Ibid.*, **194**, 401, 1913.

PLATE IX



Upper Reproduction from Untouched Copy of Widened Spectra;
Lower Reproduction from a Retouched Print

slipped back and forth behind the slit, and enables the observer to bring the star accurately upon the slit at the beginning of the exposure.

The photographs in the case of Nova Aurigae and Nova Lacertae required exposure times extending over more than one night. The observational data are given in Table I.

TABLE I

	Exposure Time	Date	Slit Width
Nova Aurigae................	16 hours	1914, March 18–22	0.062 mm
Nova Persei................	8 "	1913, November 27	0.044 "
Nova Lacertae................	11.5 "	1913, October 29–31	0.044 "
Nova Geminorum No. 2.......	2 "	1914, February 22	0.044 "

Reproductions of the enlarged and widened spectra are shown in Plate IX. In addition a photograph of the spectrum of Nova Geminorum No. 2, taken with the Cassegrain spectrograph on October 12, 1913, is added for comparison. Owing to the small scale of the photographs and the consequent enlargement of grain, many false lines were introduced in the process of widening. Accordingly, two reproductions are shown. The first is a direct copy of the widened spectra. The second is from a print on which many of the false lines have been touched out by hand. Care has been taken in this case to leave untouched all parts of the spectrum in which true lines are seen on the original negatives. In addition, all of the real lines are marked on the reproductions.

The approximate intensities of the lines in the spectra are indicated in Table II upon a scale of 0 to 10. Probably several very faint lines are lost in the continuous spectrum which is particularly strong in the case of the first three stars. A dash in the table means that the line is not seen.

The single most interesting feature of these results is without doubt the absence of the chief nebular lines λ 4960 and λ 5007 from the spectra of Nova Aurigae and Nova Persei, and of the nebular line λ 4364, which is usually associated with them. The agreement of this observation in the case of Nova Persei with that of Hartmann in 1907 makes it appear that a spectrum in which these lines are

absent must be considered as probably the permanent spectrum of this star, and from the similar result for Nova Aurigae it seems reasonable to conclude that the absence of the nebular lines is characteristic of Novae in their later history. It is about twenty-three years since the discovery of Nova Aurigae and thirteen since that of Nova Persei, but only four and two in the cases of Nova Lacertae and Nova Geminorum No. 2.

TABLE II

	Nova Aurigae	Nova Persei	Nova Lacertae	Nova Geminorum No. 2 Plate IX*a*	Nova Geminorum No. 2 Plate IX*b*
H_δ	2	2	2	3	3
4201	–	Trace?	Trace	1	1
H_γ	3	3	3	3	2
4364	–	–	Trace?	5	8
4517	–	Trace?	Trace?	2	2
4610	Trace	Trace	–	5	4
4641	1	1	3	Trace	3
4686	2	5	3	8	5
H_β	2	2	2	3	3
4960	–	–	5	4	5
5007	–	–	10	7	8
Continuous spectrum	Very strong	Very strong	Very strong	Strong	Trace

The extraordinary intensity of the principal nebular line at λ 5007 in the spectrum of Nova Lacertae is its most important feature, but a peculiar fact in connection with it is the weakness or entire absence of the nebular line at λ 4364. These two lines have been ascribed by Professor Nicholson[1] to the same vibrating atom, and their opposite behavior in the spectrum of this star is certainly an unusual feature.

The three lines at λ 4610, λ 4641, and λ 4686 show remarkable differences of intensity in the different spectra, and even on the two photographs of Nova Geminorum there are large variations. Reference has been made by one of us in an earlier communication[2] to some of these differences in the case of Nova Geminorum. The first line of the principal series of hydrogen at λ 4686 seems to be especially subject to fluctuations, and it is interesting to note the

[1] *Monthly Notices, R.A.S.*, **74**, 488, 1914.

[2] *Ibid.*, **73**, 742, 1913.

great difference in its intensity in Nova Persei as compared with Nova Aurigae.

The intensity of the continuous spectrum of these stars appears to depend in general on their age, being greatest for the older Novae. The two photographs of Nova Geminorum show how very rapidly it may change, the earlier photograph showing hardly more than a trace of a continuous background.

As Hartmann has noted in the case of Nova Persei, and as the results show for both Nova Aurigae and Nova Persei, these spectra are essentially identical with the spectra of some of the Wolf-Rayet stars. With the disappearance of the principal nebular lines and the increase in intensity of the continuous spectrum the only marked points of difference have gone. This identity of spectrum, taken in connection with the well-known agreement of distribution relative to the Milky Way of Novae and Wolf-Rayet stars, makes it probable that at least a portion of the latter are temporary stars in the later stages of their history.

In the absence of knowledge as to the physical conditions under which the principal nebular lines are produced it is, no doubt, best to ascribe their disappearance to a change in these conditions in the radiating material. In view, however, of the considerable amount of evidence of various kinds which tends to support the hypothesis that the phenomenon of a temporary star is due to the star's entering a nebula, the suggestion that the disappearance of the chief nebular lines is coincident with the emergence of the star from the nebula may perhaps deserve some slight consideration.

MOUNT WILSON SOLAR OBSERVATORY
July 1, 1914

CONSTANT DIFFERENCES IN LINE-SPECTRA

By EMIL PAULSON

In all spectra there are lines or complexes of lines, between the wave-numbers of which the differences are constant. Kayser. however, considers these differences as "eine andere Art der Gesetzmässigkeit"; other authors such as Paschen regard them as but secondary products of the series.

However that may be, the knowledge of the constant differences constitutes the first step in the ordering of a complicated spectrum. When a number of constant differences have been found, an investigation of the Zeeman effect will shed more light on the affinity of the lines, and thus it will be possible to obtain the series.

The calculation of the constant differences of the wave-numbers is exceedingly difficult for a line-rich spectrum. Kayser[1] has made such a calculation with reference to the following spectra, namely: tin, lead, arsenic, antimony, bismuth, palladium, platinum, and ruthenium. Investigations of the same nature have also been made by Rydberg[2] and Snyder[3] on the spectra of argon and rhodium respectively.

In a recently published work[4] I have described a very simple mechanical means which, without any special difficulty, will facilitate the obtaining of the constant differences, particularly in line-rich spectra. In the work referred to above, the results of an examination of 40 spectra are given, which investigation has been made in the above-mentioned manner. Having had but a short time at my disposal, I had to confine the examination to the lines of the greatest intensities in each spectrum. This way I also consider the most practical. If the differences between the lines of the greatest intensities are known, it is easy to obtain the other lines having the same differences.

[1] *Handbuch der Spectroscopie*, **2**, 573; *Abh. der Berl. Akad.*, 1897.

[2] *Astrophysical Journal*, **6**, 338, 1897.

[3] *Ibid.*, **14**, 179, 1901.

[4] *Beiträge zur Kenntnis der Linienspektren.* Diss. Lund, 1914.

In the following the results of some of the most minutely examined spectra are stated. As regards phosphorus, scandium, titanium, iron, cobalt, germanium, bromine, krypton, yttrium, and niobium, the results will be published in the *Annalen der Physik*.

The wave-lengths and intensities are naturally taken from the excellent tables in Professor Kayser's *Handbook*.[1] Where measurements of thousandths are given, these are used; the wave-length is then, however, reduced to hundredths. In other cases the very best measurements have been selected. The intensities are chiefly taken from Exner and Haschek.

The tables are written as follows: first the wave-lengths, then, in parentheses, the intensities, and lastly the wave-numbers and their differences.

I. CHLORINE

All the lines were examined. The wave-lengths and intensities are from Eder and Valenta. The results obtained are given in Table I.

TABLE I

5445.12 (1).......	18365.07		4469.57 (5)......	22373.53	
		40.92			108.02
5457.28 (3).......	18324.15		4491.25 (3)......	22265.51	
5444.41 (3).......	18367.46		4390.57 (3)......	22776.11	
		40.33			67.47
5456.39 (2).......	18327.13		4403.61 (5)......	22708.64	
4794.67 (10)......	20856.49		4323.52 (6)......	23129.30	
		67.29			108.09
4810.19 (9).......	20789.20		4343.82 (10)......	23021.21	
		40.72			
4819.63 (9).......	20748.48		4291.86 (5)......	23299.92	
					66.86
4601.19 (4).......	21733.51		4304.21 (4)......	23233.06	
		108.29			
4624.23 (3).......	21625.22		4234.14 (5)......	23617.56	
					40.66
4497.45 (½).......	22234.82		4241.44 (8)......	23576.90	
		107.99			67.02
4519.4 (½)........	22126.83		4253.53 (9)......	23509.88	

[1] *Handbuch der Spectroscopie*, **5**, 6.

TABLE I—*Continued*

4124.15 (1)........	24247.40	40.14	3845.55 (8).......	26004.08	40.37
4130.99 (4)........	24207.26		3851.53 (8).......	25963.71	
3848.03 (2)........	25987.30	40.23	3833.50 (8).......	26085.81	67.11
3854.00 (4)........	25947.07		3843.39 (5).......	26018.70	39.94
			3849.30 (2).......	25978.76	
3845.83 (8)........	26002.22	66.84			
3855.74 (2)........	25935.38				

2. MANGANESE

The number of lines examined here amounts to 42. The wave-lengths and intensities of the red lines are from Fritsch. The lines presented in Table II form pairs. The difference 173.69 is also observed by Kayser.[1] In conjunction with a third line the pairs form two triplets, which are the first lines of the two secondary series. I have observed only the pairs, the third line not being included in the investigation on account of its slight intensity.

TABLE II

6013.72 (10)	16628.64	8.68	4783.61 (10 R) ...	20904.72	173.71
6016.86 (10)	16619.96	14.24	4823.69 (10 R) ...	20731.01	
6022.02 (10)	16605.72				
			3547.94 (10 R) ...	28185.37	173.77
4030.92 (100 R) ...	24808.24	14.27	3569.95 (10 R) ...	28011.60	
4033.24 (100 R) ...	24793.97	8.67			
4034.65 (50 R)	24785.30		5341.18 (8)	18722.46	230.04
			5407.62 (6)	18492.42	
5470.83 (7)	18278.76	35.95	4018.28 (10)	24886.27	229.61
5481.61 (6)	18242.81		4055.70 (10)	24656.66	
2798.36 (50 R)	35735.21	35.97	3578.03 (10)	27948.34	229.85
2801.18 (50 R)	35699.24		3607.70 (6)	27718.49	

[1] *Handbuch*, 2, 551.

3. NICKEL

Number of the lines examined, 51. The wave-lengths and intensities of the red lines are taken from Stötting. First there was found a group of 4 lines, which was twice completely repeated, once incompletely. These are given in Table III, also the two pairs, at the foot of the table.

TABLE III

6191.39 (10)............	16151.47	
		168.27
6256.57 (10)............	15983.20	
		179.98
6327.82 (6).............	15803.22	
3472.71 (10)............	28795.96	
		168.17
3493.11 (30)............	28627.79	
		179.99
3515.21 (30)............	28447.80	
		751.65
3610.61 (10)............	27696.15	
3393.10 (20 R)..........	29471.57	
		348.81
3433.74 (15)............	29122.76	
		751.40
3524.68 (50)............	28371.36	
5035.55 (10)............	19858.80	
		2889.80
5893.10 (10)............	16969.00	
3134.22 (30)............	31905.87	
		2890.09
3446.40 (30 R)..........	29015.78	

4. MOLYBDENUM

Among the numerous lines of this spectrum only a few are intense. My examination has been extended to 50 lines and has given the result shown in Table IV.

TABLE IV

4610.06 (10)	21691.69	448.72	3158.28 (10 R) ...	31662.81	121.64
4707.44 (3)*	21242.97		3170.46 (20 R) ...	31541.17	
4326.29 (8)	23114.49	448.31	2775.74 (10)	36026.43	121.08
4411.86 (20)	22666.18		2785.10 (8)	35905.35	
3798.41 (50 R) ...	26326.80	448.89	4149.73 (8)	24097.96	719.77
3864.30 (50 R) ...	25877.91		4277.49 (10)	23378.19	
5650.40 (4)	17697.86	121.29	4107.68 (8)	24344.65	719.74
5689.39 (4)	17576.57		4232.82 (10)	23624.91	
5533.26 (6)	18072.52	121.42	3363.94 (8)	29727.05	718.84
5570.69 (6)	17951.10		3447.30 (10)	29008.21	

* Hasselberg.

5. PALLADIUM

In this case all the lines in the arc spectrum and also some in the spark spectrum, in all 348, were examined. The intensities are taken from Kayser, whose measurements of the wave-lengths have also been used. Table V gives the differences discovered.

Later on I found that 6 of these pairs have also been observed by Kayser. A third line, however, belongs to the pairs, so that they in reality form two triplets. This fact I had not observed, my examination not being extended to so great intervals. Two other constant differences were discovered. The results of this work are given in Table VI.

TABLE V

4817.66 (9)	20756.96	1191.11	3002.78 (4 R)	33302.47	1191.08
5110.94 (6)	19565.85		3114.16 (5 R)	32111.39	
3634.84 (10 R) ...	27511.53	1191.10	2922.62 (7 R)	34215.88	1191.11
3799.33 (5 R)	26320.43		3028.03 (4 R)	33024.77	
3460.89 (7 R)	28894.29	1191.24	2577.20 (1)	38801.80	1191.13
3609.71 (9 R)	27703.05		2658.82 (2)	37610.67	
3287.38 (5)	30419.36	1191.30	2433.19 (2)	41098.31	1190.90
3421.37 (8 R)	29228.06		2505.80 (2)	39907.41	
3242.82 (10 R) ...	30837.36	1191.39	2254.40 (1)	44357.70	1190.99
3373.14 (6 R)	29645.97		2316.60 (0)	43166.71	

TABLE VI

4875.58 (7)	20510.38	1627.60	3251.75 (5 R)	30752.66	1628.54
5295.83 (10)	18882.78		3433.58 (5 R)	29124.12	
4170.01 (5)	23980.76	1628.25	2461.41 (1 U)	40627.12	1628.60
4473.77 (6)	22352.51		2564.2 (1 U)	38998.52	
3832.45 (10)	26092.97	1628.26	5529.66 (6 u).....	18084.29	402.79
4087.52 (6)	24464.71		5655.63 (5 u).....	17681.50	
3719.06 (4 R)	26888.52	1628.21	5497.06 (2)	18191.54	402.75
3958.78 (5 R)	25260.31		5621.52 (2)	17788.79	
3481.31 (7 R)	28724.82	1628.15	3441.55 (6 R)	29056.68	402.81
3690.49 (6 R)	27096.67		3489.93 (4 R)	28653.87	
3302.25 (6 R)	30282.38	1628.51	3258.91 (6 R)	30685.11	402.73
3489.93 (4 R)	28653.87		3302.25 (6 R)	30282.38	
3258.91 (6 R)	30685.11	1628.43	3028.03 (4 R)	33024.77	402.81
3441.55 (6 R)	29056.68		3065.42 (4 R)	32621.96	

6. LANTHANUM

Only 25 strong lines were examined in which were found the pairs shown in Table VII.

TABLE VII

4238.56 (20)	23592.91	519.16	4077.49 (10)	24524.89	381.21
4333.93 (20)	23073.75		4141.87 (10)	143.68	
3949.24 (20)	25321.32	518.81	3988.67 (15)	25071.01	381.11
4031.85 (20)	24802.51		4050.24 (10)	24689.90	

7. SAMARIUM

In this line-rich spectrum the 48 strongest lines were examined. The results obtained were as presented in Table VIII.

TABLE VIII

4424.55 (20)	22601.37	173.31	4391.03 (10)	22773.70	929.53
4458.70 (10)	22428.06	421.60	4577.88 (10)	21844.17	
4544.12 (10)	22006.46	615.03	4152.38 (10)	24082.58	929.72
4674.77 (10)	21391.43		4319.12 (10)	23152.86	
4225.48 (10)	23665.95	172.69	4454.84 (10)	22447.50	548.31
4256.54 (10)	23493.26	421.59	4566.38 (10)	21899.19	
4334.32 (10)	23071.67	614.49	4329.21 (10)	23098.91	548.55
4452.92 (10)	22457.18		4434.52 (20)	22550.36	

8. EUROPIUM

First, 4 pairs of lines with a very great intensity were discovered, the oscillation differences of which in the mean are 1669.71. Table IX shows the results of this work.

TABLE IX

4205.20 (100)...........	23780.08	1669.88
4522.80 (20)............	22110.20	
4129.90 (100)...........	24213.66	1669.50
4435.74 (50)............	22544.16	
3725.10 (30)............	26844.92	1669.70
3972.16 (50)............	25175.22	
3688.57 (20)............	27110.77	1669.75
3930.66 (50)............	25441.02	

I have more recently examined all the lines in the arc spectrum, 280 in all, in order to ascertain how often this difference occurs in the whole spectrum. Table X gives the pairs with the same difference which were then also discovered.

TABLE X

3744.35 (2)	26706.90	1669.66	2820.90 (4)	35449.68	1669.78
3994.05 (1)	25037.24		2960.34 (3)	33779.90	
3687.93 (1)	27115.48	1670.19	2729.46 (5)	36637.29	1669.69
3930.00 (2)	25445.29		2859.79 (3)	34967.60	
3678.41 (1)	27185.66	1670.18	2705.36 (2)	36963.66	1669.88
3919.19 (1)	25515.48		2833.36 (2)	35293.78	
3511.20 (3)	28480.29	1669.56	2701.21 (3)	37020.44	1669.89
3729.85 (2)	26810.73		2828.81 (4)	35350.55	
2906.80 (5)	34402.09	1669.62	2685.74 (3)	37233.68	1670.03
3055.07 (4)	32732.47		2811.86 (2)	35563.65	
2862.69 (3)	34932.18	1669.70	2559.30 (1)	39073.18	1669.59
3006.39 (2)	33262.48		2673.54 (3)	37403.59	

It is here, as often happens, a question of a complex of lines, the wave-numbers of which have constant differences in the horizontal and vertical rows. Two differences in question are set forth in Table XI.

TABLE XI

4435.74 (50)	22544.16	433.96	4383.36 (3)	22813.55	265.67
4522.80 (20)	22110.20		4435.01 (2)	22547.88	
4181.06 (2)	23917.38	433.72	4182.42 (4)	23909.61	265.98
4258.28 (2)	23483.66		4229.47 (1)	23643.63	
4129.90 (100)	24213.66	433.58	3930.66 (50)	25441.02	265.80
4205.20 (100)	23780.08		3972.16 (50)	25175.22	
3943.21 (2)	25360.05	433.89	3916.93 (2)	25530.19	265.16
4011.85 (3)	24926.16		3958.04 (1)	25265.03	
3425.19 (3)	29195.46	433.13	3688.57 (20)	27110.77	265.85
3476.77 (1)	28762.33		3725.10 (30)	26844.92	
			3213.84 (3)	31115.42	265.98
			3241.55 (3)	30849.44	

9. CERIUM

The examination has included the 30 strongest lines and has given the results presented in Table XII.

TABLE XII

4562.55 (8)	21917.57	78.55	4367.15 (4)	22898.23	1131.27
78.96 (5)	839.02		4594.12 (4)	21766.96	
4471.38 (5)	22364.46	78.14	4306.89 (4)	23218.61	1131.42
87.06 (5)	286.32		4527.51 (5)	22087.19	
4152.17 (9)	24083.71	78.49	4246.07 (8)	23551.19	1131.37
65.76 (10)	005.22		4460.34 (8)	22419.82	
4150.11 (10)	24095.74	78.52	4186.76 (10)	23884.82	1131.43
63.68 (6)	017.22		4394.95 (4)	22753.39	
			3952.75 (8)	25298.84	1131.34
			4137.79 (9)	24167.50	

10. XENON

In the first spectrum of this element the triplets given in Table XIII have been discovered (measurements are taken from Baly).

TABLE XIII

4658.94 (1)	21464.11	759.62	3985.39 (3)	25091.64	3684.87
4829.87 (4)	20704.49	3684.08	4671.42 (10)	21406.77	760.29
5875.30 (1)	17020.41		4843.44 (2)	20646.48	
4109.84 (5)	24331.84	3685.36	3951.16 (10)	25309.03	3684.88
4843.44 (2)	20646.48	759.52	4624.46 (15)	21624.15	759.39
5028.42 (2)	19886.96		4792.77 (1)	20864.76	

11. IODINE

An investigation of 50 lines has resulted in the pairs given in Table XIV (measurements are taken from Konen).

TABLE XIV

5739.61 (6)	17422.79	107.09	4221.24 (10)	23689.73	958.07
75.11 (6)	315.70		4399.15 (8)	22731.66	
5464.60 (10)	18299.60	107.83	3686.79 (8)	27123.86	956.95
96.99 (10)	191.77		3821.62 (8) ?.....	26166.91	
4423.98 (8)	22604.08	107.09	4512.80 (8)	22159.19	1355.59
45.04 (8)	496.99		4806.86 (10)	20803.60	
4342.25 (8)	23029.53	107.53	3561.32 (8)	28078.47	1356.00
62.62 (6)	922.00		3742.03 (8)	26723.47	
5338.31 (8)	18732.52	957.46	3512.82 (6)	28467.16	1355.72
5625.86 (10)	17775.06		3688.48 (8)	27111.44	
4292.14 (6)	23298.40	958.12			
4476.22 (6)	22340.28				

12. DYSPROSIUM

Among the 25 lines examined the pairs shown in Table XV were found.

TABLE XV

4111.50 (8)	24322.02	828.54	4050.73 (9)	24686.91	136.67
4256.50 (8)	23493.48		4073.28 (10)	24550.24	
3968.55 (10)	25198.12	828.39	3978.70 (10)	25133.84	137.52
4103.45 (9)	24369.73		4000.59 (10)	24996.32	
3944.83 (10)	25349.64	828.48			
4078.11 (10)	24521.16				

13. ERBIUM

There were twenty lines examined with results as presented in Table XVI.

TABLE XVI

4301.78 (8)............	23246.19	440.29
4384.83 (8)............	22805.90	
3938.79 (10)...........	25388.51	440.34
4008.31 (8)............	24948.17	
3830.69 (10)...........	26104.96	440.24
3896.40 (15)...........	25664.72	

14. THULIUM

There were 30 lines examined with the results embodied in Table XVII.

TABLE XVII

3751.98 (5)	26652.59	71.62	3958.21 (8) ...	25263.95	243.00	909.28
3762.09 (20) ...	26580.97		3996.65 (5) ...	25020.95	666.28	
3734.29 (15) ...	26778.86	71.02	4105.99 (15) ..	24354.67		
3747.22 (20) ...	26707.84					
			3718.07 (20) ..	26895.68	243.09	909.03
3453.82 (15) ...	28953.45	71.50	3751.98 (5)....	26652.59	665.94	
3462.37 (20) ...	28881.95		3848.13 (50) ..	25986.65		
3700.41 (15) ...	27024.04	245.18				
3734.29 (15) ...	26778.86					
3668.24 (5)	27261.03	245.25				
3701.54 (15) ...	27015.78					

15. LUTETIUM

All the lines in the arc spectrum, 48 in all, were included in the investigation, which has given the results shown in Table XVIII.

TABLE XVIII

3876.82 (2)	25794.34		2754.29 (2)	36306.99	
		5771.55			1764.31
4994.31 (3)	20022.79		2894.97 (3)	34542.68	
		1764.39			
5476.93 (6)	18258.40				
			3118.56 (4)	32066.08	
					2630.51
2613.50 (1)	38262.87		3397.25 (5)	29435.57	
		5771.29			639.45
3077.72 (4)	32491.59		3472.69 (6)	28796.12	
		1764.43			
3254.45 (4)	30727.16				
		2216.83	2798.34 (1)	35735.47	
3507.50 (3)	28510.33				2630.89
			3020.73 (3)	33104.58	
					639.79
2900.44 (3)	34477.52		3080.26 (2)	32464.79	
		1764.21			
3056.86 (2)	32713.31				
		2217.23	2963.46 (3)	33744.34	
3279.11 (2)	30496.08				639.76
			3020.73 (3)	33104.58	

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TELESCOPIC VISION OF AN ILLUMINATED SURFACE

By FRED W. VORHIES

INTRODUCTION

An interesting aspect of the subject of telescopic vision has been discussed in a series of articles written by Stoney.[1] In these articles Stoney discusses the subject from a theoretical standpoint but makes some suggestions as to how apparatus could be set up and used in verifying his theories and assumptions. It is the purpose of this paper to show how this apparatus works in practice and to give the results obtained by its use. In view of the fact that there has been so much discussion during the past, concerning the ability to see details upon such astronomical objects as Mars, the author trusts that this contribution will be of value. Reference should be made to Stoney's articles for the theory of his experiments, for it is too long to reproduce in full here.

As a basis for his theories Stoney makes the following assumptions which I adopted and proved to be correct:

(1) In studying an illuminated surface or object—the planet Mars, for example—the real object, No. 1, can be removed and a substitute, No. 2, can be used in its stead. No. 2 is to be a transparent object with the details of No. 1 delineated upon it.

(2) When No. 2 is illuminated from behind with diffused light, the same sort of image will be formed in the telescope as when No. 1 is illuminated from in front with diffused light.

(3) A lens can be placed immediately in front of No. 2 and the image which it casts of a point-source of light, placed at some distance (the light from this source passing through object No. 2), is called the concentration image.

(4) The introduction of this lens does not alter the optical conditions.

(5) One method of analysis permits us to consider that the ordinary telescopic images of illuminated surfaces are made up of an infinite number of the above concentration or partial images.

[1] *Philosophical Magazine*, 16, 318, 796, 950, 1908.

The above assumptions enable us to reduce the highly complex problem of image-formation by diffused light to a simple diffraction problem. It should further enable us to place, at least, an inferior limit to the image-forming power of our telescopes.

EXPERIMENTAL METHODS AND THEORY

Fig. 1 is a diagram of the experimental apparatus in its final form. M is a monochromatic illuminator which gets its light from an arc lamp. Light from M is focused by the lens L_1 on the pin-hole S, which acts as the point-source mentioned above, that illuminates the object (called No. 2 above) O. Behind and touching O is the achromatic lens L_2 which focuses the light from S upon the aperture A which is made separate from the camera C. This focused image of the point-source with its accompanying diffraction pattern, caused by the object O, is the concentration image which has been mentioned above. The camera represents our telescope and the aperture A represents the rim of the object glass.

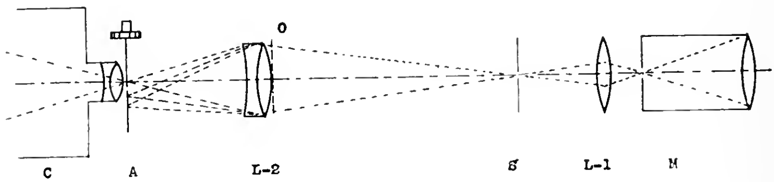


FIG. 1

The object O was made by drilling a hole 4.5 cm in diameter in a plate of copper. Across this, wire was placed, varying from 0.08 mm to 0.62 mm in diameter. This wire was placed in such a manner as to form details which could be studied in a photograph. The distance OA is determined by the focal length of the lens L_2. In this case OA was 120 cm.

If we now suppose that the object O is to represent the planet Mars, we can obtain by means of Airy's formula,[1] which gives the diameter of the diffraction rings formed at A by a circular aperture of given size at O, when illuminated by a point-source at S, the size of the apertures that must be used at A to represent given

[1] *Wood's Physical Optics*, 2d ed., p. 237.

telescope objectives. Further, as I shall now show, the distance from O to A, the size of the object O, and the size of the aperture A can be so chosen as to give on the photographic plate an image of the same optical excellence as that obtained by a telescope of given size at a given distance from a heavenly body which is represented by the object O. In other words we can duplicate astronomical conditions in the laboratory.[1]

In order to use Airy's formula, which refers to diffraction rings formed with a point-source of light, and apply it to an astronomical object seen by reflected diffused light, use must be made of the five assumptions made in the first part of this paper. Airy's formula for the fifth maximum diffraction ring is $\sin\theta=\frac{2.361}{\gamma}\lambda$, where θ is the angle of diffraction, λ the wave-length of the light used, and r the radius of the lens casting an image of the point. In this work light of wave-length 0.000055 cm was used, and r, the radius of the planet Mars, was taken as 2,100 miles. Then $\sin\theta=\frac{2.361\times 0.000055}{2100}$. Again remembering, according to the assumptions, that the lens is immediately in front of the body (No. 2) substituted for the actual planet (No. 1), we get another expression for $\sin\theta$ which is given by the radius of the fifth diffraction ring divided by the distance, say 35,050,000 miles, from the earth to the planet Mars. From these two expressions the value of x is found to be 2.138 cm. $2x=4.276$ cm, the diameter of the fifth maximum ring in the diffraction pattern from the object representing Mars.

According to Stoney's theory it is the relation of the aperture of the telescope to the diameter of the diffraction pattern that determines the excellence of the image. Perhaps it would seem from this that a telescope 4.28 cm in diameter should give an image of the planet. If we were dealing with ideal conditions, a telescope of this size might possibly give an outline of the planet, but the image would contain no details. It must be remembered that in nature we have an overlapping of a great many partial

[1] Optical conditions only have been duplicated. The assumption was made that the details on the planet Mars contrast against the surface as well as dark wires would against white paper.

images, and that the small telescope would not admit enough of these various patterns to form a satisfactory image.

We shall designate by the constant K the ratio of the diameter of the aperture of the telescope to the diameter of, say, the fifth bright ring in this diffraction pattern. If a 24-inch telescope is used, the value of K is found to be $\frac{24\times 2.54}{4.276}$, or 14.27. This means that the diameter of the telescope is 14.27 times greater than the diameter of the fifth bright diffraction ring from Mars. This value of K we shall now use in determining the diameter of A, the aperture in the laboratory apparatus that is to replace the actual objective aperture of the telescope considered.

If the diameter of O is 4.5 cm and OA equals 120 cm, $\sin\theta = \frac{2.361\times 0.000055}{2.25}$

Also as above, $\sin\theta = \frac{x}{120}$

Eliminating $\sin\theta$, $x = .006928$ cm,

$2x = .013856$ cm, the diameter of the fifth bright diffraction ring from O.

$.013856\times 14.27 = .1976$ cm, or about 2 mm, which is used as the diameter of the aperture which represents a 24-inch telescope in the experimental apparatus. Using this relationship, the sizes of the other apertures to be used are easily determined.

Table I shows the sizes of the apertures used and the diameters of the telescopes which these apertures represent in the experimental apparatus.

TABLE I

Aperture	Telescope
4 mm	48 inch
3	36
2	24
1.25	15
1	12
.6	7.2
.4	4.8

Returning now to the object of O, if 4.5 cm equals 4200 miles, the wires on the object which were 0.62 mm, 0.38 mm, and

0.08 mm, respectively, would equal 58 miles, 35.5 miles, and 7.5 miles respectively, on the planet Mars. These calculations were made because a great deal of discussion has taken place among astronomers concerning certain details upon the surface of the planet. So-called canals varying in width from 60 miles to 20 miles have been viewed by different astronomers. In order to investigate this point, I made the smallest details represent objects 7.5 miles in width. The pattern formed by these wires was chosen quite at random and in no way is supposed to represent the map of Mars. The things that are faithfully represented are the disk of Mars at this given distance, and dark objects of the sizes specified.

If an incandescent lamp behind a ground-glass screen were placed at S, instead of the point-source of light, there would be an infinite number of point-sources. Each of these sources or points would be capable of illuminating O and would thus give its diffraction pattern at A, or, as we have said, a concentration image. The advantage, then, in using the point-source S is simply to cut down the complexity of the diffraction pattern at A and in turn to simplify the image produced in the camera C. Then again, if this diffused white light were used, there would be a large number of different wave-lengths in operation. This is simplified by using the illuminator M. By the use of these two devices the image is greatly simplified and a study of it is much more easily made.

Upon examining the diffraction pattern at A, it is found that it consists not only of the usual bright central spot surrounded by bright and dark rings caused by the circular form of the object that represents Mars, but also in the field there are bright and dark patches which are spectra caused by the wires used to represent superficial details on the planet. Now any part of this pattern, when admitted into the camera or telescope, is capable of giving some kind of an image of the object O. This fact permits one more step in the analysis and gives a means of obtaining partial images on the sensitive plate. Under ordinary conditions most of the light is contained in the central bright spot and the first five or six bright rings. This being true, if this part of the pattern is admitted into the telescope, a good image should be obtained. If this part of the pattern does not enter the telescope, the image

will not be so good, the quality depending on the particular part of the pattern admitted. It is an interesting fact that if only the central maximum is admitted by the telescope aperture, we get no image of the object, although a large amount of light will be admitted. This is in conformity with Abbe's theory of image-formation.[1]

The above discussion now permits the insertion of two important definitions. The image obtained on the sensitive plate when the diffraction pattern from a point-source is admitted into the telescope or camera is called a partial image. If the central bright spot falls symmetrically within the aperture of the camera objective, an optimum partial image is formed. This seems to be the best image that it is possible to obtain of a given object with a given aperture, and is theoretically better than any image of a similar object taken under similar conditions with diffused light. In other words, with this laboratory apparatus we not only can imitate astronomical conditions, but can obtain even better images than are possible under natural conditions, because we can control the number of partial images that go to make up the final image. If the aperture A is moved to one side, the central bright spot will not be admitted into the camera and the image obtained will be formed by light from some part of the outer diffraction pattern. This image is one of the inferior images which, along with all the other partial images, good and inferior, go to make up the natural image of the object. From this it is easily seen that telescope images of bodies illuminated by diffused light are simply the result of an infinite number of partial images, and that instead of having one diffraction pattern falling within the camera, there is an infinite number of these patterns. It is then quite obvious that a natural image cannot be as good as an optimum image, because the natural image can be considered to be made up of the optimum and a number of other images which are very inferior. As the experimental results show, a natural image may approach the optimum in excellence but it cannot equal or surpass it.

EXPERIMENTAL RESULTS

The experimental results of this work are found in the accompaning photographs. Plate X contains four series of partial images

[1] *Wood's Physical Optics*, 2d ed., p. 223.

taken with different sized apertures. Series 1 was taken with a 2-mm aperture and represents the partial images that go to make up the actual image that would be seen with a 24-inch telescope. No. 1 was taken with the central bright spot of the diffraction pattern focused in the center of the aperture. This is what we call an optimum image and is better than any that can be obtained in actual practice under the same conditions. Nos. 2–8 were taken by moving the aperture of the camera to one side by means of the micrometer screw. The aperture was moved one half-millimeter each time. This means that No. 8 was produced by that part of the diffraction pattern 3.5 mm out from the central bright spot. Now it can be seen that in actuality there would be an infinite number of these partial images, and that the images we obtain with optical instruments are produced by an infinite number of these partials, interfering as they are superposed by a lens or by the eye. The increasing inferiority of the partial images as we go from 1 to 8 is to be noted.

The phenomenon of the change in appearance of the lines from black to white can be accounted for by the laws of diffraction. It will be noticed in the last part of each series that the horizontal wires almost disappear. This is caused by the movement of the aperture *A* to one side. After it has been moved 2 or 3 mm, it is in such a position that the parts of the diffraction pattern that enter the camera are altogether those caused by the perpendicular wires. This part of the pattern is not capable, therefore, of giving images of the horizontal wires. Investigations on a similar problem have been made with ordinary wire screens.[1]

No. 9 is what we might call the composite image. It is to be taken as an approximate imitation of a natural image. It is obtained by superposing the exposures 1–8 on the same plate, giving equal times of exposure to all eight. From the discussion above the conclusion would be drawn that this image, No. 9, should not be as good as No. 1. In series 1 a very little difference can be seen between 1 and 9. This is because the excellence and large number of the good images in the series outweigh the defects contributed by the poorer images. This point is made more clear by the following series. Series 2, 3, and 4 were taken exactly as

[1] A. B. Porter, *Philosophical Magazine*, 11, 154, 1906.

series 1, with exactly the same lateral movement of the aperture A, and show the effects of smaller apertures. Series 2 was taken with a 1.25-mm aperture representing a 15-inch telescope, series 3 with a 1-mm aperture representing a 12-inch telescope, and series 4 with a 0.6-mm aperture representing a 7.2-inch telescope.

It will be noted that as the aperture decreases in diameter, the number of good images decreases and that No. 9, the composite image, gradually gets poorer than the optimum No. 1. In series 4 the optimum itself is not distinct. This is on account of the fact that the aperture was so small that the main part of the diffraction pattern was not admitted. The subject of partial images will be discussed again in connection with Plate XII.

Series 1 of Plate XI shows eight optimum images represented by Nos. 1–8 inclusive, obtained with apertures of the sizes given in Table II.

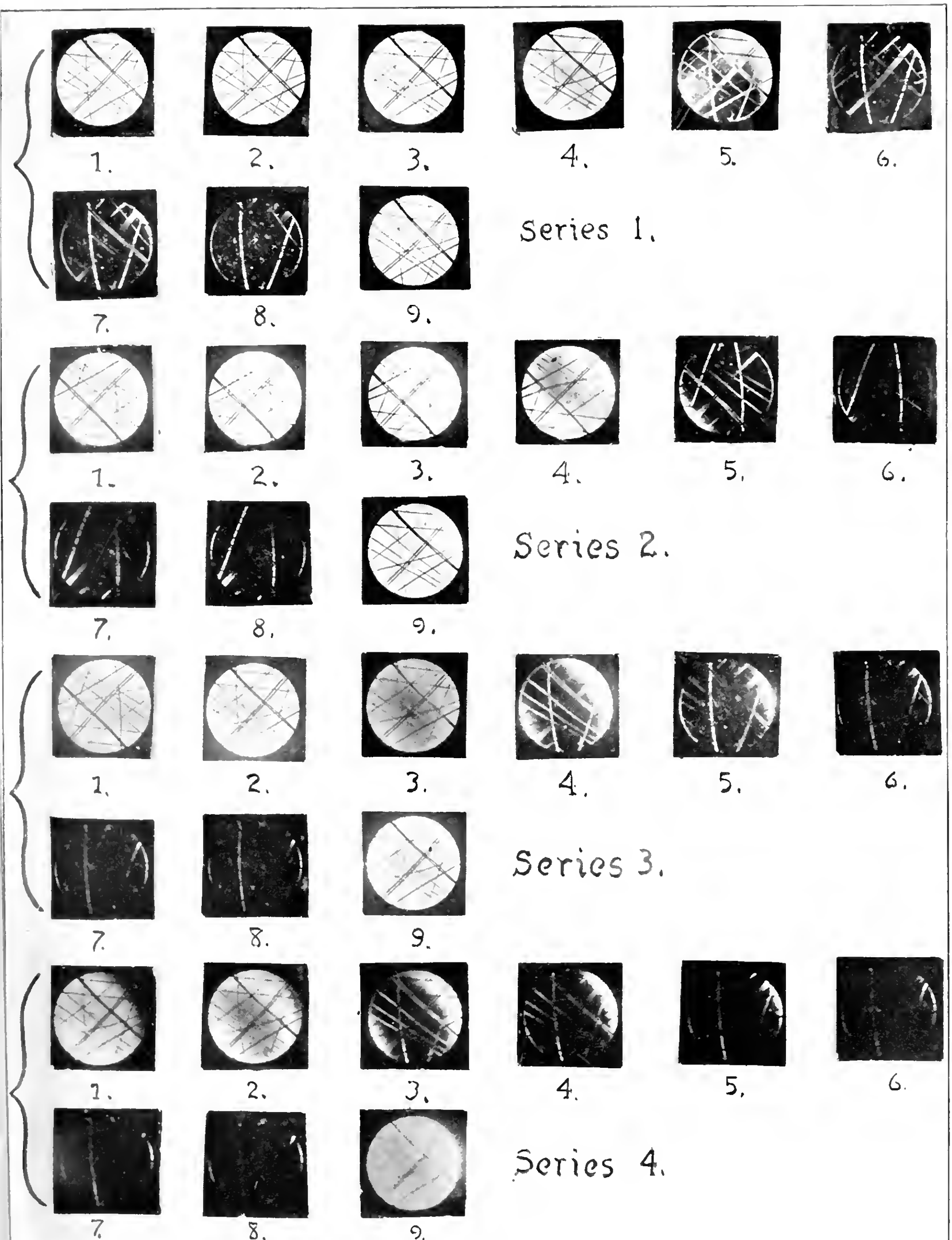
TABLE II

No.	Aperture	Telescope
1	4 mm	48 in.
2	3	36
3	2	24
4	1.25	15
5	1	12
6	0.6	7.2
7	0.4	4.8
8	0.3	3.6

Series 2 is taken with the same apertures and under the same conditions except that the object O was illuminated with diffused light instead of monochromatic light. This was done by removing the point-source S (Fig. 1) and placing in its stead an incandescent light in front of which was a ground glass. This series represents, then, the actual telescope images.

Series 3, Plate XI, was taken with the same apertures as were used in series 1 and 2, but with reflected light. To accomplish this a white cardboard was placed just back of object O and directly against the wires. Four incandescent lights were then placed in front and a little to one side of the object. The lens L_2 was removed while this series was being taken, so that the illumination on the object would be more intense. Nos. 7 and 8 are missing in series 3,

PLATE X


1. 2. 3. 4. 5. 6.
7. 8. 9.
Series 1.
1. 2. 3. 4. 5. 6.
7. 8. 9.
Series 2.
1. 2. 3. 4. 5. 6.
7. 8. 9.
Series 3.
1. 2. 3. 4. 5. 6.
7. 8. 9.
Series 4.

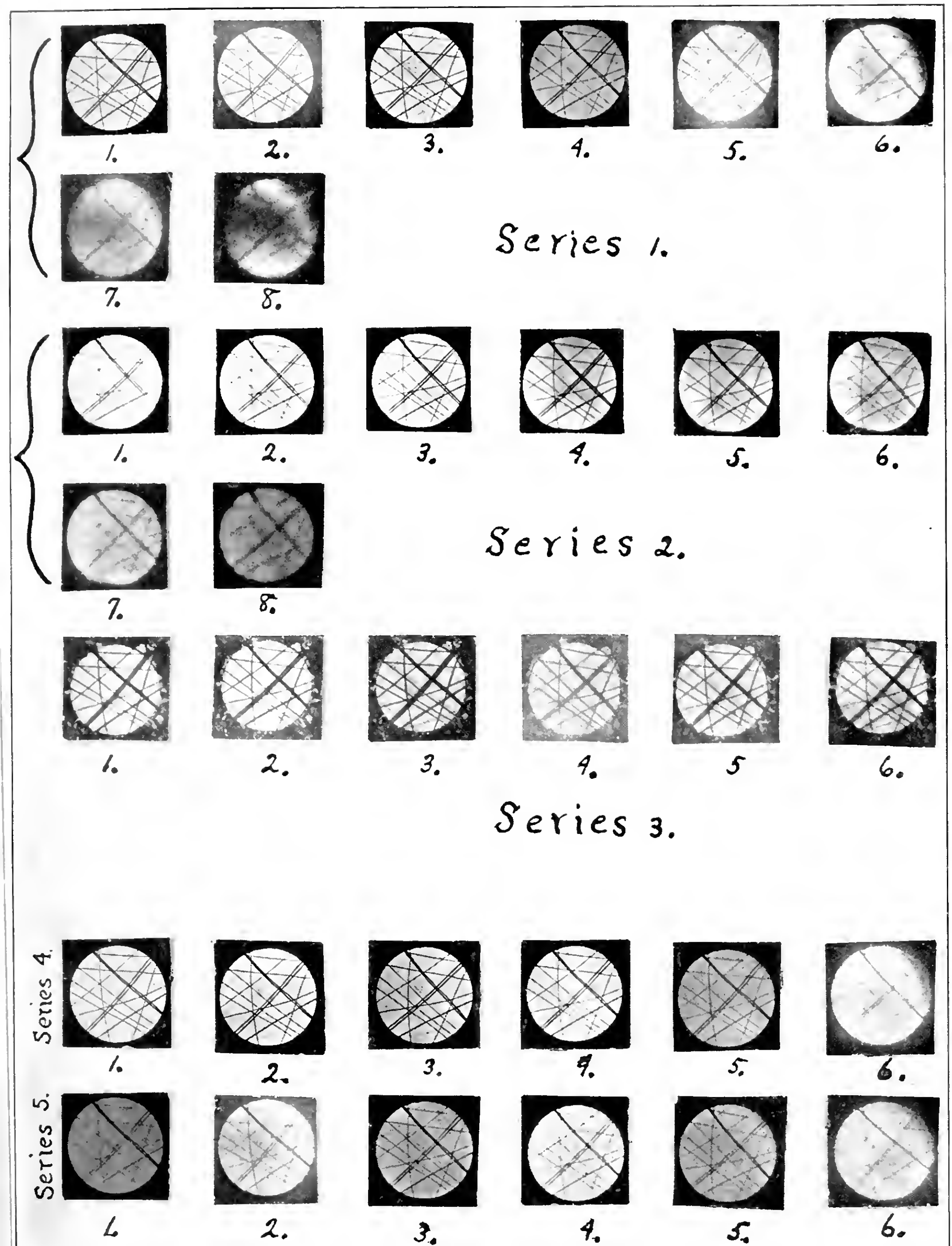

1.
2.
3.
4.
5.
6.
7.
8.
Series 1.
1.
2.
3.
4.
5.
6.
7.
8.
Series 2.
1.
2.
3.
4.
5
6.
Series 3.
Series 4.
1.
2.
3.
4.
5.
6.
Series 5.
1
2.
3.
4.
5.
6.

PLATE XII

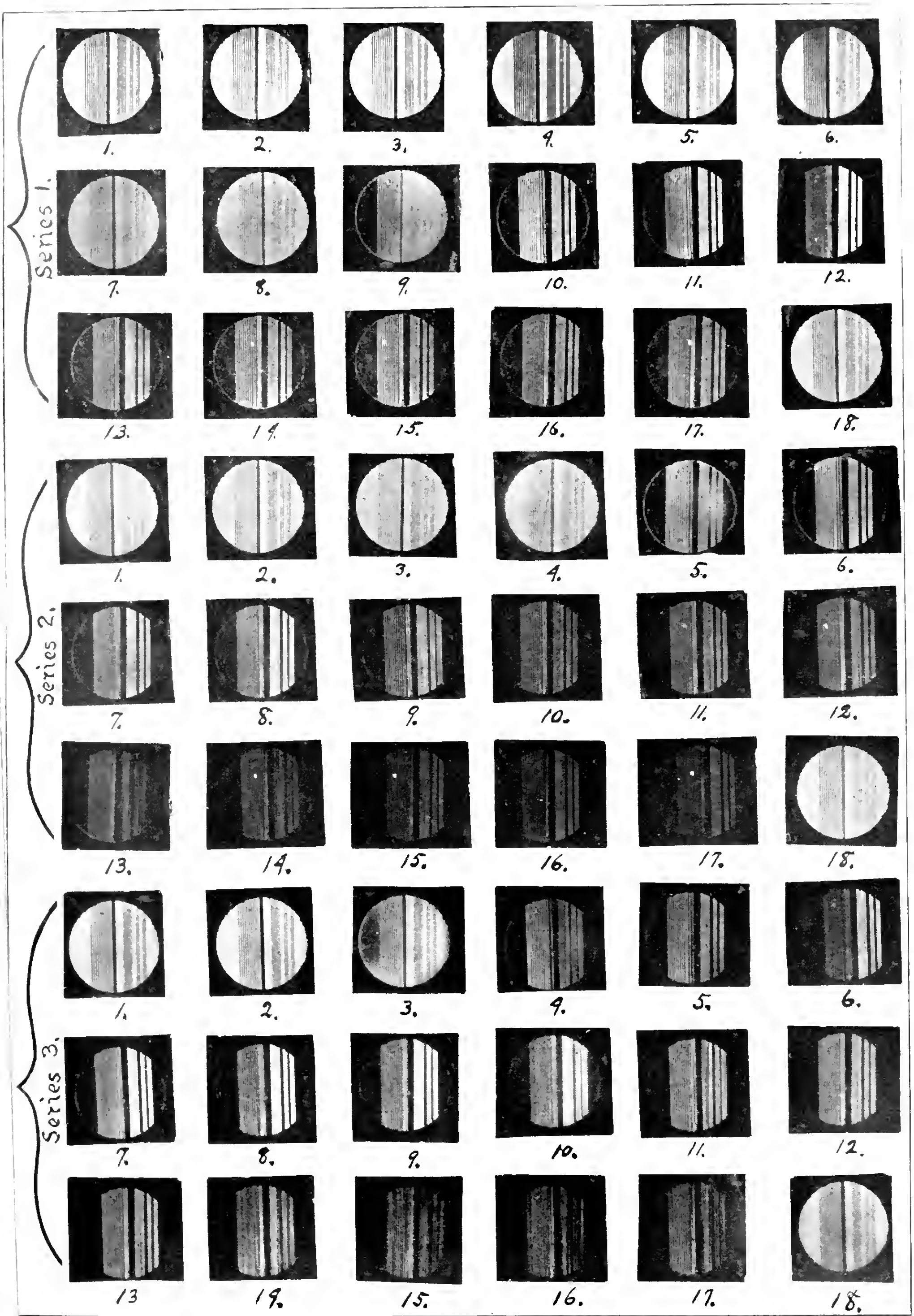
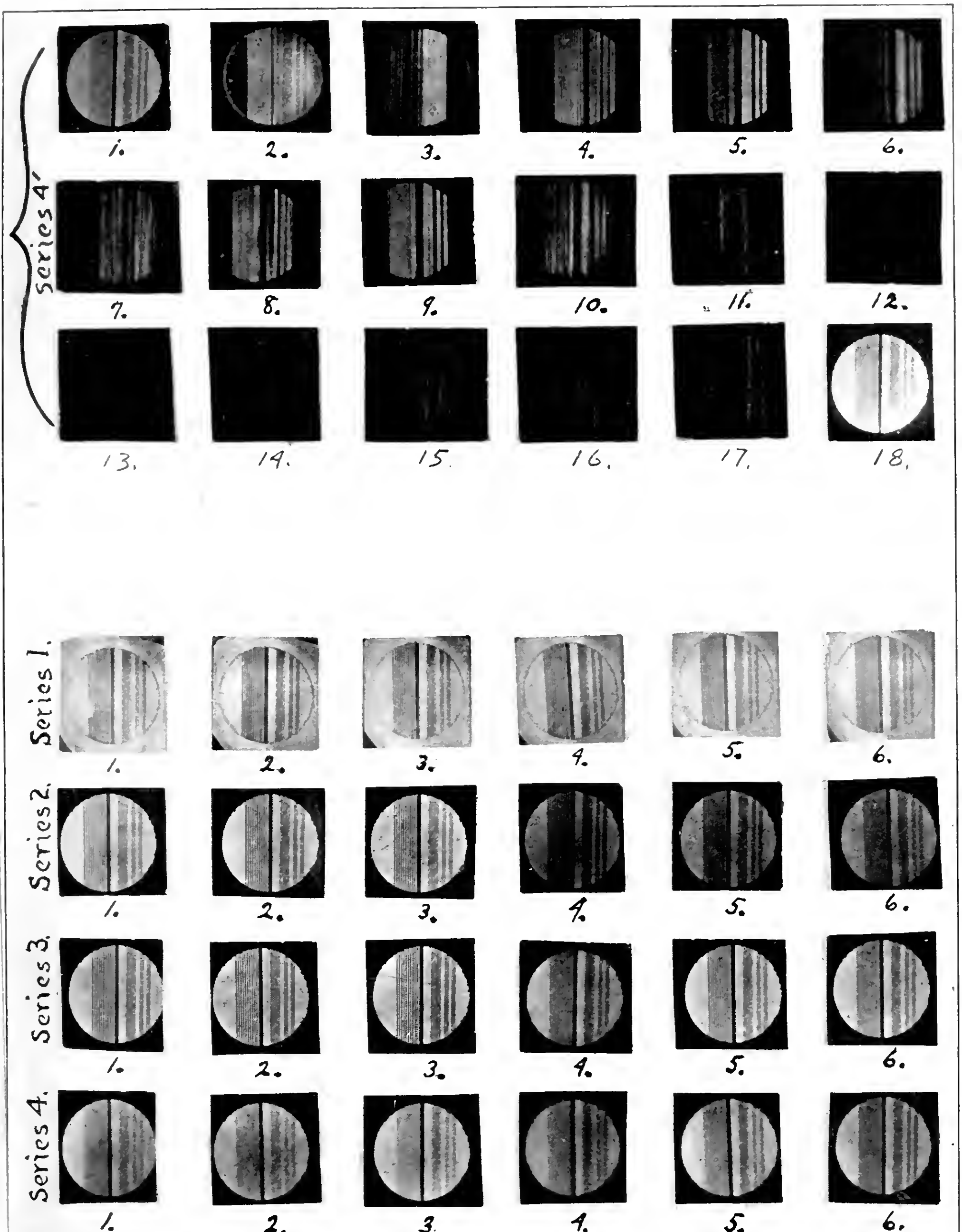


PLATE XIII


Series 4'
1. 2. 3. 4. 5. 6.
7. 8. 9. 10. 11. 12.
13. 14. 15. 16. 17. 18.
Series 1.
1. 2. 3. 4. 5. 6.
Series 2.
1. 2. 3. 4. 5. 6.
Series 3.
1. 2. 3. 4. 5. 6.
Series 4.
1. 2. 3. 4. 5. 6.

because the illumination could not be made intense enough to affect the photographic plate within a reasonable time.

The points to be noticed in comparing number with like number in these three series are as follows: (1) With the exception of the cases of the very small apertures, the optimum images are better than the diffused light images. (2) There is very little difference between series 2 and series 3. The lines in series 3 are possibly a little broader, owing to the effect of shadows. This point will be discussed again in connection with Plate XIII. The similarity of series 2 and series 3 shows that we were justified in using Stoney's assumptions in the substitution of object No. 2 for object No. 1.

Realizing that I was dealing with ideal atmospheric conditions I arranged for disturbing the atmosphere while an exposure was being made. To accomplish this, two bunsen burners were placed between the object *O* and the camera *C*. I do not claim that this gave real atmospheric conditions, but it is quite evident that there would be some similarity between movements of the air in the laboratory and movements of the atmosphere when a telescope is being used in actual practice. Series 4 and 5, Plate XI, show the results obtained. Series 4 is made up of optimum images taken with the apertures used above, while series 5 was taken under the same conditions as series 4, except that the burners were lighted and the air was in motion. It is quite evident that motions or disturbances in the atmosphere affect the quality of the image. The final conclusions from these results will be left for the latter part of this paper.

Realizing that the results obtained by use of the complex object might be questioned, I constructed a more simple object and photographed it under the same conditions as the first. The second object used was a simple wire grating made by winding wire around two screws which were fastened in a frame. The screws were made with fifty threads to the inch. Wire of two sizes (0.15 mm and 0.37 mm) was wound upon these screws and soldered. The wire on one side was then cut away, and that which was left was tightened by turning two nuts in the frame. This grating was then placed in the position of the original object. The advantage in using the grating with vertical wires came from

the fact that its diffraction pattern consisted of a row of horizontal spectra, rather than confused and promiscuously scattered spectra, as was the condition in the case of the first object. As the camera aperture is moved to one side, nearly all of the diffraction pattern is admitted (a part at a time) into the camera. The composite photograph would be a truer representation of the actual image in this instance, for more of the pattern is used than in the series already discussed.

Plate XII contains three series, showing the different partial images obtained with apertures of varying diameters, the first one in each series representing the optimum image. Series 1 was taken with a 4-mm aperture, representing a 48-inch telescope; series 2 with a 2-mm aperture, representing a 24-inch telescope; series 3 with a 1-mm aperture, representing a 12-inch telescope; and series 4^1, Plate XIII, with a 0.6-mm aperture, representing a 7.2-inch telescope. It will be noticed in series 1 that as the aperture was moved sidewise across the field, one half-millimeter at a step, six good images were obtained, while series 2 contains four, series 3, two, and series 4^1, Plate XIII, only one. It would be expected, then, that the composite image would be much better in series 1 than in the other series, and this is what is observed when an examination is made of No. 18 in each series. In series 1, No. 18 is practically as distinct as the optimum No. 1. In series 4^1, Plate XIII, even the optimum No. 1 is indistinct. This would indicate that the aperture did not admit enough of the diffraction pattern to give a good image.

Plate XIII, lower half, shows the same effects as were shown by Plate XI. These photographs were taken with the following apertures: 4 mm, 3 mm, 2 mm, 1.25 mm, 1 mm, and 0.6 mm, representing respectively, the same sized telescopes as were mentioned in the previous tables. Series 1 was taken with reflected light, series 2 with transmitted diffused light, series 3 with light from a point-source which gives us optimum images, and series 4 was taken under the same conditions as in series 3, except that the atmosphere between the camera and the object was disturbed by means of two bunsen burners. There seems to be very little, if any, difference between series 1 and series 2. This again shows

that it makes no difference where the illumination is located and is a further vindication of Stoney's theory. The optimum images in series 3 are better than the corresponding images in series 1 and series 2. This is possibly more evident in Plate XIII than in Plate XI. Series 4 shows conclusively that movements in the atmosphere affect the quality of the image to a marked degree.

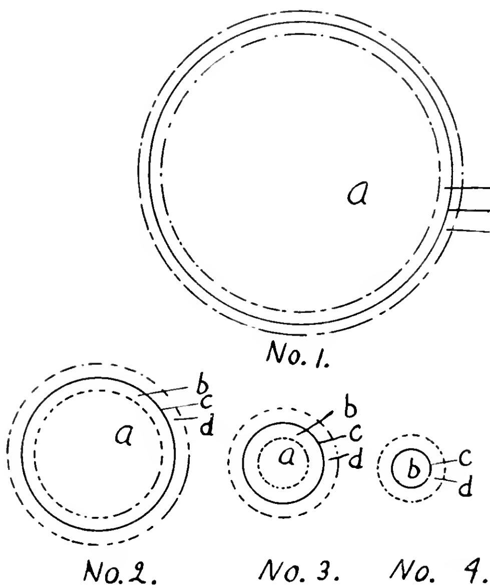


FIG. 2

In order to add clearness to the subject of partial images and further to explain the conclusions drawn from the photographs, Fig. 2 is inserted. The full line in these drawings indicates the relative size of the different apertures used. No. 1 represents a 48-inch, No. 2 a 24-inch, No. 3 a 12-inch, and No. 4 a 6-inch telescope.

The radii of the dotted circle are so chosen that the difference between each of these radii and the radius of the full circle would be equal to the relative radius of the fifth bright ring from the diffraction pattern of Mars. We can consider, then, that these three circles divide the space in and around the telescope objective

into three parts or divisions, a, b, and d. We then see that if the central bright spot of any of the infinite number of diffraction patterns from Mars falls within the space a, a good image will be formed. If the central bright spot falls within the space b, an image will be formed which will not be quite as good as those found in class a. As the central bright spot of the diffraction pattern passes over the edge of the telescope another class of images will be formed which will be much poorer than those of either class a or class b. This group of images is called class c. This corresponds to what has already been said in the previous paragraphs. If the central bright spot is not admitted into the camera, a very poor partial image is formed by the parts of the pattern that are admitted. As the central bright spot falls within the space d, only the outer diffraction rings enter the telescope and the images formed by them are poorer than those in either of the other classes. If the central bright spot falls outside of the space d, the essential parts of the pattern do not enter the telescope, and so we are not interested in them.

As has already been shown, the image formed in a telescope is the result of the superposition of the images from these four classes. If class a predominates the resultant image should be a comparatively good one, while if the poorer or inferior classes predominate, the resultant image will be of an inferior quality and thus will be indistinct.

By finding the areas of the different sections in each of the drawings of Fig. 2 some interesting comparisons can be made. Table III shows the relative values.

Now if we remember that the images formed by class b are fairly good partials and that those formed by class a are optimum partials, we can make use of Table III. When a 48-inch telescope is used, 75 per cent of the partial images making up the actual image are found in class a, while 86 per cent of the total are included in classes a and b. This would indicate that a 48-inch telescope should give a very good image of the disk of the planet Mars. These calculations are, of course, based on the image of the disk of Mars. The excellence of the image of the smaller details of the planet will be dependent on the relative size of these details as

compared with the size of the disk of the planet. No. 2 shows that when a 24-inch telescope is used, 56 per cent of the partials that make up the image are included in class *a* and 75 per cent in classes *a* and *b*. When we remember that the light from class *a* and class *b* is more intense than that from the other classes, we might

TABLE III

No.	Area sec. a	Area sec. b	Area sec. d	Area total	Area $a+b$
1	435	65	78	578	500
2	93	32	40	165	125
3	16	15	20	51	31
4	1.5	6.3	11	20.8	7.8

	Per cent a is of total area	Per cent $a+b$ is of total area
1	75	86
2	56	75
3	31	60
4	7	37

expect a 24-inch telescope to give a reasonably good image. Nos. 3 and 4 indicate how far the quality of the image falls off when a 12-inch or a 6-inch telescope is used. In the latter case the aperture is so small that nearly all of class *a* is excluded. This is also in accord with experimental results, for it is found that when the aperture would not admit the whole of the bright part of the diffraction pattern, the image obtained was very indistinct.

CONCLUSIONS

1. The assumptions made at the beginning of this paper have been proved to be correct. This is first shown in Plates X and XI and later by Plates XII and XIII. There seems to be no difference between the photographs taken with transmitted diffused light and with reflected light. The sum of the partial images gives a composite image which seems to be of the same quality and shows the same details as the images obtained with diffused light. This point is of great importance because it shows that the resolution of the image into its different partials is permissible. It also shows that

the introduction of the lens in front of the object did not alter the optical conditions. If there is any difference between the composite image and the diffused light image, it may be accounted for by recalling that the composite image was taken with light of one wave-length, while white light was used when the diffused light images were obtained.

2. In order to obtain a good image the aperture of the telescope must admit all of the brighter parts of the diffraction pattern from the object. The essential part of the pattern would usually consist of the central bright spot and the first five or six bright rings. This conclusion is based on series 4, Plate XI, and series 3, Plate XIII. No. 6 in each of these series shows the effect of using an aperture that would not admit all of the brighter parts of the pattern.

3. Airy's formula can be used in making an estimate of the size of a telescope necessary to give a distinct image of a heavenly body. This is shown particularly in Fig. 2. The calculations and estimates made by use of this formula agree quite closely with the experimental results.

4. The quality of the image is not greatly improved by increasing the size of the telescope aperture indefinitely. Smaller details may be seen, however, with the larger telescopes. This is shown especially in series 3, Plate XIII. It will be observed that Nos. 1 and 2 are equally good. No. 1 was taken with a 4-mm aperture and No. 2 with a 3-mm aperture. No. 1, however, might have been able to show details that would not appear in No. 2, if smaller details had been present in the object.

5. Under ordinary conditions in nature, perfect images are not obtained. It has been shown that better images can be obtained with this experimental apparatus than can be obtained under normal conditions. In every case it was found that when an aperture large enough to admit the essential parts of the diffraction pattern was used the optimum images were better than those secured by the use of diffused light. This is shown in Plates XI and XIII.

6. If atmospheric conditions were perfect, details as small as 7 miles across, on the planet Mars, could be detected with a 24-inch telescope. This seems to be in agreement with the conclusions of Professor Lowell. He has detected certain details upon the sur-

face of the planet as readily with his 24-inch telescope as a number of other astronomers have seen with the larger instruments. This was due, not only to the undeniably good atmospheric conditions found in Arizona, but also, in accordance with the results of this paper, to the fact that his telescope is sufficiently large for these details. Whether these details represent canals or not is outside the scope of this paper.

7. Movements in the atmosphere affect the quality of the image to a marked degree. While real atmospheric conditions were not produced in the laboratory, yet the results obtained can be relied on to a certain extent.

PRACTICAL APPLICATIONS OF EXPERIMENTAL APPARATUS

1. The apparatus as set up in this experiment can be used in obtaining a photograph of any transparent object, such as lines on a glass plate. A better photograph can be obtained in this way than can be secured in the ordinary way with diffused light for illumination.

2. This apparatus can be used in studying the partial images from any transparent body and the contribution of each of these to the total image formed naturally with diffused light.

3. As advocated by Stoney and as shown in the results obtained in this paper, this apparatus can be used as a practical check on the work of astronomers, and, in fact, on any persons who view or photograph distant or minute objects.

In conclusion I wish to acknowledge indebtedness to the staff of the physical laboratory of the State University of Iowa for their interest in the work, and, especially to Professor L. P. Sieg for suggesting the problem.

Physical Laboratory
State University of Iowa
June 1914

A PHOTOGRAPHIC PERIODOGRAM OF THE SUN-SPOT NUMBERS

By A. E. DOUGLASS

In 1898 Professor Arthur Schuster suggested the construction of a periodogram which should display rhythmic time intervals in any series of measures, in the same manner that a spectrogram shows the space vibrations or waves in a beam of light. He analyzed the sun-spot numbers as an example, and showed how several periods, 4.38, 4.80, 8.36, 11.125, and 13.5 years, seemed to be operating,[1] some more intensely than others.

In 1913 Kimura[2] performed an analysis of the same by a method of successive approximations and derived the amplitudes of a large number of different periods, which when combined produce a result remarkably like the sun-spot curve since 1750. Some of his chief periods are: 82.2, 54.2, 20.03, 12.05, 11.114, 10.48, 9.99, and 8.55 years. Last year also Professor Turner took up the problem with consummate skill and untiring energy.[3] On thorough mathematical analysis in the first three papers, he finds that the main features of the sun-spot record can be represented by four periodic terms of approximately 8.3, 10.2, 11.4, and 14.7 years, but that their coefficients do not remain constant, and only the 11.4-year period is sensible at the present time. Meanwhile A. A. Michelson[4] applied the harmonic analyzer and determined the amplitudes of a considerable number of periods, such as 8.6,

[1] "On the Investigation of Hidden Periodicities, with Application to a Supposed 26-Day Period of Meteorological Phenomena," *Terrestrial Magnetism*, **3**, 13–41, 1898; "On the Periodogram of Magnetic Declination at Greenwich," *Cambridge Philosophical Society Transactions*, **18**, 107–135; "The Periodogram and Its Optical Analogy," *Proceedings of the Royal Society*, **77** A, 136–140, 1906; "On the Periodicities of Sun-Spots," *Philosophical Transactions of the Royal Society of London*, **206** A, 69–100, 1906.

[2] "On the Harmonic Analysis of Sunspot Relative Numbers," *Monthly Notices, R.A.S.*, **73**, 543, 1913.

[3] *Monthly Notices, R.A.S.*, **73**, 549, May 1913 (discussion of Kimura's paper); **73**, 714, sup. number, 1913 (applying the Fourier sequence); **74**, 16, November 1913 (continuing the last); **74**, 82, December 1913 (discussing discontinuities and the meteoric hypothesis).

[4] "Determination of Periodicities by the Harmonic Analyzer with an Application to the Sun-Spot Cycle," *Astrophysical Journal*, **38**, 268, 1913.

PLATE XIV

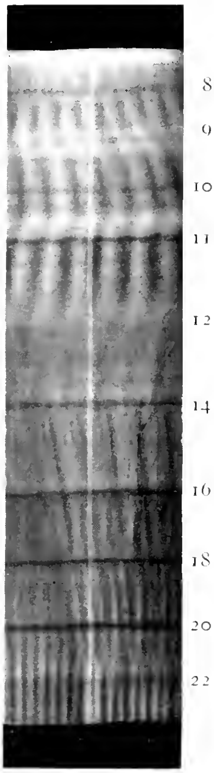


FIG. 1.—Periodogram of the sun-spot numbers. Corrugations show periods. The numbers give time of vibrations in years. The white line is the year 1830 and shows phase.

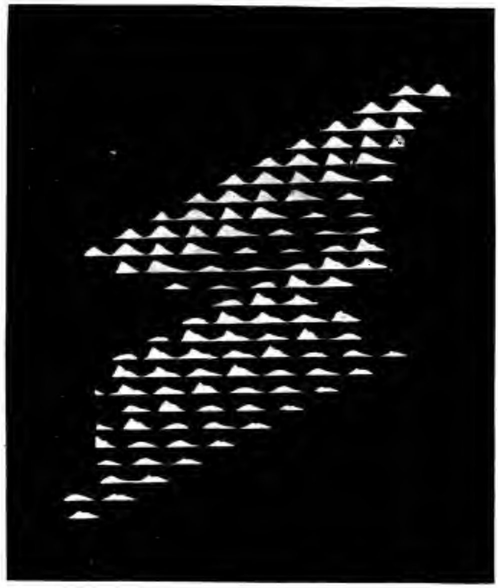


FIG. 2.—Diagram used in making the periodogram, consisting of the sun-spot curve mounted in multiple.

FIG. 3.—Figure 2 photographed out of focus to show discontinuities in the vertical lines.

10.3, 11.4, 15.1, 58, and 105 years, but most of these appear to him doubtful. He concludes, "Indeed it would seem that with the exception of the 11-year period and possibly a very long period (of the order of 100 years), the many periods found by previous investigators are illusory."

But Turner in his last article (December 1913) presents the matter from a new viewpoint. He investigates discontinuities in the series and finds that the breaks in the sun-spot series "are near the dates 1766, 1796, 1838, 1868, 1895, which are sufficiently close to those of Leonid periphelia to suggest a *vera causa*," and he discloses his hypothesis of the meteoric origin of sun-spot periodicity, a hypothesis of the greatest interest but not pertinent to the present subject.

The work described below adds little to the information obtained in the investigations alluded to, but it presents the sun-spot history in a new way and suggests perhaps a rapid means of carrying on preliminary studies of periodicity in any series of continuous records.

In Plate XIV, Fig. 1 is a periodogram of the sun-spot curve from 1755 to 1911, made by a photographic process in which the camera has done the additions for all the periods named beside it. The existence of a rhythm in any specified period is indicated by a beaded or corrugated effect. The corrugations are in fact the rhythmic vibrations of the curve. On a moment's examination this periodogram shows much of the information referred to above. The 11-year period is the most pronounced, yet not so superior to all others as would be expected. It may be of any duration from 11 to 11.8 years; 11.4 is a good average. There is obviously a period somewhere between 9.5 and 10.5 years and one between 8.0 and 8.8, but it is less conspicuous. Faint indications of periods are found near 14 years. The double of 8.4 is seen between 16 and 17 years. The double of the 10-year period shows near the 20, and at 22 the double of the 11 begins. Other photographs made at the same time[1] show the same periods extremely well, but were less satisfactory in some of the mechanical or photographic details.

[1] May 1913. H. H. Clayton had produced a periodogram by mechanical means some years previously. The writer here wishes to acknowledge his indebtedness to Professor E. C. Pickering for permission to use some of the facilities at Harvard College Observatory in this work.

In all cases the phase is located by the central white line which represents very closely the year 1830. In this photo-periodogram therefore, we have at once pictured to the eye some of the general results of the mathematical work. The method has the disadvantage that we cannot get absolute amplitudes, but perhaps some photometric work could give approximate values if it were worth trying. But relative amplitudes show at once; and we can judge, not only of the relative importance of different periods, but of the precision, or its lack, with which any period can be stated.

Fig. 2 shows a part of the process. It is the diagram from which the periodogram was photographed. In order to produce it the sun-spot curve was cut out in white paper and pasted in multiple, as is seen, on a black background. The left end of each of the upper 10 lines is the date 1755. Each successive line is moved 10 years to the left, so that passing from above vertically downward each successive line represents a date 10 years later than its predecessor. This continues until the whole period from 1755 to 1911 is covered; and the lower 10 lines show the latter date at their right ends. It is not necessary that any of the lines should be full length, as we use only a part of each. Now, by passing the eye downward from the top, a period near 10 years will show itself at once by the successive crests in vertical alignment. If the crests form a line at some angle to the vertical, then the period they indicate is not exactly 10 years; it is more than 10 if the slant is to the right and less than 10 if the slant is to the left. The horizontal lines are spaced the equivalent of 5 years, hence if we measure the angle A made by a vertical line and a line joining two crests in successive horizontal lines, we find the period indicated is

$$P = 10 \text{ years} + 5 \tan A$$

where A is positive if measured on a slant to the right.

Since each angle with the vertical represents a different period, it was only necessary to mount this diagram on an axis with clock work and slowly rotate it, in front of a camera with a cylindrical lens over its objective and a narrow horizontal slit in the focal plane and a sensitive plate passing slowly downward across the slit, to produce the periodogram. The cylindrical lens with

horizontal axis summates the duplicate curves in whatever line happens to be vertical. Of course there is a practical limit to the different angles at which the diagram may be viewed. An angle too far in one direction, making the tested period very small, would require a great number of duplications of the curve, while too great an angle the other way, making the tested period very large, catches the curve used here in a non-symmetrical form and introduces error. In the periodograms actually made of the sun-spot curve the minimum period tested was 7 years and the maximum 24 years. One notes especially that this is a continuous process and that all periods from the minimum to the maximum are tested.

In the arrangement described above, there are several limitations to the accuracy of results. First, the curves are non-symmetrical about their horizontal bases, and, when summated in a slant line far from the vertical, crests are thrown to one side of their proper place. Plotting the curves above and below a line would improve the result. Secondly, the resolving power decreases as the slant either way increases: the plot must be on larger and larger scale to overcome this. Thirdly, the wider the slit in the focal plane, the less the resolving power. Fourthly, there is a photographic limitation due to the failure of the photograph to show slight contrasts, and, fifthly, the eye cannot ordinarily detect contrasts under about 10 per cent.

When one applies this method as here described to other curves, a new condition presents itself, for the percentage variation in most curves is far less than in the sun-spot numbers. One must therefore cut off the lower part of the plot between the zero line and the lower extremes of the curve; for example, it would be thus in a series of barometer readings. However, in spite of these limitations the photographic periodogram does give important information quickly and plainly to the eye and at little cost, and thus can serve as a guide, showing when more refined methods are desirable.

But in common with any single mathematical treatment, the periodogram has one defect, it assumes the variables to be continuous throughout the series. It seems to the writer that Turner

has caught an important secret of the sun-spot problem, namely, that the variables are discontinuous. One can see from his papers how great a labor it has taken to reach that conclusion. This accounts for much of the discordance between investigators and the disappointment one has felt in the lack of definite result and a basis of prediction for the future.

Now it would take several periodograms of the type here presented to show this discontinuity, but the method here given, with a slight change, does show the whole history of the sun-spot discontinuities at a glance. Fig. 3 is a photograph of Fig. 2, taken out of focus for the purpose of calling attention to certain general features. In Fig. 2 the eye naturally turns to the sharp outlines and notes its minute details. In Fig. 3 the crests of 2 are changed into large blotches connecting somewhat with their nearest neighbors and varying in intensity. The sun-spot sequence appears in each nearly vertical line of blotches. Having a number of exactly similar lines side by side, the irregularities are repeated in each and thus strike the consciousness with the effect of repeated blows. These irregularities are the discontinuities referred to by Turner in connection with his hypothesis. It is evident at a glance that the sun-spot sequence divides itself into three parts, namely, a 9.3-year period, 1750–1790, then an interval of readjustment, 1800–1830, with a 13-year period, and lastly an 11.1-year period lasting to the present time (values approximate).[1] But the latter is not perfectly constant, for after 1870 there is a change in intensity. The breaks thus shown and Turner's dates of discontinuity are here compared.

Periodogram	Turner
	1766
Between 1788 and 1804	1796
" 1830 and 1837	1838
" 1870 and 1884	1868
	1895

[1] In discussing the periodicities of sun-spots (pp. 75–78) Schuster divided his 150 years, from 1750 to 1900, into two nearly equal parts. He found in the first part two periods of $9\frac{1}{4}$ and $13\frac{3}{4}$ years acting, successively, and in the second part, a period of 11.1 years. This is shown graphically in Fig. 3.

By means of this diagram, one can discover at a glance the origin of many of the periods which Michelson thought were illusory, and in which he was largely right. We can plainly see a 9.3-year period in the early part of the curve; let us call this part of the sequence A_n; and let us call its broken continuation near the center B_n, and the lower and later part giving the 11.1-year period, C_n. Thus we get at once three periods 9.3, 11.1, and something over 13 years. If now we bring the average A_n into line with the average C_n as the periodogram does we get 11.4 years. If we bring the average A_n into line with the average C_{n-1} we get close to 10 years. If we bring into line A_n and the heavier parts of C_{n-2} we get 8.4 years or thereabouts. And at 5.6 years we find a period which is just half of C_n and at 4.7 we find the half of A_n, and so on. It is like a checker board of trees in an orchard; they line up in a number of directions with more or less intensity. But the diagram in Fig. 3 helps remove some of the complexity of the sun-spot problem. It shows us that while these various periods are apparent, yet many are illusory as Michelson said. The diagram supplies a basis for profitable judgment in the matter. Hence to avoid just such awkward cases as the sun-spot curve, it is here presented as a necessary accompaniment of the periodogram in any doubtful cases.

University of Arizona
July 1914

WAVE-LENGTHS OF THE CHIEF LINES OF NITROGEN AND OXYGEN IN THE REGION λ 3880 TO λ 4700

By JOHN S. CLARK

In a note to the Astronomical and Astrophysical Society of America in August 1911, Sebastian Albrecht drew attention to the urgent need of additional investigation in the laboratory of the wave-lengths of the lines due to nitrogen, oxygen, and silicon, in order to obtain sufficiently accurate data for his work on stellar radial velocities. It was pointed out that the two best laboratory determinations differed from each other by many times the probable errors of the wave-lengths of these lines in stellar spectra relative to the standard lines of carbon (λ 4267.280), hydrogen (λ 4340.634), helium (λ 4471.693), and magnesium (λ 4481.375), and that this uncertainty as to the wave-lengths caused the radial velocities of the B-type stars to be uncertain by possibly as much as 4 or 5 km per second. The present investigation was carried out with a view to determining accurately the wave-lengths of the chief lines of nitrogen and oxygen in the region usually studied in stellar spectrograms, viz., λ 3880 to λ 4700.

It was found that suitably sharp nitrogen and oxygen lines were obtained when a highly condensed discharge was passed through dry air at rather low pressure in a capillary vacuum tube, and this method was used as a source of the lines. The spectra were photographed by means of a 10-foot concave grating, mounted according to the method described by Eagle,[1] and the third-order spectrum, giving a mean dispersion of 1.7 A per mm, was used throughout. The internal shutter of the instrument permitted two comparison spectra to be partially superimposed on the spectrum to be measured, and so the iron arc was exposed twice, once before and once after the long exposure. Thus it was arranged that any displacement, due to change of temperature or other cause, occurring during the exposure, might be detected. The temperature of the air near the grating was measured by means of a resistance thermometer and recorded on the chart of a Callendar recorder. Now the

[1] *Astrophysical Journal*, 31, 120, 1910.

coefficient of linear expansion of speculum metal is 0.0000193 per degree C. Hence at λ 4100, the middle of the region examined on one plate, a temperature change of 0°17 C. (which was the total observed rise in temperature during the particular four hours' exposure) would correspond with a displacement of 0.0134 A. The observed displacement, as deduced from the measurements with reference to the two iron comparisons, was in the correct direction, but its average value was only 0.005 A, and as the greater part of the rise in temperature occurred at the beginning of the exposure (as shown by the temperature chart) the wave-lengths obtained in this particular case from the second iron comparison were given greater weight (in the proportion of 3 to 2) in estimating the final wave-lengths.

In measuring the negatives the lines were divided into groups extending over not more than 15 A, and these groups were measured with red to the right and left in succession, first with one iron comparison, and then with the other, two independent values for the wave-length thus being obtained from each plate. The standard iron lines used were in every case "A lines" given by Burns,[1] and the measured wave-lengths were corrected where necessary by means of parabolic curves of errors obtained from the measured iron lines.

The results are given in the appended tables, and the previous measures of Neovius[2] and of Frost and Adams[3] are included for comparison. The latter are given in terms of Rowland's units, and to facilitate comparison the present measures have also been converted from the International to the Rowland scale by means of a curve taken from Kayser's *Handbuch der Spectroscopie* (**6**, 890).

Column 4 in the tables gives the probable errors derived from successive measures of the lines. The probable errors due to faulty setting of the cross-wires were worked out from the actual settings by means of the usual formula, viz.,

$$\text{Probable error of mean} = 0.6745 \sqrt{\frac{\Sigma(d^2)}{n(n-1)}}$$

[1] *Zeitsch. für wiss. Phot.*, **12**, 6, 1913, and *Lick Observatory Bulletin*, **8**, 247, 1913.

[2] *Bihang. till K. Svenska Vet.-Akad. Handl.*, Bd. **17**, No. 8, 56, 1891.

[3] *Astrophysical Journal*, **16**, 120, 1902.

TABLE I

NITROGEN

CLARK				PREVIOUS MEASURES			REMARKS
λ (International Units) 1	λ (Rowland) 2	Intensity (Max. 10) 3	Probable Errors 4	Neovius 5	Intensity 6	Frost and Adams 7	8
3914.138	3914.28	< 1	±.004	....	....	....	Not tabulated by Neovius
14.301	14.45	2	.001	....	....	....	
19.003	19.15	3	.001	3919.2	10	....	
55.851	56.00	4	.004	56.1	7	....	
94.995	95.15	10	.005	95.2	12	....	Broad line
4026.087	4026.23	1	.004	4025.9	2n	....	
35.090	35.24	3	.001	35.2	5n	....	
41.325	41.47	6	.01	41.4	6n	....	
97.327	97.48	10	.004	97.4	8n	....	Lockyer's enhanced lines (4097.45, 4103.5)
4103.393	4103.54	6	.004	4103.4	4n	....	
19.222	19.37	4	.003	19.3	9	....	Also oxygen line according to Neovius
....	24	< 1	....	24.0	5	....	Not measured
33.654	33.80	1	.005	34.2	6	....	
45.759	45.91	2	.001	45.8	7	....	
....	52	< 1	....	52.0	5	....	Not measured
76.163	76.31	2	.001	76.7	5n	....	
4227.738	4227.90	1	.001	4228.5	6n	....	
36.99	37.15	1n	.01	37.0	6n	....	Close diffuse doublet
41.787	41.94	8	.003	42.0	6n	....	
4348.078	4348.24	1	.003	4347.9	6	4348.13	
....	....	....	....	4426.1	5	....	
4447.035	4447.20	10	.003	47.2	12	4447.16	
....	....	....	....	60.0	3	....	
4510.91	4511.08	1	.01	....	....	....	
14.865	15.04	2	.008	4514.8	3	4515.08	
30.421	30.59	2	.005	30.3	6n	....	
4601.490	4601.66	4	.002	4601.3	9	4601.63	
07.167	07.34	3	.004	07.2	8	07.31	
13.884	14.06	2	.002	14.2	8	14.03	
21.405	21.58	3	.002	22.0	9	21.55	
30.551	30.73	8	.003	30.9	12	30.70	
34.165	34.34	1	.01	34.0	3	....	Diffuse line (possibly poor measure)
40.649	40.82	3	.005	40.5	1	....	
43.106	43.28	4	.002	43.4	9	43.24	
50.853	51.03	1	.01	51.0	4	50.93	Also given as an oxygen line

TABLE II

Oxygen

Clark: λ (International Units) 1	Clark: λ (Rowland) 2	Clark: Intensity (Max. 10) 3	Clark: Probable Errors 4	Previous Measures: Lunt 5	Previous Measures: Intensity (Max. 10) 6	Previous Measures: Frost and Adams 7	Remarks 8
3882.194	3882.34	1	±.004	3882.37	6	....	
3911.951	3912.10	2	.001	3912.19	7	....	
45.033	45.18	< 1	.007	45.26	6	....	
54.368	54.51	3	.003	54.64	7	....	
73.266	73.41	5	.001	73.51	9	....	
82.725	82.87	1	.007	82.93	6	....	
4069.635	4069.78	3	.01	....	....	....	Not given by Lunt or Neovius, but probably due to oxygen
69.903	70.05	4	.005	4070.07	10	....	
72.156	72.31	6	.002	72.45	10	....	
75.869	76.02	8	.001	76.09	10	....	
4105.001	4105.15	1	.004	4105.15	6	....	
19.222	19.37	4	.003	19.51	8	....	Also given as nitrogen
20.267	20.42	1	.01	20.61	6	....	
32.99	33.14	1	.01	33.06	5	....	Diffuse line—possibly poor measure
53.310	53.46	1	.001	53.64	6	....	
85.453	85.61	3	.001	85.72	6	....	
89.793	89.95	3	.003	90.06	7	....	
4317.160	4317.32	2	.01	4317.25	6	4317.27	
19.647	19.81	2	.005	19.79	6	19.76	
45.570	45.73	2	.006	45.75	6	45.68	
47.429	47.59	2	.008	47.54	6	47.62	
49.435	49.60	4	.007	49.52	8	49.54	
51.275	51.44	3	.004	51.35	7	51.50	
66.906	67.07	2	.01	67.03	6	67.01	
4414.888	4415.05	5	.01	4415.08	10	4415.08	
16.974	17.14	4	.005	17.02	9	17.12	
4590.983	4591.16	3	.01	4590.91	8	4591.07	
96.189	96.36	2	.005	96.05	7	96.29	
4638.865	4639.04	1	.005	4638.88	5	4638.94	
41.827	42.00	3	.005	41.84	8	41.89	
49.148	49.32	6	.001	49.06	9	49.25	
50.853	51.03	1	.01	50.95	5	50.93	Also given as a nitrogen line
61.650	61.83	2	.002	61.64	5	61.73	
76.246	76.42	1	.008	76.22	5	....	

but these errors were always less than 0.001, and in many cases less than 0.0005 for the better lines. The figures tabulated are estimates of the probable errors derived from successive measurements of the wave-length, due consideration being paid to the character of the line and the difficulty of measuring it accurately. As a matter of fact, however, the lines, being produced under the conditions of low pressure and high-tension discharge, were in general exceedingly sharp and thus well adapted to accurate measurements.

The foregoing measurements were carried out in the spectroscopic laboratory of the Imperial College of Science and Technology, London, under the direction of Professor Fowler, F.R.S., to whom the author is indebted for constant interest during the measurement, and advice in the preparation of the paper.

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FIVE LITHIUM LINES AND THEIR MAGNETIC SEPARATION

By NORTON A. KENT

INTRODUCTION

In the Zeeman effect given by narrow, double, and complex lines Michelson,[1] Gmelin,[2] and Wendt[3] observed assymmetries. The interest involved in these facts arose from the knowledge that the complicated magnetic separation of series doublets and triplets was dependent upon the close relationship of the vibrating systems within the atom (Paschen,[4] 1909), and further from the proof that narrow series doublets and triplets suffered a magnetic change, the final result of which was a single normal triplet (Paschen and Back[5]). Examples suitable for a study of these changes, Na λλ 3303 and 2853, were treated by Back[6] and Fortrat[7] respectively.

Zeeman[8] had shown Li λ 6708 a double line and Back had succeeded, at 29,800 gausses, in obtaining with this and other lithium lines, as a final effect, a single normal triplet.

[1] *Astrophysical Journal*, **8**, 46, 1898.

[2] Dissertation, Tübingen, 1909.

[3] Dissertation, Tübingen, 1911.

[4] *Annalen der Physik*, **30**, 746, 1909.

[5] *Ibid.*, **39**, 897, 1912; **40**, 960, 1913.

[6] Anhang., *Annalen der Physik*, **39**, 926, 1912; also an unpublished paper on the same line.

[7] *Comptes rendus*, **156**, 1607, 1913, and **157**, 639, 1913.

[8] *Proc. Royal Acad. Amsterdam*, May 30, 1913; September 3, 1913.

Moreover, mainly upon the work of Paschen and Back, as a basis, Voigt[1] had formulated a mathematical theory of the Zeeman effect for close pairs homologous to Na λ 3303, the final result predicted being a quintuplet with simple parallel component and two perpendicular components, each a close pair. In this work he dealt particularly with unpublished experiments of Back on Na λ 3303.

As a logical sequence, then, of these results and of their theoretical explanation and because the magnetic change of a doublet such as D_1 and D_2 of sodium is a simple one and further had been treated theoretically by Voigt, it was suggested by Professor Paschen that the writer make a study of Li λ 6708. As the work progressed it became evident that other lithium lines could be obtained and these too were briefly studied.

APPARATUS AND METHOD OF PROCEDURE

1. The spectroscopic system consisted of a Hilger echelon and constant-deviation prism placed in juxtaposition. The echelon constants were: number of plates, 33; thickness, 9.487 mm; step, 1.01 mm; aperture, 39×34.3 mm. The collimator and telescope lenses were Zeiss achromats of 5-cm aperture and 70-cm focal length. Between the telescope objective and the photographic plate was inserted a simple, double-concave lens, so situated that the resultant magnification was approximately fourfold.[2] This arrangement was used in all the photographic and most of the visual work. This optical system gave about 20 times as intense a spectrum as the second order of a 21-foot concave grating, and a dispersion and resolving power on the average, in the visible region, equivalent to the fourth and fifth orders respectively. During exposures a delicate thermometer, inserted in the case covering the echelon, was held constant in temperature to at least 0.3°, and usually to less than 0.1° C.

The single-order position was generally used—obtained by altering either the temperature of the room or the angle of the

[1] *Annalen der Physik*, 41, 403, 1913; 42, 210, 1913.

[2] Twofold magnification was also used. See Table II.

instrument. The difference in scale between the two positions was less than the error of measurement of the plates.

2. As sources of light there were used:

a) Vacuum tubes of various forms into which were introduced fused lithium chloride,[1] ground fine by mortar and pestle. In order of trial these tubes are represented in Fig. 1. The walls of (*a*) soon blacken and become too opaque for use "side on," while "end on" in the magnetic field a poor image is given. Form (*b*), in which the cathode is an aluminium cup filled with the salt, serves better with no field, but with field the image is lost entirely; (*c*) works well, but the salt is soon exhausted and the process of filling laborious; (*d*) serves excellently.[2] The first tube of this kind, of ordinary glass was short lived: quartz proved very durable. Nearly all the published results were obtained with Heraeus quartz tubes such as this.

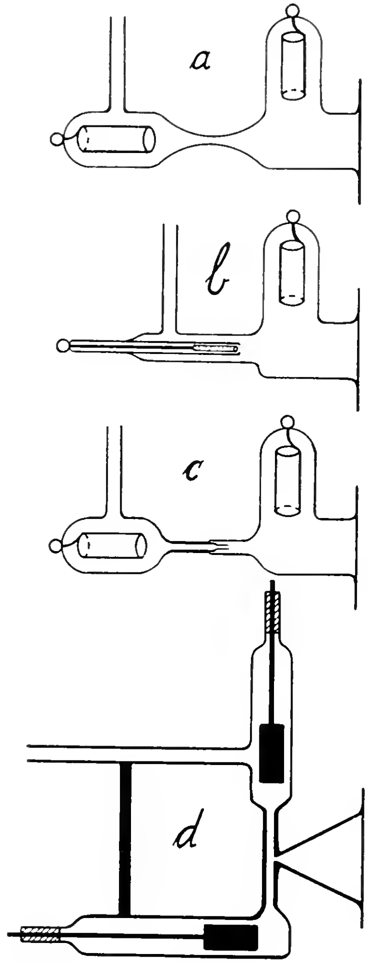

FIG. 1

The lithium salt, introduced through the window with the capillary held horizontal, was fused to the walls of the tube with a small flame. Such a tube lasts many hours, and, when the stream of ions has cut the quartz, the capillary may be cleaned and re-fused with an oxygen flame.

[1] Lithium nitrate and lithium oxide were also tried but found unsuitable.

[2] A similar form was used by Runge and Paschen: see *Astrophysical Journal*, 15, 238, 1902.

The tubes were excited by a Klingelfuss induction coil, the primary current varying from 1 to 6 amperes, generally being about 1.5, and the frequency from 45 to 60 per second. With high fields larger currents and higher gas pressures were necessary. To the evacuating system were added two important accessories—a large bulb (0.5 liter capacity) to prevent rapid decrease of pressure by self-evacuation, and an apparatus for generating and drying hydrogen. The strength and character of the lithium lines are very sensitive to changes in pressure and kind of gas.[1] The tube was washed repeatedly with dry hydrogen before and during the discharge. The most favorable conditions were then found by pumping out the hydrogen, observing the tube meanwhile with a pocket spectroscope. During long exposures this process was several times repeated.

b) The spark in air between carbon terminals soaked in a fused alloy of lithium and sodium. Capacity and inductance were inserted in the spark circuit.

c) Trembleur *in vacuo*. A lithium-cadmium alloy of any desired strength was prepared electrolytically by running a known current for a given time through melted cadmium covered with fused lithium chloride, the negative terminal being an iron wire inserted in the cadmium and insulated above where it passed through the lithium chloride; and the positive being a carbon rod, touching only the lithium salt. Pieces of the alloy were then later melted upon brass or manganin terminals. The vacuum used was generally about 5 mm. The peculiar form of trembleur employed was due to Dr. Back[2] with whom the trembleur-echelon part of the present work was performed. His able assistance, both in this regard and in other minor ways, the writer sincerely appreciates. Certain high-field plates were also taken by him alone, with the trembleur and the 21-foot Rowland grating.

3. As electromagnets there were used a small instrument; a large magnet made at the Institute; and, with the trembleur, a large Weiss instrument. The values of the fields used with the

[1] Air and oxygen were also tried, the latter proving especially disadvantageous.

[2] A description of this instrument will shortly be published by Dr. Back in the *Annalen der Physik*.

first two were measured by a galvanometer calibrated by standard coil and exploring spool. Errors of determination are certainly less than 2 per cent. With the Weiss instrument the fields were measured by the known Zeeman separation of certain cadmium lines, these photographs being taken by Dr. Back with the trembleur and grating under conditions as nearly as possible those obtaining in the echelon exposures. Moreover, as Cd λ 6439 appeared on the echelon plates with Li λ 6708, measurements thereon served as a check, although the images were somewhat overexposed. The two sets of measurements agree to about 1 per cent.

4. The plates used were Wratten and Wainwright, "Panchromatic B, backed" for $\lambda\lambda$ 8127, 6708[1] and 6104; Hauff "Orthochromatic" for λ 4972; and Hauff "Extra-rapid" for λ 4602. For development, glycine in rather weak solution was used.

5. The plates were measured on a Zeiss comparator, four settings on each line, red-right and four, red-left. The error of measurement, varying according to the character of the plate and of the line, was on the average, about 0.003 A on vacuum-tube plates, and less than 0.006 A on the trembleur.

In determining the separation of the doublet components, the values of the constant k for the echelon were calculated from the measurements of Gmelin.[2] The provisional value of the wave-length difference is: $\Delta\lambda_p = k\frac{dn}{dn'}$, where dn is the number of divisions of the comparator between the two lines of the pair and dn' that between the same component in adjacent orders. The prism scale was determined from lines of known wave-length, generally those of a helium tube, and the ratio of the prism dispersion to that of the echelon (varying from 1 to 3.4 per cent) was then applied as a positive[3] correction to $\Delta\lambda_p$.

6. To separate the parallel and perpendicular components, a double-image calc-spar crystal (generally oriented so that the two images lay one above the other) was set before the slit.

[1] These plates, stained with dicyanine, were tried with λ 6708, but proved inferior. The stain was indispensable however with λ 8127.

[2] Inaugural Dissertation, Tübingen, 1909.

[3] The direction of longer wave-lengths in the echelon spectrum (determined by the known position of the satellite of He λ 5875.87) lay toward that of shorter as given by the prism.

RESULTS

The results obtained comprise a study of (*a*) five lines of the lithium spectrum with no magnetic field, and (*b*) their magnetic separation.

(*a*) Of these five lines Zeeman[1] had already developed the doublet λ 6708 as an absorption line and determined the separation of its components as 0.144 A. The other four had not previously been shown double. The writer has obtained all as narrow emission lines and found them double. Tables I, II, and III present the results. The sources were vacuum tubes of form (*c*) (Fig. 1) for plate 13 and form (*d*) for the others. Professor Paschen very kindly measured plates 80 and 75. His values are given under P (Table I), the writer's under K. The average deviation from the mean of measurements given in these two tables is about 0.001 A, or 1 per cent. The second series was taken to test the validity of the values obtained in the first. Table III presents the weighted means of both series and, in addition, measurements made upon certain λ 8127 plates taken by Professor Paschen and Mr. Ignatieff at zero and $3\frac{1}{4}$-fold magnification.

It seems, then, that the frequency variations must be real. There is substantial agreement between the values obtained for the second subordinate series (.340, .336, .339), but those for the first subordinate (.309, .328) differ both from each other and from those of the first series. Zeeman has already stated that according to known series laws the frequency difference for the red lithium line is too small compared with that of the other alkalies. The same was noted before by Popow[2] for certain beryllium lines.

For reproductions of these pairs see Plate XV, A. The enlargements (negatives) therein shown are, unless otherwise stated, 4.5 the size of the photographed image, or 18 times that given without the divergent lens. The scale is 1 A = 1 cm. Longer wave-lengths lie to the left. In section A in order from left to right are: λ 6708. The very strong single line is He λ 6678. Note the satellite (faint) between it and the doublet; λλ 6104, 4972, and 4602—

[1] *Proc. Royal Acad. Amsterdam,* September 3, 1913.

[2] *Verhandlungen der Schweizerischen Naturforschenden Gesellschaft,* September 9, 1913.

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PLATE XV

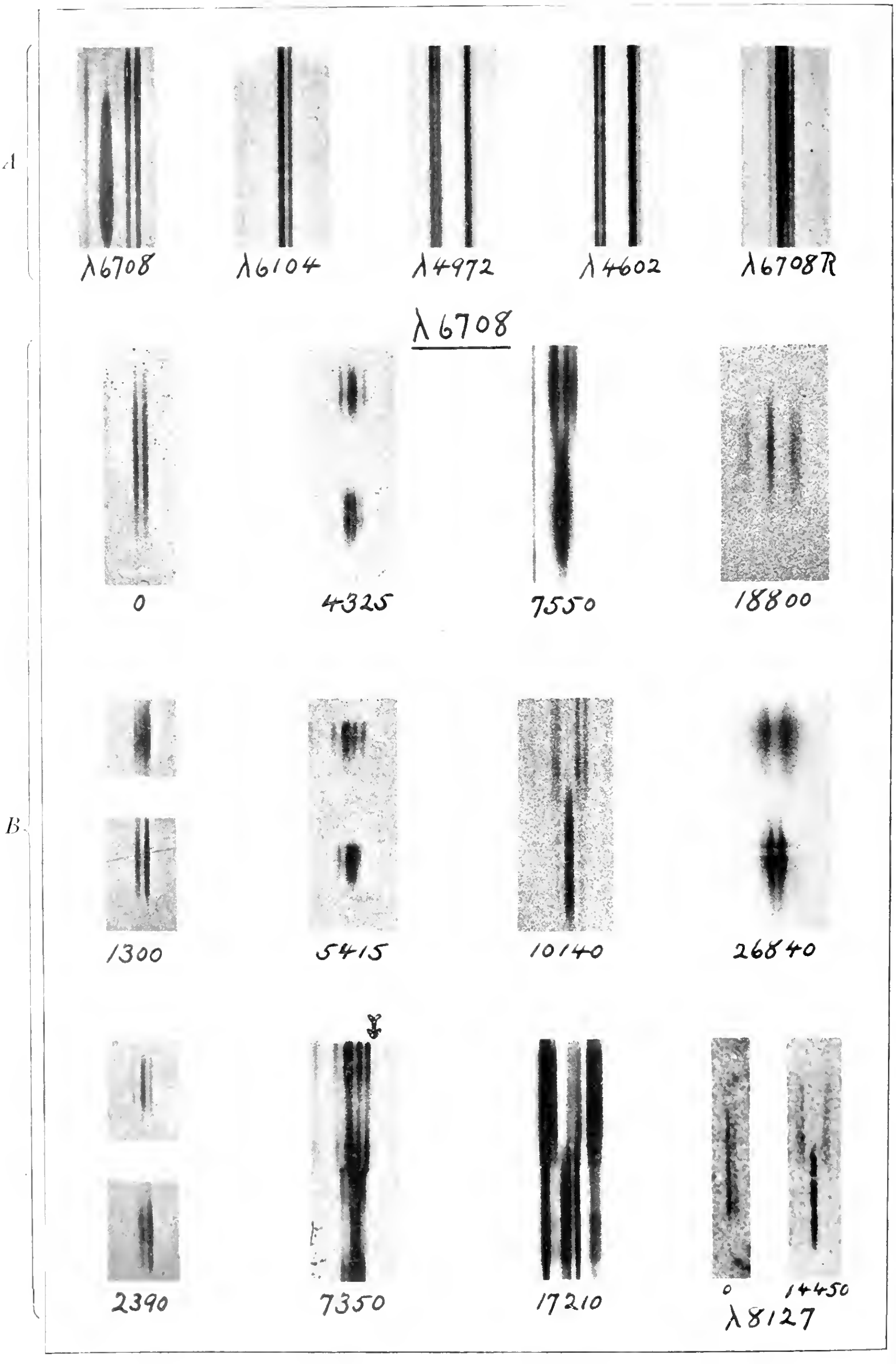
A
λ6708
λ6104
λ4972
λ4602
λ6708R
λ6708
0
4325
7550
18800
B
1300
5415
10140
26840
2390
7350
17210
0
14450
λ8127

TABLE I

FIRST SERIES, FOURFOLD MAGNIFICATION

Number of Plate	Time of Exposure in Minutes*	Character of Plate	Wave-Length of Line λ, in A	Series Classification†	Provisional Value of Wave-Length Difference $\Delta\lambda p$ in A		Correction for Prism Dispersion in A		Final Value of Wave-Length Difference $\Delta\lambda$, in A			Frequency Difference $\frac{\Delta\lambda}{\lambda^2}=\Delta\nu$, in cm.$^{-1}$		
					K		K		K		Mean	K		
13*b*	8	Excellent	6708.2	1.5s−2p_i	.1485		.0019		.1504‡		.151	.336		
					.1478		.0019		.1497§					
13*a* and *b*	5 and 8	Excellent			.1482		.0019		.1501¶					
84	11	Excellent			.1508		.0021		.1529					
					P	K	P	K	P	K	Mean	P	K	Mean
80	12	Excellent	6103.77	$2p-3d$	.1105	.1124	.0033	.0015	.1138	.1139	.114	.3054	.3057	.306
75	45	Very weak	4972.11	$2p_i-3.5s$	.0809	.0839	.0016	.0017	.0825	.0855	.084	.3337	.3458	.339
					K		K		K			K		
77	5	Excellent	4602.37	$2p_i-4d$	.0674		.0017		.069			.326		

* The exposure times here given are typical of those of the entire investigation, the limits being about as here shown.

† See F. Paschen, *Jahrbuch der Radioaktivität und Elektronik*, 1911, Bd. 8, Heft 1, p. 174.

‡ First set of measurements.

§ Second set.

¶ Third set; values obtained from 13 *a* and *b* combined.

TABLE II

SECOND SERIES, TWOFOLD MAGNIFICATION

Number of Plate	Time of Exposure in Minutes	Character of Plate	λ	$\Delta\lambda_p$ in A	Correction for Prism Dispersion in A	$\Delta\lambda$ in A	Mean $\Delta\lambda$ in A	$\Delta\nu$ in cm^{-1}
87	5	Excellent	6103.77	.1129	.0023	.1152	.115	.310
89a	3	Excellent		.1162	24	.1186		
b	3	"		.1134	23	.1157		
91a	3	Fair		.1131	18	.1149		
b	3	"		.1135	18	.1153		
93a	10	Fair		.1112	16	.1128		
100	55	Excellent	4972.11	.0812	23	.0835	.084	.339
95a	4	Excellent	4602.37	.0671	21	.0691	.070	.329
b	4	"		.0661	22	.0683		
97a	4	Excellent		.0690	23	.0713		
b	5	"		.0678	.0023	.0701		

TABLE III

λ	Means of Series				Weights Assigned to Values in Series		Weighted Means	
	I		II					
	$\Delta\lambda$	$\Delta\nu$	$\Delta\lambda$	$\Delta\nu$	I	II	$\Delta\lambda$ in A	$\Delta\nu$ in cm^{-1}
8127.1*....	—	—	—	—	—	—	.225	.340
6708.2.....	.151	.336	—	—	—	—	.151	.336
6103.77....	.114	.306	.115	.310	1	6	.115	.309
4972.11....	.084	.339	.084	.339	1	1	.084	.339
4602.37....	.069	.326	.070	.329	1	4	.070	.328

* Series Classification, $2p_i$-$2.5s$.

in double order condition; λ 6708, Li_1 and Li_2 *reversed* and the satellite (unreversed) 0.146 A from Li_1 or 0.222 A from the center of symmetry. λ 8127 is shown in the next to the last cut of Plate XV, section B, and this cut and the one following are tenfold enlargements of negatives made without any divergent lens. The stronger line in λ 6708 lies toward the violet; in the other four to the red. In confirming this statement the reader must bear in mind that the relative intensities as photographed depend upon the position of the echelon, whether accurately or only approximately in single or double order. No real change in the relative intensity of the lines of λ 6708 was noted. Zeeman found that with increased vapor density the violet component was less intense than the red. This may have been due to the coming up of the weak satellite beyond the red component. However, Zeeman does not offer this explanation, although he does note that "with still greater density new absorption lines appear in the vicinity of the lithium pair."

(*b*) The magnetic resolution was studied both photographically and visually for λ 6708 in detail, briefly for λλ 8127 (by photograph only) and 6104; and briefly, visually, for λλ 4972 and 4602. In the visual work Professor Paschen, Dr. Back, and the writer all shared, the greater part of the observing, however, being done by Professor Paschen who, moreover, together with Mr. Ignatieff, at the cost of much labor, very kindly made the previously mentioned exposures on λ 8127 and certain others also on λ 6708 to obtain plates unquestionably free from reversal. This was accomplished with λ 6708 at low fields and incidentally the sixth perpendicular component was found present although very weak (see p. 350). With a field of 12,000, reversal became so hard to avoid that resort was had to λ 8127.

The advantage of visual observation with λ 6708 lay in the fact that the presence of reversal could thus easily be detected. Both the photographic and visual results here given were, unless otherwise stated, obtained with quartz vacuum tubes of form (*d*), Fig. 1. Statements based on the visual work are indicated by (v).

λ 8127: See p. 353.

λ 6708: This doublet exhibits a magnetic separation shown in Plate XV, B, and represented schematically in Fig. 2.

In the plate the perpendicular structure lies above, the parallel below. The values of the fields in gausses are given beneath each negative. The exposure at 18,800 gausses was made without the calc-spar crystal. Concerning Figs. 2, 3, and 4 certain general remarks may here be made.

Fig. 2: This schematic representation is composed of sixteen sections, the first representing the theoretical structure and approximate relative intensity of D_1 and D_2 of sodium in an imaginary field and the remaining fifteen that shown by Li λ 6708 from 1300 to 44,200 gausses. In these last sections the positions, in zero field, of the components of the lithium line (Li_1 and Li_2, section I), and their geometrical mean are shown by short lines at the top of each section. The series of lines following show successively (1) the components of the theoretical structure perpendicular to the lines of force, according to Preston's rule, (2) the observed perpendicular components, (3) the Preston theoretical parallel, and (4) the observed parallel components (series 3 and 4, broken lines). In all cases the theoretical components are designated by letters (as in sections I and II), the observed by numbers (section II). At XI, however, to as much of the theoretical structure of D_1 and D_2 as still falls within the limits of the plot is added that of a normal triplet, components I, II, III. This is plotted to a point, *p*, 0.020 A to the violet of the geometrical center of Li_1 and Li_2, the mean position of the line 8, 9 as measured on eleven plates.

The approximate relative intensities are represented by the lengths of the lines, the scale of the theoretical being smaller however than that of the observed.

The positions of the observed lines of sections II, III, VI, VIII, IX, and XIV are located arbitrarily as a whole (each line, of course, retaining its measured distance from its neighbors) without regard to a reference line, while those of IV, V, and VII are set accurately by λ 6563 of hydrogen as a standard, those of X to XIII by λ 6678 of helium and those of XV and XVI by λ 6439 of cadmium.

Sections II to XIV show vacuum-tube plates; XV and XVI trembleur plates.

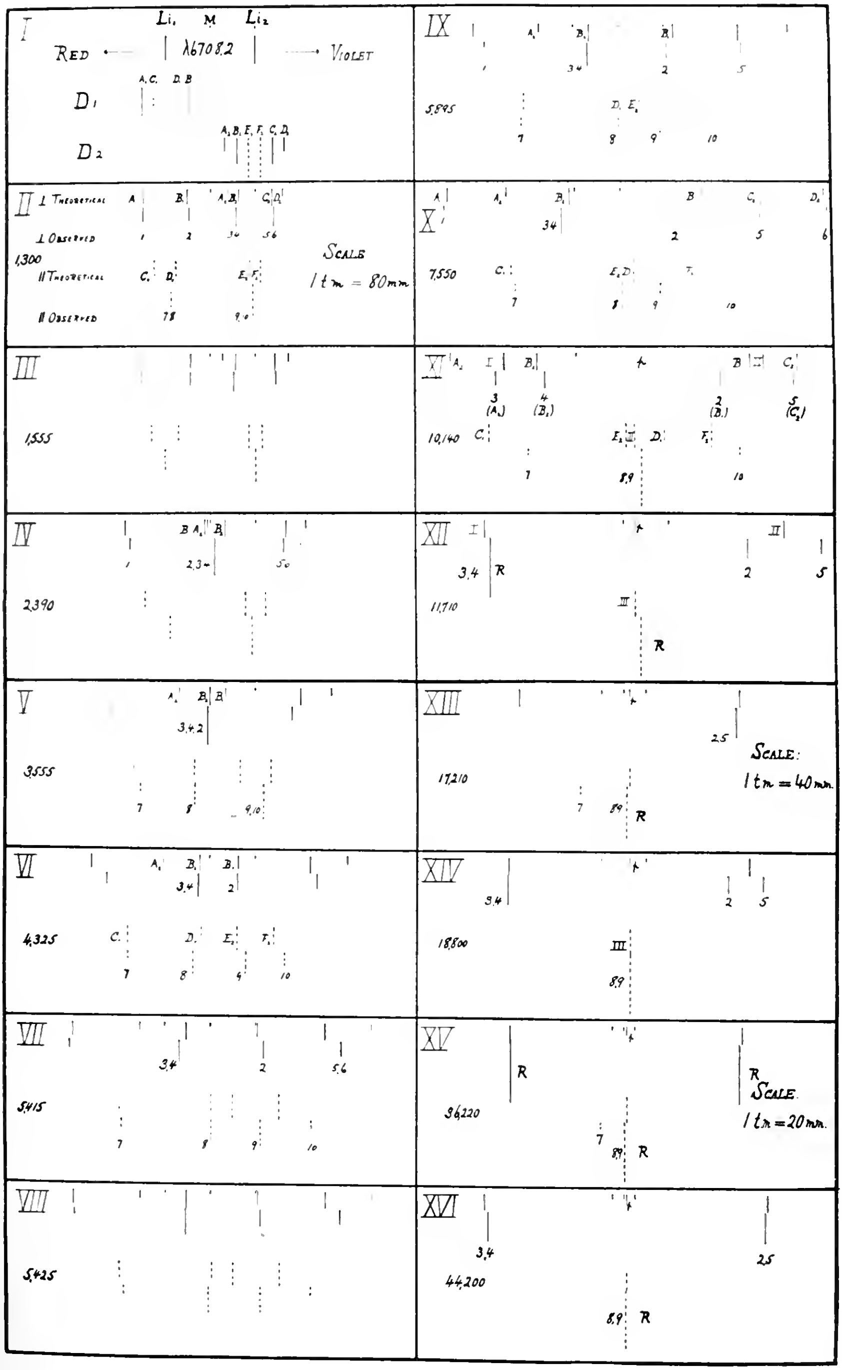

FIG. 2

The wave-length scales are: sections II to XII, 1 A=80 mm; XIII and XIV, 40 mm; and XV and XVI, 20 mm. The greatest error of measurement is not more than 1, 0.5 and 0.25 mm for the three scales, and the average error is but a fourth as great, or 0.003 A.

If a plate does not show a certain line we must not conclude that it does not exist; it may be either weak or unresolved.

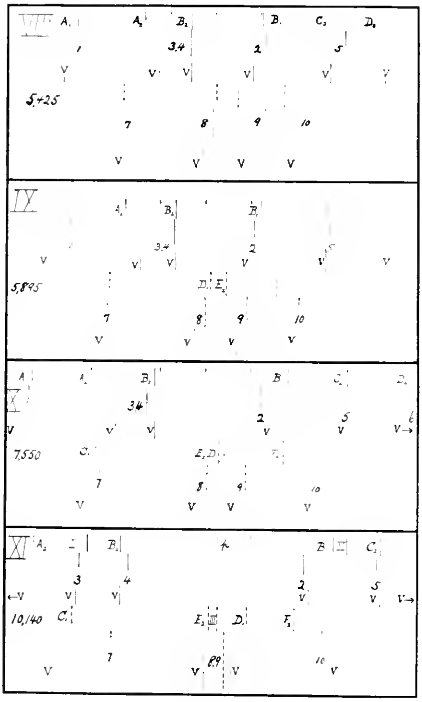

FIG. 3

In X, lines 1 (too weak to measure) and 6 (not visible) are approximately placed by data from another plate at a slightly different field. The same statement holds for lines 7 and 10 in XI.

In XII to XVI, *R* indicates a measurement made to the center of reversal. The lines are here drawn full length.

In XIII the helium line interferes with 3,4 which is therefore omitted; 2,5 is a broad line.

Fig. 3, composed of sections taken from Fig. 2, shows in addition the positions and intensities of the components, marked V, of λ 6708 according to Voigt's theory (see p. 352).

In Fig. 4, for λ 6104 the same general method as that of Fig. 2 is employed.

These three sketches were made with care on a scale five times as great as the reproductions and the errors of plotting are less than those of measurement—in fact the sketches may be regarded quantitively as substitutes for tables of wave-lengths.

For λ 6708, then, experiment shows that the components *parallel* to the lines of magnetic force are at first one line each; later

Li_1 appears double at 3555 (2700 v) gausses (Fig. 2, V), and Li_2 follows at 4325 (Fig. 2, VI). Of these four lines, the two inner increase greatly in intensity but remain clearly separated to 5500 (v) and are just distinct at 8300 (v). They fuse before 10,140 (9300 v) and appear as a strong, broad line which narrows with increase of field. Reversal occurs easily with large current in the tube and is almost unavoidable with the trembleur. The two outer lines weaken continually from even as low a field as 4325 gausses and become progressively harder to measure. This loss of intensity is by no means wholly due to the echelon. However, these "residuals" both appear on a strong tube plate at 18,070 and a trembleur plate at 19,500, while the one of longer wave-length occurs on a trembleur plate at 36,220.

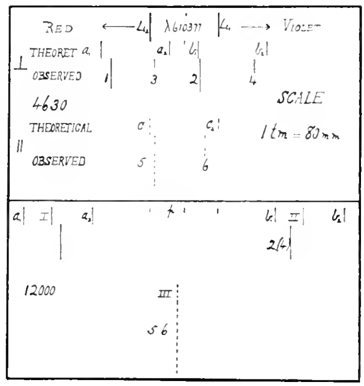

FIG. 4

The plates show certain interesting wave-length changes the first steps of which are in part similar to those noted by Dr. Back with Na λ 3303. These changes are first evident at 4325 gausses and consist of an approximate agreement with the theoretical (assuming the structure that of D_1 and D_2 of sodium) of the wave-length difference of the lines in each of the two pairs of parallel components but a progressive drawing apart of their centers of symmetry—consult Fig. 2 (fields 4325, 5415 or 5425,[1] 5895, and 7550) and Table IV. Thus, lines 7, 8 and 9, 10 preserve a distance which is approximately the theoretical, C_1-D_1 and E_2-F_2 (9,10 quite accurately so); but the centers of symmetry of the pairs should remain 0.151 A apart. They actually lie at a distance, d, as follows:

TABLE IV

Field in Gausses	d in Angstroms
4325	0.174
5415	0.198
5425	0.198
5895	0.193
7550	0.210

[1] Notice the agreement in the measurements of two plates taken under like conditions.

During this recession, the inner lines gradually draw together and finally fuse at 9300 as previously stated. By consulting Fig. 2 (sections VII to XI, XIII, and XV), it will be seen that the residual 7 maintains a constant wave-length difference (by calculation 0.183 A) from the center of 8 and 9 after a field of 5415 (note changes of scale in XIII and XV), while the residual 10 (last plotted in XI) continually recedes from 8,9, this law holding true until later it fades out entirely. Plate XV, B, 10,140, shows these residuals, 7 and 10.

Although all these wave-length changes are undoubtedly real, they are small; and the most striking change is that in the relative intensities of the four lines. Finally it is to be noted that the two inner never cross each other, as they theoretically should just previous to 7550.

The behavior of the *perpendicular* components is, however, quite different. Those of Li_1 and Li_2 not only preserve a better agreement with their theoretical position up to about 6000 gausses but cross each other (VI). Four lines are present at 4325, and five at 5415 (or 5425), one of these, line 3, not being distinct from its stronger neighbor; five are quite resolved at 6300 (v) and at 7350 six components are present; but lines 3 and 4 are not resolved and 6 is extremely weak. The residual is visible just below the arrow in 7350, Plate XV, B. This component is difficult to obtain because (*a*) it is not separated from 5 until a field of such magnitude is reached that the line lies in the outer and weaker region of the echelon diffraction pattern; (*b*) it is intrinsically weaker than its companion line, 5; and (*c*) it is decreased in intensity by the action of the field. It is, however, in its theoretical position while 3 and 4, which are first resolved at 10,140, are at a distance much less than the theoretical (A_2-B_2). This is precisely the effect observed by Dr. Back with the homologous component in sodium λ 3303.

X, 7550, shows a tendency to be noticed even in IX, namely, 2 has drawn in and away from B_1.

Thus lithium λ 6708 in low fields shows the types of the sodium D-lines in both perpendicular and parallel components.

To return to the previous discussion: The photograph at 10,140 shows four perpendicular components (a stronger plate at 12,000

shows five, line 6 having disappeared and line 1 being extremely weak). The theoretical normal triplet, components I, II, III, with p as a center is here first plotted. Observe that the distance I–II is greater than that between the centers of symmetry of 3,4 and 2,5—in other words the system has not yet reached the separation of a normal triplet.

At 18,800 (XIV), a high-field, unreversed, vacuum-tube plate, obtained with much difficulty, shows 3,4 as an almost structureless, broad line; while 2 and 5 are not yet one, but are closer than at 10,140 (0.575 as against 0.600 A). In placing these lines coincidence was assumed between 8,9 and III at p.

At 22,600 (V), 2 and 5 also have joined. The two resultant lines sharpen progressively to 44,200 and, with the trembleur, under suitable conditions, have been seen to flash out narrow and unreversed—not so sharp, however, as the single parallel component. Moreover, the three components have been observed at 31,300 gausses, with trembleur and Rowland grating, all reversed and again, narrow and unreversed.

The reproduction of Plate XV, B, 26,840 gausses, shows a reversed trembleur photograph. In the parallel structure the red residual may be noted; in the perpendicular, the reversal of the two components and the outward flaring of the ends of the lines—this because the field is uniform over only the central region. From the form of the curvature we infer that the two components are approaching each other from adjacent orders, as the lesser field shows the greater separation.

At 36,220 (XV) the triplet is normal. The two perpendicular components lie superposed, the separation having advanced to such an extent that these lines have not only receded from the field of view, but have returned from the adjacent orders. A simple calculation gives the true separation with fair accuracy.

At 44,200, 3,4, and 2,5 are broad and structureless. In calculating the separation account is taken of the fact that the perpendicular components have not only returned from adjacent orders but are again separating.

Thus, in conclusion, we may state that *in low fields the doublet* λ *6708 exhibits the types of* D_1 *and* D_2 *of sodium* and that *an increase*

of field produces a simple normal triplet. The parallel component, formed from lines 8 and 9, is well-nigh complete just beyond 8000 gausses; the red perpendicular, formed from 3 and 4, near 10,000; the violet, formed from 2 and 5, beyond 19,000; and the separation is approximately normal at this point. During this process the noteworthy facts are: (1) the enhancement of the inner components and the decrease in intensity of the outer; (2) the repulsion of the centers of symmetry of the two parallel pairs and the non-crossing over of their two inner lines; (3) the crossing over of the perpendicular component 2 of Li_1 and, later, (4), its drawing in toward the center of the perpendicular system; (5) the distorted position of component 3, it lying much nearer 4 than the theoretical; (6) the constant position of 7; and (7) the increasing distance of 10 from the center of the parallel system. The center of this triplet is shown by measurement to lie 0.020 A from the geometrical center of Li_1 and Li_2 toward Li_2—the stronger line. The foregoing numbered statements confirm Back's experiments and qualitatively all but (6) are in accord with Voigt's theory. The latter's statement[1] that the displacement of the parallel doublets is "oppositely directed" and "approximately equally great" is true to about 4 per cent, if point p be regarded as the center of the repulsion. Fig. 3 shows the good agreement between this mathematical theory and experiment. The positions and intensities of the Voigt lines (marked V) were calculated from a formula kindly given the writer by Professor Voigt and both are plotted on the same scale as the "observed" lines of Fig. 2. In VIII to X, if the parallel components were displaced about 0.020 A, the distance to p from the geometrical center of Li_1 and Li_2, the coincidence would be well-nigh exact. For all four fields shown the Voigt component for line 2 shows a shift almost the same as the experimental. Lines 3 and 4 when first resolved are at approximately the Voigt distance. On the other hand in all cases the outermost Voigt components show too great a separation, line 7 especially. That the intensities agree only approximately is largely due to the echelon.

[1] See β, *Annalen der Physik*, **42**, 213, 1913.

Thus for low fields Voigt's statements are in the main confirmed, but not in the final result predicted for high fields.[1] The perpendicular components have not indeed been photographed as single sharp lines because in high fields it is necessary to use large currents in the vacuum tube or employ the trembleur, in both of which cases the tendency to reverse is so strong that the parallel component, visually observed sometimes as simple, generally appears reversed and the perpendicular components often so. Thus the photograph, a composite of various states, always shows a reversed line for the parallel component and generally a broad but single line for each of the perpendicular (the coefficient of absorption of which is half as great).

On many strong, vacuum-tube plates, notably between 10,000 and 19,000 (and even one plate at 31,000), also a trembleur plate at 19,500, there is shown incomplete polarization of both parallel and perpendicular components. Plate XV, B, 10,140, shows incomplete polarization in the center of the perpendicular structure. See also, in Plate XV, 17,210 a very strong, reversed, vacuum-tube photograph which exhibits this phenomenon.[2] It is possible that this result was in part due to an incorrect orientation of the double-image crystal; but the phenomenon is in the main undoubtedly real, for, when the original component is reversed, its counterpart in the region of opposite polarization often appears unreversed.

A few exposures made with the spark in air for fields from 10,000 to 16,000 gave results in accordance with, but far inferior to, those of the vacuum tube; while the trembleur plates proved eventually of less than expected value owing to reversals.

λ 8127: At 13,300 and 14,450 gausses (see Plate XV, B, last photograph) the structure is that of λ 6708 at 10,140 but turned about, the violet perpendicular component being here completely formed while the red is still two lines—a result to be expected owing

[1] It is possible that the addition of the quadratic terms alluded to on p. 211 of Voigt's paper will both eliminate the slight disagreements with the theory at low fields and also produce the correct final result.

[2] In Fig. 2, XIII, the incomplete polarization is not shown.

to the inversion of intensities of the components of the two pairs. This makes absolutely certain the like results obtained with λ 6708, for λ 8127 does not reverse in the vacuum-tube discharge.

λ 6104: At 4630 gausses the two components of this doublet give what appear to be two triplets. The abnormalities in both positions and intensity are rendered clear by Fig. 4. Lines 1 and 4, also 5 and 6 are nearer each other than the theoretical (assuming two normal triplets). At 9000 (v) 5 and 6 have fused into a fairly sharp line, 3 has disappeared and 4 exists as a faint residual, the image at this stage resembling the helium doublet λ 5876 at 31,710 gausses.[1] At 12,000 both perpendicular components are simple lines, 4 having either faded out or drawn into 2, and the parallel component lies at p', 0.011 A from the center of symmetry. The separation 1–2 is less than 1–II, that of the normal triplet. At higher fields all the lines sharpen and at 30,000 appear of equal breadth. Grating plates taken by Dr. Back at 40,000 show the separation normal.

λλ 4972 and 4602 were studied very briefly and the only statement which can be made is that high fields produce triplets.[2] These Dr. Back has lately, with grating and trembleur, shown normal at about 34,000 gausses.

In the latter part of the work an interesting fact was observed. With about 6 amperes in the primary of the coil, the tube (form *d*) being quite hot after a continuous run of over forty minutes, at rather high pressure, owing to the introduction of fresh hydrogen, λ 6708 vanished almost entirely from the spectrum and λλ 6104, 4972, and 4602 attained great brilliancy. To the naked eye the tube presented the unusual color of a bright and strong yellow.

SUMMARY

1. Lithium λλ 8127, 6708, 6104, 4972, and 4602 have been photographed as narrow emission lines, shown to be close pairs and their difference of wave-length measured.

[1] Paschen and Back, *Annalen der Physik*, **39**, 916, 1912.

[2] Hansen (*Annalen der Physik*, **43**, 25, 1914) showed λ 4972 normal at 29,800 to within $3\frac{1}{2}$ per cent.

2. The frequency differences are unequal and this small variation is probably real.

3. The intensity relation of the components is the same as that of homologous lines of the doublet series of other alkali elements.

4. The magnetic resolution in low fields shows the doublet λ 6708 to be exactly the same as D_1 and D_2 of sodium, while λ 6104 gives what appears to be two triplets.

These five lines are thus true and regular series doublets and further,

5. Four of them have been shown to give in high fields simple normal triplets as was formerly shown by Dr. Back with a Rowland grating and a spark in air.

6. For λ 6708 it has been shown just which of the ten, low-field components disappear and which combine to form the three lines of the final, high-field, normal triplet (see p. 352), the change having been traced from beginning to end.

To both Dr. Back and Dr. Gerlach of the Institute hearty thanks are due for willing and able assistance given at various times; also to Mr. Ignatieff for the valuable facts added to the work from the plates taken by Professor Paschen with his assistance.

Finally, the writer must express his deep feeling of indebtedness to Professor Paschen himself, not only for the suggestion of this problem but also for his enthusiastic and invaluable co-operation, and for the opportunity of enjoying all the facilities of his wonderful laboratory.

TÜBINGEN
June 30, 1914

ON THE DISTRIBUTION OF THE ELEMENTS IN THE SOLAR ATMOSPHERE AS GIVEN BY FLASH SPECTRA[1]

By CHARLES E. ST. JOHN

The remarkable eclipse spectrum obtained by Professor Mitchell in 1905, the results of the reduction of which appeared in the *Astrophysical Journal* of December 1913, furnishes a body of observational data that lend themselves to the statistical method of discussion. In sharpness of definition this spectrum without doubt surpasses any other published, and probably no plate has been obtained more nearly at the proper epoch of the eclipse.

The general scheme of this discussion is to form numerous groups of lines on a simple plan. Each element has been considered by itself, and the lines of like solar intensity assigned to it have formed the ultimate group. This is necessary when the purpose is to bring into relief characteristics that are shown only by small differences.

The publication of a discussion of the data by Professor Mitchell in the *Astrophysical Journal* of March 1914 gives an opportunity for a consideration of the data from the point of view brought out in my papers on "Radial Motion in Sun-Spots."[2] The material used in that discussion was furnished by 506 lines, each line being measured, on an average, upon 14 plates of high dispersion, so that some 7000 differential measures were involved. The number of lines in Mitchell's Table I[3] is 2841. The present discussion is confined to the plate ending at λ 5879, and for the most part to the lines not appearing as blends in Mitchell's table, which reduces the number by approximately one-half; but what is lost in quantity is more than balanced by the precision obtained when the purity of the lines is assured. This is particularly true when characteristics of

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 88.

[2] *Mt. Wilson Contr.*, Nos. 69 and 74; *Astrophysical Journal*, **37**, 377, and **38**, 157, 1913.

[3] *Ibid.*, **38**, 424, 1913.

individual elements and lines are under investigation, as a treatment of the data *en masse* obscures the finer distinctions.

LEVEL IN GENERAL

An indication of the low level reached by Mitchell can be obtained by comparing the relative numbers of weak Fraunhofer lines obtained by different observers as shown in Table I.

TABLE I

FLASH SPECTRA LINES CORRESPONDING TO SOLAR LINES OF INTENSITY<1

	0000	000	00	0	Total	Region	Percentage
*Frost 1900	......	......	......	5	5	350 A	......
†Mitchell 1901	......	......	......	5	5	1100	......
‡ Evershed 1900	......	......	5	20	25	1500	......
§Jewell 1900	......	1	12	26	39	2500	......
Mitchell 1905	20	87	220	278	605	2500	21
¶Hale and Adams without eclipse	34	46	17	8	105	180	80

* *Astrophysical Journal*, **12**, 307, 1900.
† *Ibid.*, **15**, 97, 1902.
‡ *Phil. Trans. Roy. Soc.*, **201** A, 457, 1903.
§ *Publications U.S. Naval Obs.*, **4**, Part 4, Appendix 1, 121, 1906.
¶ *Mt. Wilson Contr.*, No. 41; *Astrophysical Journal*, **30**, 222, 1909.

It is seen that 21 per cent of the lines observed by Mitchell are weaker than 1 on the Rowland scale of intensity and that the proportion reported by other eclipse observers is vanishingly small, while 80 per cent of those measured on the Hale and Adams plates are weaker than unity. It is especially noticeable that the ratio of the number observed by them to the number observed by Mitchell increases with decreased Fraunhofer intensity. In obtaining flash spectra without an eclipse, the slit of the spectrograph is held rigorously tangent to the edge of the photosphere by guiding upon a chromospheric line. This can be done with great precision and constancy by the method employed, and assures the possibility and the probability of the appearance upon the Hale-Adams plates of lines originating in the lowest levels of the solar atmosphere. From the clever method of Mitchell for fixing the critical instant of exposure, it is clear that the spectrum corresponding to practically the entire depth of the solar atmosphere could impress itself upon the

plate, and the richness of his plate in weak lines and the paucity of such lines upon the previous eclipse plates are probably accounted for by their exposures being ended too soon—that is, before the advancing moon had uncovered the whole depth of the atmosphere; or perhaps too late—that is, after the continuous spectrum of the photosphere had obliterated the weaker lines in the flash spectrum.

The above comparison appears to sustain the assumption, not only that the weaker Fraunhofer lines originate at lower depths, but also that these depths increase regularly with the decreased intensity of the solar lines.

THE BEHAVIOR OF SOME TYPICAL ELEMENTS

Iron.—From the long list of lines, 356 were selected as due only to iron, the ground of selection being that no other identifications were suggested by Mitchell; that is, they were not blends nor were they bracketed with other lines. From these, which include all the pure iron lines, the enhanced lines were excluded and the remaining lines were classified according to the Rowland intensities with the result shown in Table II.

TABLE II

FE LINES AND LEVEL AS SHOWN BY FLASH SPECTRA

	Number of Lines									
	4	19	30	55	72	49	40	28	24	25
Solar intensities..	00	0	1	2	3	4	5	6	7–8–9 (7.8)	10–40 (16.4)
Flash intensities..	0.25	0.26	0.33	0.8	1.4	1.6	2.0	3.5	3.8	7.0
Heights in km...	275	279	288	344	369	397	425	488	590	806

The regular progression shown by the flash intensities and the mean of the heights assigned to iron lines corresponding to like Rowland intensities, as these intensities increase from 00 to 16.4, lead to the conclusion that in a quite definite sense one is justified in saying that the lines of iron of a given intensity in the Fraunhofer spectrum extend to quite definite elevations. The regular march of the elevations with solar intensities is more plainly shown when the intensities are plotted as abscissae and the heights as ordinates. As a convenient scale, in lieu of a better, the numerical intensity

intervals are used as abscissae. Owing to the uncertainty in the Rowland intensity intervals the smoothness of the resulting graph (Fig. 1, *a*) cannot be interpreted, of course, as meaning that intensity and height are directly proportional, but only that the heights and the intensities progress together.

Titanium, scandium, and yttrium.—These related elements are interesting components of the solar atmosphere and may conveniently be considered together. A common characteristic is the high proportion of enhanced lines. As clear a separation as possible has been made between the normal or unenhanced lines and the enhanced lines. As there are several degrees of enhancement, it is probable that some enhanced lines still remain. For titanium, the table given by Reese[1] has been of service, and the information given by Mitchell's tables has rendered it possible to make a rather close separation for all elements. The large number of enhanced lines in the titanium spectrum shows so clearly the behavior of this class of lines that it has furnished criteria for separating the lines of the other elements.

TABLE III

TITANIUM, SCANDIUM, AND YTTRIUM

	Number of Lines								Element
	7	19	18	18	21	14	9	4	
Solar intensity......	000	00	1	0	2	3	4	5	Titanium
Flash intensity......	0.1	0.25	0.4	0.9	1.5	1.9	2.8	2.8	Fig. 1, *c*
Height............	271	290	347	353	389	429	494	487	

	Number of Lines						Element
	5	9	5	10	3	3	
Solar intensity.......	000	00	0	1	2	3	Scandium and yttrium
Flash intensity.......	0	0.25	1.0	1.6	1.3	1.0	Weighted means
Height.............	290	295	360	385	483	433	Fig. 1, *e*

Lanthanum group = La, Ce, Dy, Er, Eu, Gd, Nd, Pr, Sa.—The elements La and Ce, according to the conclusions of Adams from his investigations on the spectrum of the limb and center of the sun and from his study of solar rotation, are at a low level in the solar

[1] *Astrophysical Journal*, **19**, 322, 1904.

atmosphere, and the same conclusion was reached in the writer's discussion of the data obtained from the displacements in the penumbrae of sun-spots.

Carbon.—It is probable that the lines referred to carbon are due to cyanogen[1] or to nitrogen, but for convenience of reference, the older designation will be used. Grotrian and Runge have recently obtained the cyanogen bands in the presence of nitrogen, when both cyanogen and carbon were evidently absent.[2]

The low level of carbon was remarked by Adams in his rotation work, and by the present writer in the second paper on radial motion where reference was made to the weakening of the carbon or cyanogen lines in the arc in the presence of metallic vapors.[3] The high proportion of the weakest carbon lines on the chromosphere plates of Hale and Adams and upon the eclipse plate of Mitchell is evidence of the low-level origin of these lines.

TABLE IV

La Group and Carbon

	Number of Lines							Element
	1	7	27	18	8	10	2	
Solar intensities	0000	000	00	0	1	2	3	La group
Flash intensities	0.0	0.7	0.9	1.1	1.8	0.8	2.0	Fig. 1, *g*
Heights	250	329	354	382	406	395	550	

	Number of Lines						Element
	6	13	10	7	7	4	
Solar intensities	0000	000	00	0	1	2	Carbon
Flash intensities	0.7	0.5	0.2	0.9	0.7	1.8	Fig. 1, *i*
Heights	342	342	335	443	479	738	

The curves showing the relation between height and the Rowland intensities for the elements Ti, Sc-Y, the La group, and CN are given under *c*, *e*, *g*, and *i* in Fig. 1. When one considers the number of observations upon which the points depend, the uncertainties that occur in estimating the lengths of arcs, and the

[1] Kayser, *Handbuch der Spectroscopie*, **5**, 229.

[2] *Physikalische Zeitschrift*, **15**, 545, 1914.

[3] *Mt. Wilson Contr.*, No. 74, pp. 6–7; *Astrophysical Journal*, **38**, 162–163, 1913.

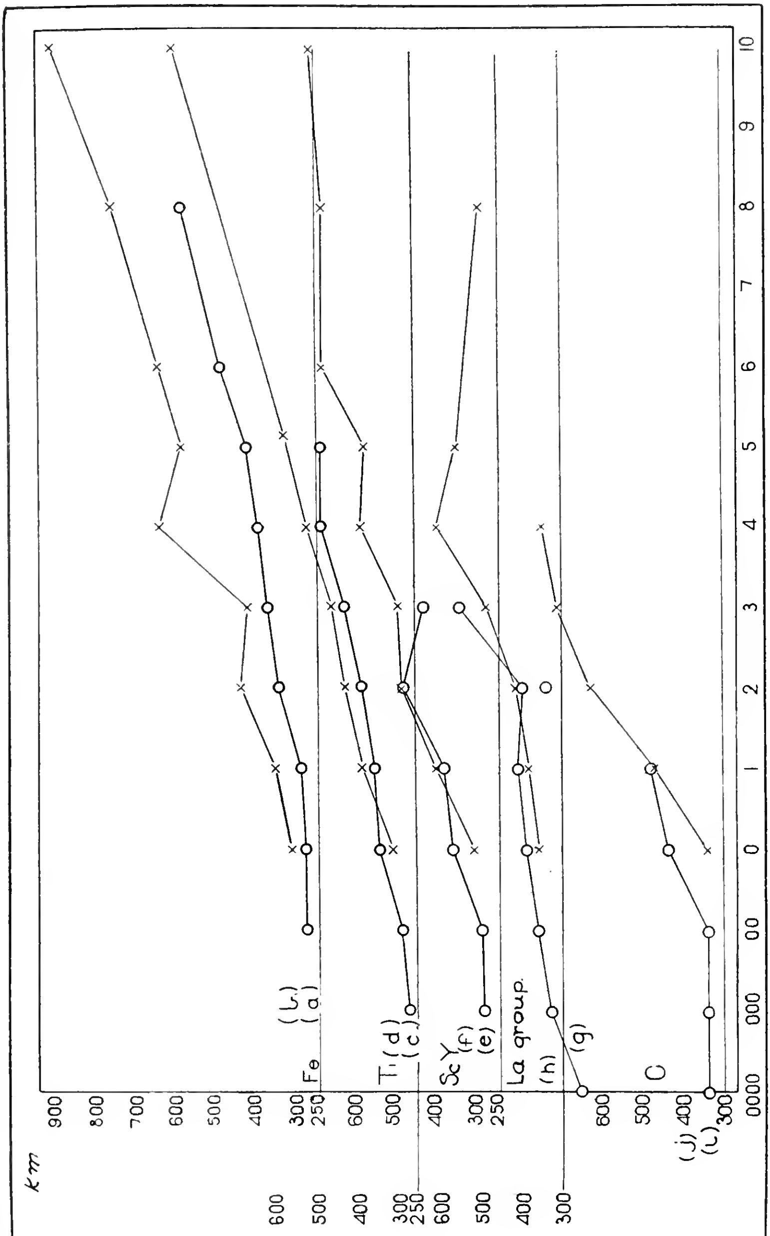

Fig. 1

O relation between solar intensity and height
× relation between flash intensity and height

increased regularity that accompanies increased weight, the connection between Rowland intensity and height is quite evident.

An estimation of the accuracy to be expected in the reduction of eclipse plates may be had from considering the heights and flash intensities. Mitchell says that "equality in height must entail an equal intensity in the flash spectrum."[1] In Fig. 1 are shown graphically the relations between heights and flash intensities, and though more observations are at our disposal, the regularity of the graphs is no greater than when Rowland intensities are used as abscissae. There are discordant observations in both cases, lines of equal flash intensity giving widely different heights, lines of very unequal flash intensities giving equal heights, and lines of the same solar intensity giving different heights—and this last is particularly true for elements with strong lines to the violet of λ 3850, in which region the apparatus was much less sensitive, owing to the low reflecting power of silver and speculum metal and to the decreased sensitiveness of the photographic plate. These discordant observations do not weaken the general statements that lines of equal mean intensity correspond to equal heights, and that lines of a given flash intensity mean equal heights, but rather tend to strengthen them, for if, in spite of these discordances, the means show that heights progress with both Rowland and flash intensities, it seems there must be a fundamental truth underneath the statements.

In saying that the Fraunhofer lines of intensity 4, for example, are at a definite elevation in the solar atmosphere, it is meant that when a sufficiently large mass of data is treated statistically it results that the lines of intensity 4, as a group, are of a higher level than those of an intensity less than 4, and at a lower level than groups of intensities greater than 4. The case is quite similar to that of stellar distances and proper motions. It cannot be said of a definite star with large proper motion that it is a near star, but out of a hundred stars with large proper motions it can be quite safely assumed that a very large majority are of the near-star group. There are undoubtedly differences between the effective levels of lines of equal solar intensity, just as the writer showed that there

[1] *Astrophysical Journal*, **39**, 128, 1914.

are differences in the radial displacements given by lines of like solar intensity, and that these differences are related to the classification of iron lines on other grounds. A thing that is generally true must first be established in its broad outline, and later, when some individual members of a group show themselves as definite exceptions, they offer a new means of obtaining added information and of pushing the analysis yet a step farther. It is yet too early to emphasize the exceptions, though data are accumulating that promise the added information. In the writer's opinion the criterion demanded by Mitchell in discussing this question is too exacting, when he requires that all lines of a given solar intensity must always be represented in the flash spectrum by lines of equal height.[1]

Some idea of the weight to be given to the means of the groups containing lines of like solar intensity may be had from an analysis of the group made up of solar intensity 3. This contains 72 lines and has a mean height of 369 km. Mitchell's estimates are made in 50 km steps so that heights of 350 and 400 km come nearest to this mean. A high degree of precision in any single measurement is an error of one unit of the scale, which in this instance is 50 km, so that heights of 300 and 450 km would still be within the range of error. Ninety per cent of the lines are within these limits. The one line below the lower limit is assigned a height much less than the average for lines of the same flash intensity, and the six above the upper limit are assigned heights greater than the average for lines of their flash intensities.

TABLE V

HEIGHTS AND RADIAL DISPLACEMENTS OF IRON LINES

Solar intensities....	00	0	1	2	3	4	5	6	7–9	10–40
Heights...........	275	279	288	344	369	397	425	488	590	806
Displacements.....	+0.034	0.030	0.028	0.025	0.023	0.021	0.019	0.016	0.010	0.002

A comparison of the eclipse results for the iron lines with the radial displacements for lines of the same solar intensities is of interest, as the increasing displacements with decreased intensity were interpreted as consequences of greater depths.

[1] *Astrophysical Journal*, 39, 128, 1914.

The progressive decrease of displacements with increase of heights is a phenomenon that appears to find its straightforward interpretation in differences of effective level; that is, at the lowest height given by the iron lines, the vapors are flowing out of the spot tangential to the surface with the highest velocity, at greater heights the velocity is less, and the lines of greater solar intensity originate at these higher levels.

THE DISTRIBUTION OF THE ELEMENTS

When one considers the maximum height at which the different elements can be detected by flash spectra, a vertical distribution is shown that confirms the distribution obtained from the consideration of the sign and magnitude of the radial displacements; that is, the highest level is given by the H and K lines of calcium, the next by the hydrogen lines, while the heavy elements appear only in the lower portions of the solar envelope.[1] The vertical section of the solar atmosphere, in Fig. 2, is drawn to scale from data taken from Mitchell's first paper, the mean values being used where there are several lines of the same solar intensity. Some related radial displacements are also given. As in the terrestrial atmosphere, the percentage composition of the heavy elements is greatest in the lower portions, the lighter elements gaining in percentage, but decreasing in absolute density with height.

The distribution of the elements in accordance with their atomic weights, and the march of heights with intensities indicated by eclipse data are in satisfactory agreement with the interpretation of radial displacements as phenomena of level. In the case of the heavy elements, lanthanum and cerium, the displacements in the penumbrae of spots are greater than for iron lines of equal solar intensities, and an interpretation on the basis of level places the origin of a line due to the heavy metal at a lower level than a line of the same solar intensity due to iron.

The flash spectra heights given by lines due to the heavy metals are greater than for iron lines of the same solar intensity, as Mitchell has shown in his second paper. In discussing the matter he says:

> It therefore seems that St. John and the writer both use the differences in heights or differences in level to explain differences in the lines of the solar

[1] *Mt. Wilson Contr.*, No. 74, pp. 3–8; *Astrophysical Journal*, 38, 159–164, 1913.

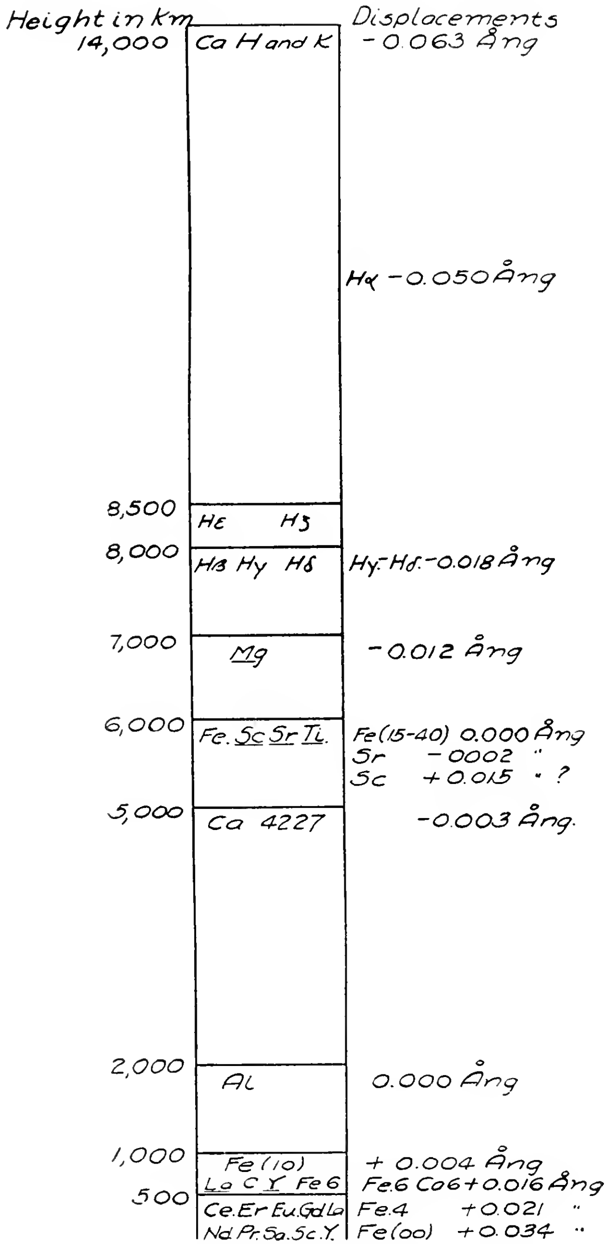

FIG. 2.—Absolute heights from flash lines; relative heights from displacements

spectrum. According to St. John's ideas, however, the rare earths are found in the low-lying regions of the reversing layer. Since they are found comparatively close to the photosphere, they are at a rather high temperature. As a result of this high temperature there is little absorption by the rare-earth vapors, and the Fraunhofer lines are not strong. These high temperatures, however, make more brilliant the lines of the flash spectrum, with the result that the intensities of the rare earths in the flash spectrum are much greater than in the Fraunhofer spectrum. Measures of the 1905 eclipse spectrum confirm the increased intensities in the flash spectrum demanded by St. John's theory, *but these measures do not show that the rare earths are found in shallow layers*, but exactly the reverse.[1]

The contradiction implied in the paragraph is only in appearance. Professor Mitchell informs the writer that he himself is not referring here to the absolute heights to which the rare-earth vapors extend, but to the fact that for lines of the same solar intensity the heights are greater in the case of the rare earths than for iron. My own conclusion was that, in the main, the heavy and rare-earth elements occur in detectable amounts only in the lower portions of the solar atmosphere, a conclusion that does not depend upon whether the lines of the heavy elements have their origin somewhat above or below the lines of iron of the same solar intensity, since none of the lines of the heavy elements reaches an elevation exceeding a few hundred kilometers, and with this conclusion Professor Mitchell writes that he is in entire agreement.

In Mitchell's second paper, the heavy elements (mean atomic weight, 148) are grouped with the light elements Sc (atomic weight, 44) and Y (atomic weight, 89) which are rich in enhanced lines, and it is possible that the heights reached by these elements have been mainly responsible for the high elevation of the group. In his Table III, p. 176, under solar intensity 3 to 5, there are 243 lines of the Fe group, with a mean height of 505 km, and 29 lines of the rare earths, with a mean height of 785 km. It is difficult to identify completely the 29 lines, but there are 22 lines attributed to Sc or Y only with a mean height of 864 km. There remain 7 lines with a mean height of 529 km to be accounted for. There are 4 lines of mean height 625 km, in which Sc or Y is the chief element, and 11 lines of mean height 491 km, in which one of the heavy-earth elements is the only or chief component. It is quite evident that the

[1] *Astrophysical Journal*, **39**, 128, 1914.

high elevation of 783 km is not due to the heavy elements. As to the higher elevation of the group when lines of solar intensity 6 to 10 are considered, it needs only to be said that no line of solar intensity 6 or more is known for the heavy elements, so that they have no influence here. The high elevation and flash intensity shown by the lines of the heavy elements relative to other lines of the same solar intensity appear prominently with decrease in solar intensity. The explanation seems to be found in the increased intensity of the flash lines of the heavy elements relative to the solar lines, which Mitchell says his measures confirm. An increased flash intensity due to the higher temperatures at the lower level carries with it an increase in height deduced from the length of the line. Hence it would seem that the heights given by flash spectra may be quite fictitious in the case of lines of very low level. The question is important in the interpretation of eclipse spectra and deserves further consideration.

FLASH SPECTRUM HEIGHTS FROM WEAK SOLAR LINES

The intensity assigned to a flash line is a complicated function of the length, width, and blackness of the line. If exposures of the same length could be given to the entire depth of the solar atmosphere and of sufficient duration to photograph the relatively cool H and K level, something like the following would be expected: a long, broad arc of weak intensity for the upper calcium, and short, narrow, and very black arcs for the lines of the La group, their blackness being due to the extremely high temperature of their vapors. For these lines the blackness would play an increasing rôle in the estimation of their intensities. A similar influence would make itself felt only in a less degree when determining the intensities on an ordinary eclipse plate; that is, the short arcs produced by the hot, low-lying vapors would be blacker in proportion to their length than the long arcs, and their intensities relative to that of the solar lines would increase with decrease of level. This increase of relative intensity with decrease of solar intensity, or increase of depth, is a marked feature of Mitchell's observations.

The exposure on the Mitchell eclipse plate was progressive, and hence the exposure at high levels was longer than at low levels. The highest level photographed is 14,000 km; the unenhanced

lines of the heavy elements are all below 500 km. Had the exposure begun at the instant the projected limb of the moon was 14,000 km above the photosphere, the exposure for the outer envelope would have been nearly 30 times as long as for the region where the unenhanced lines of the La group originate. The actual case differed from this only in degree. Had the same time of exposure been given for the lowest levels as for the highest, the lines of very weak solar intensity would have been still stronger, and had the exposure for all levels been as short as for the very lowest, it is a question whether the relatively cool high-lying vapors would have photographed at all. In the case of eclipse plates, a strong emission of the low-lying vapors is shown by the presence of the flash reversals of the weakest Fraunhofer lines, though the exposure time for them is practically instantaneous.

To obtain an idea of the general case, the mean flash intensity was found for all lines of solar intensity 6 or less assigned to a single element and the difference F−S taken as shown in Table VI. The regularity of the increase of flash intensity relative to the Rowland intensity is very marked from a flash strength of 3 intensity intervals less to a strength of 3 intervals more than the Rowland intensities in passing from solar intensity 6 to 0000. A low level in the solar atmosphere appears to be favorable to a high flash intensity relative to solar intensities of the corresponding lines, carrying with it increased length of arc, and this high relative strength is characteristic of carbon and the La group of heavy elements, in which cases it reaches its highest value.

TABLE VI

FLASH INTENSITY RELATIVE TO SOLAR INTENSITY

	Number of Lines									
	18	69	164	166	189	185	166	109	71	44
Solar intensity....	0000	000	00	0	1	2	3	4	5	6
Flash intensity ...	0.44	0.3	0.38	0.55	1.1	1.3	1.9	1.9	2.3	2.8
F−S	+3.44	+2.3	+1.38	+0.55	+0.1	−0.7	−1.1	−2.1	−2.7	−3.2

Another interesting fact appears when the heights for the same flash intensities of different groups of elements are compared with

the heights for the same flash intensities of iron. For 124 lines of the heavy elements and carbon, the heights average 76 km higher than for Fe lines of the same flash intensities; for 159 lines of Ti it is 5 km and for 73 lines of Sc and Y 2 km higher than for iron. It seems clear that some character of the flash lines of the heavy elements and carbon—the increased blackness, perhaps—has made it possible to follow the vanishing tips of the lines with less difficulty than in the case of the other elements and this has resulted in a greater length of arc for the flash lines of the heavy elements than for lines of the same flash intensity due to iron. When the heights for lines of the same solar intensities are compared, the average height for the heavy elements and C is 104 km, for Ti 54 km, and for Sc and Y 69 km greater than for the Fe lines. Since the flash lines of Ti, Sc, and Y yield the same heights as the flash lines of Fe of like intensity, the level of the lines of these elements relative to Fe lines of the same solar intensity is indicated with greater probability than for the heavy elements and carbon, in which case the high level relative to iron lines of like solar intensity is mainly accounted for by the greater height given by lines of the same flash intensity. An indication of this effect of low level appears in the graphs showing the relation between heights and solar intensities. For lines of medium intensity the slope of the curves is regular when the points depend upon a sufficient number of lines; but for lines of low intensity the heights decrease less rapidly or not at all. In the case of these lines, the actual differences in level are smaller than for stronger lines, and the increased emission due to the higher temperatures of the lower levels would tend to increase their blackness and to give, as shown above, a slightly anomalous height on eclipse plates, the effect becoming more apparent with decreasing intensity or increasing depth.

The relative intensity of the lines on a flash spectrum plate plainly depends upon the phase of the eclipse photographed, the exposure times, and the sensitiveness of the apparatus for different wave-lengths. Bearing upon this question the following examination was made of the probable sensitiveness of Professor Mitchell's apparatus: A Seed orthochromatic plate was exposed in a concave grating spectrograph of 1 m focus to sunlight reflected from a

freshly burnished silver mirror, a combination similar to that used by Mitchell. The exposure and development were such as to end the spectrum between λ 5800 and λ 5900. The opacity of the plate was measured by a Hartmann microphotometer and the results plotted against wave-length. The maximum sensitiveness is near λ 4500, and at λ 3300 it is about one-third of the maximum. The combined reflecting power of silver and speculum metal was found from the data of Hagen and Rubens;[1] at λ 3300 the reflecting power is 0.23 of what it is in the visual region, so that the apparatus was approximately one-thirteenth as sensitive at λ 3300 as in the visual region.

This would give lower heights for lines in the ultra-violet. In a certain sense it would be like giving a shorter exposure to the ultra-violet lines of the high-level vapors, as the photographic effect of the radiation of the relatively cool high-level layers would be more decreased than that from the intensely hot low-lying layers, and the relative intensities of the high- and the low-lying lines would be greatly changed.

Evidence that the relative flash intensities are greatly altered in the ultra-violet region and change in such a way as to increase the intensities of the weak solar lines relative to the stronger one is found by comparing the values of F−S in the case of very weak medium, and very strong solar, lines for the two regions.

TABLE VII

		F−S	
		Violet	Ultra-Violet
Intensity..........	000–0	+1.1	+ 1.1
Mean intensity......	3	+1.52	− 1.82
Intensity..........	20	+8.4	−16.2

The intensity of the weak lines relative to the solar lines remains practically unchanged, but the relative intensity decreases progressively with increased solar intensity. The absolute flash intensity of ultra-violet lines of solar intensity 20 is a third of that for the violet lines of the same solar intensity, or the intensity of the

[1] Wood, *Physical Optics*, p. 466, 1911.

weak lines relative to that of the strong lines has increased threefold, entailing a great difference in their relative heights.

That the ultra-violet weak solar lines were photographed at all under these conditions is an indication of the enormous radiation from these low-lying layers, and the weakening of the flash lines which are due to the vapors extending to high elevations is what would be expected when the source of the flash line is a widely extended surface of relatively low intensity. In the one instance we have an intense emission concentrated in a small area; in the other, the source is a very extended surface of cooler vapor with a relatively feeble emission.

In view of the preceding considerations, the usual method, in discussing eclipse plates, of determining heights from length of arc appears of doubtful value in the case of low-level lines.

There are several facts established by observation, relative to the members of the group of heavy elements, all of which should be considered in an attempt to fix the relative level at which the lines of these vapors occur.

1. The intensity of the La and Ce lines at the sun's limb is greatly decreased in comparison with the intensity at the center, while that of most lines is increased. (Adams.)

2. The rotation value given by La and carbon is lower than for Fe lines of the same solar intensity. (Adams.)

3. The displacements given by the La and Ce lines and those of the rare-earth metals in the penumbrae of sun-spots are very large, larger than for the Fe lines of the same solar intensity. (St. John.)

4. The heights given by Mitchell are greater for the flash lines of the La group and carbon than for the Fe lines of the same flash intensity.

5. The heights found by Mitchell are greater for the lines of the La group and carbon than for iron lines of like solar intensities.

Mr. Adams has shown that the behavior of the La and Ce lines at the sun's limb finds an explanation in their low level, that the angular velocity of the solar vapors decreases from that of the high-level hydrogen to a minimum value for lanthanum and carbon, and that the low rotation value for these lines relative to that for

Fe lines of the same solar intensity finds its obvious cause in the low level of these lines relative to the Fe lines of the same solar intensity. The radial displacements of the Fraunhofer lines in the outer edge of the penumbrae of spots appear to be a Doppler effect and the regular increase with decrease of solar intensity, a phenomenon of level. Under this interpretation, the greater displacements given by the lines of the heavy elements than by Fe lines of the same solar intensity mean obviously a low level for these lines relative to those of Fe of like solar intensity.

The greater heights given by the flash spectrum for the lines of the heavy elements relative to those of iron of like solar intensity appear to Mitchell to be in contradiction to these three lines of observation. A large part of this excess in height comes, however, from the character of their flash lines, which has allowed their vanishing tips to be followed farther than in the case of the lines of iron of the same flash intensity. This characteristic of their flash lines, rather than their greater strength relative to the Fe lines of the same solar intensity, contributes probably the major part of the excess in height.

HEIGHT AND WAVE-LENGTH

In the discussion of the radial displacements, it was noticed that the displacements were greater in the red than in the violet for lines of the same solar intensity. This was attributed to our seeing into the sun to a greater depth in the red, owing to the greater scattering of the violet light—that is, the effective levels are lower for the longer wave-lengths. Inasmuch as it is the upper border of the effective layer, or the height to which the vapor emitting a given line extends, that determines the length of the arc, a similar effect might be expected from the flash spectrum. A large number of lines—666—of solar intensity 1 to 6, identified with single elements, were divided into groups of 100 A each. The mean heights for the groups were plotted as ordinates. The result is a curve, (Fig. 3, *a*), with a maximum at λ 3850, a slow drop to the red, and a rapid fall to the ultra-violet.

The maximum height given by the flash lines is at λ 3850, while the photographic maximum of the apparatus is at λ 4500. The

curve. Fig. 3, *b*, represents approximately the effective sensitiveness of the apparatus for various wave-lengths as it is derived from the photographic sensitiveness and the reflecting powers of the surfaces.

It will be noticed that the heights do not follow the effective sensitiveness, that in the region λ 3700 to λ 4350 the heights are greater than in the region λ 4350 to λ 5000, though the sensitiveness is greater in the latter case, and that in the region λ 5000 to λ 5800 the heights are not low because of low sensitiveness of the apparatus, which for this region averages slightly higher than at λ 3850. Apparently the reported intensities and heights are fully as great as photographed, for Mitchell says that in estimating intensities allowance was partially made for the decrease in the sensitiveness of the plate in the green and yellow regions. With due allowance for plate sensitiveness and reflecting power of the surfaces, the fact still remains that the heights decrease with increasing wave-lengths.

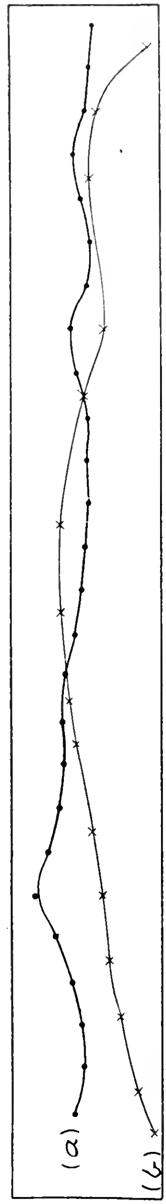

FIG. 3

(*a*), heights as ordinates, wave-lengths as abscissae. All single lines 1–6 solar intensity
(*b*), sensitiveness of apparatus derived from plate sensitiveness and reflecting power of the surfaces

EFFECTIVE LEVELS

The effective level of a line may be defined as that portion of the entire depth of a vapor that is mainly concerned in the production of the line. The light in a Fraunhofer line of intensity 4, for example, comes from a lower depth than in a line of intensity 10 because of the greater selective absorption in the latter. The light from the photosphere and from the lower, hotter layers of the vapor under consideration is selectively absorbed in

both instances and fails to reach the surface, so that the emitted light of the respective wave-lengths comes from more or less sharply bounded shells of vapor having a larger radius for the line of intensity 10 than for the line of intensity 4.

The idea of effective levels has been tacitly employed, though not so definitely expressed in all spectroheliograph work. There is no question among solar physicists but that the light employed in H_3–K_3 photographs comes only from the highest level when a very narrow slit is used; that light forming the H_2–K_2 photographs comes from a low level, but not from the lowest levels; these are successively reached by setting the slit on the broad H_1–K_1 shading at increasing distances from the center of the line. It is clear that here we are dealing with successive shells of calcium vapor. It is probable that a similar succession of shells would obtain for the calcium lines of decreasing intensities, and that is what appears in the radial displacements and in the heights from the flash spectrum when a sufficient number of lines is present to obtain means of weight. For the calcium lines the succession is indicated as follows:

	H_3–K_3	H_2–K_2	4227	8–6	5–3	2–1
Radial displ....	−0.063	−0.044	−0.003	+0.017	+0.019	+0.028 A
Flash sp. hgt........	14,000		5,000	583	491	350 km

The negative signs indicate an inflow, the positive signs an outflow, from the spots, tangential to the solar surface.

What may be called the radiation centers of gravity of the effective layers are at not greatly different levels in the reversing layer. Lines of solar intensity 0000 to 10 occur below 1000 km, allowing an average distance of about 70 km for each layer. If such be the conditions in the solar atmosphere, it is clear that flash spectra measurements of heights made by 50 km steps could show the facts only by taking the means of a very large number of lines, that individual departures from the mean would be numerous, and that small differences between the short arcs would be difficult of determination. In point of view of precision there is a wide difference between the estimates of flash intensities and arc lengths upon a single flash spectrum plate and differential measurements of line displacements upon many plates of high dispersion.

GENERAL CONCLUSIONS

The facts determined by observations that bear upon the question of the distribution of the elements in the solar atmosphere are gradually increasing in number and are already sufficient for the deduction of some probable conclusions. The salient facts shown by flash spectra and displacements in the penumbrae of sun-spots, based upon mean values, are as follows:

From Flash Spectra	From Radial Displacement
The heights and flash intensities increase progressively with the solar intensity of the lines.	The displacements vary progressively with the solar intensity, from large negative values for the strongest lines to large positive values for the weakest lines.
The flash intensity of unenhanced lines relative to the solar intensity increases with decreased solar intensity as follows in the case of iron:	The displacements increase with decrease of solar intensity as follows in the case of iron:

Solar int.	00	0	1	2	3	4	5	6	7–8–9	10–40
F−S	+1.25	+0.26	−0.67	−1.2	−1.6	−2.4	−3.0	−2.5	−3.9	−9.4
Displ.	0.034	0.030	0.028	0.025	0.023	0.021	0.019	0.016	0.011	0.002

From Flash Spectra	From Radial Displacement
The vapors of the elements with the strongest solar lines extend to the greatest heights.	The elements with the strongest solar lines give small positive or negative displacements.
Enhanced lines are higher than the normal or unenhanced lines of the same solar intensity.	The displacements are smaller for enhanced lines than for normal or unenhanced lines of the same solar intensity.
The heights are greater for lines in the violet than in the yellow-green when lines of the same solar intensity are compared.	The displacements are less for lines in the violet than in the yellow-red when lines of the same solar intensity are compared.

These facts are all harmonized by the consideration that the vapors of the elements ascend in detectable amounts to different heights, that the lines of any one element originate at depths increasing with decrease of solar intensity, that the enhanced lines are

higher than unenhanced lines of equal solar intensity, and that we see into the sun to greater depth at the red end of the spectrum than at the violet.

The general distribution of the elements in the solar atmosphere and the relative levels or heights for the lines of a given element, such as iron, were found by interpreting radial displacement in spots as Doppler effects varying with the depth. This interpretation finds strong confirmation in Mitchell's eclipse data, which give a similar distribution of the vapors of the elements and show that the heights of the lines of an element rich in lines, such as iron, increase progressively with their solar intensities. Thus we have added evidence that in the Evershed effect we have a means of sounding the solar atmosphere with a high degree of precision.

The writer finds no better expression for summing up this investigation than that used at the conclusion of the discussion of the displacements in the penumbrae of sun-spots, excluding the specific reference to the H_α line which does not appear in Mitchell's flash spectrum: "The resulting distribution shows that the H_3 and K_3 lines of calcium are the lines of highest level, followed by the H_α line of hydrogen, and that, in the main, the heavy and rare elements occur in detectable amounts only in the lower portions of the solar atmosphere."

Mount Wilson Solar Observatory
June 15, 1914

NEW "VAPOR LAMPS" AND SOME PRELIMINARY OBSERVATIONS OF THEIR SPECTRA IN THE SCHUMANN REGION

By FREDERICK A. SAUNDERS

It is now many years since mercury arc lamps became familiar to physicists. Similar lamps working with the vapors of cadmium or of zinc have more recently been made.[1] Five years ago, Professor Paschen[2] succeeded in running a lamp with the vapor of thallium. At his suggestion, and with his active personal co-operation, I undertook to make a calcium vapor lamp. The work was done in Professor Paschen's laboratory, and my very best thanks are due to him, not only for the design of the vapor lamp and for his skilful help in innumerable experimental difficulties, but most of all for the extraordinary generosity and kindness with which he placed the entire resources of his wonderful laboratory at my disposal. In the end, vapor lamps were successfully run in a quartz tube both with calcium and with magnesium. The spectra of these lamps were examined from λ 9000 to λ 2000, and, in a preliminary sort of way, in the Schumann region. The first part of this work will be reported on later; the present paper concerns itself with (1) the new vapor lamps, and (2) the experiments in the Schumann region.

THE NEW VAPOR LAMPS

A quartz tube was secured of the shape shown in the figure. Its dimensions were, roughly: length, 30 cm; internal diameter, 1 cm; distance between terminals, 7 cm. The ends were ordinarily closed by plates of quartz, Q_1 and Q_2, attached by white sealing-wax and kept cool by wet cotton. The terminal-tubes at A and B were each filled with about 1 cc of small lumps of calcium (or magnesium) metal, into the under part of which were inserted copper wires. These wires were led out through long, narrow, bent

[1] Stark and Küch, *Physikalische Zeitschrift*, 6, 438, 1905.

[2] *Annalen der Physik*, 29, 628, 1909.

tubes, dipping under water (for cooling) and sealed air-tight at the ends by wax. Connection with an air-pump was established through a third tube. A low pressure was easily produced by a pair of pumps in tandem, driven by a motor—one, a rotating oil pump, the other, a Gaede mercury pump. The terminal wires were attached to 220-volt d.-c. mains, and the secondary of a small induction coil was connected between the negative terminal of the lamp and an idle terminal in the air-pump connections (an external electrode is sufficient).

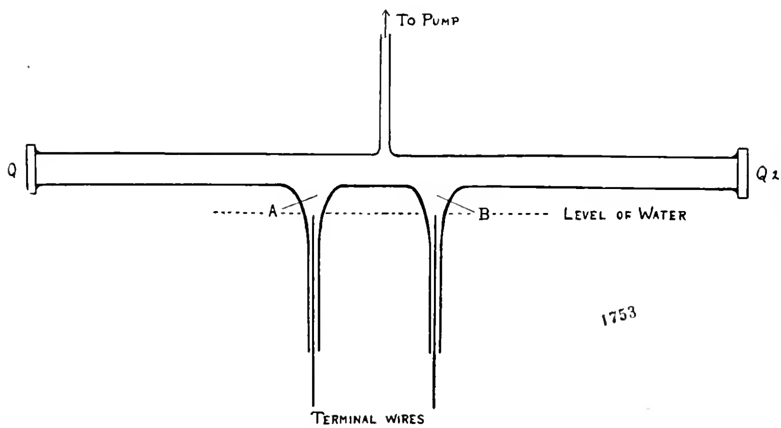

Fig. 1

With a blast lamp the middle portions of the tube and the metal in the electrodes were heated red hot and exhausted until green fluorescence showed itself in the glass connections of the apparatus when the induction coil was operated. After this process was thoroughly carried out, the arc usually started easily; sometimes it was sufficient merely to close the 220-volt connection, and ionize for a moment the residual gas in the tube by a discharge from the induction coil; but more often a certain amount of preliminary heating was also necessary.

The arc itself was intensely brilliant; in the case of calcium, of a bluish-white color, in magnesium greenish-white. Almost immediately after the arc began, the metal covered the inside of the tube with an opaque deposit, extending, however, over the

central portion of the tube only. The quartz windows at the end, through which the observations were made, remained clear for a long time if the tube was air-tight and the current not more than 4 or 5 amperes; in one case, they were still clear after a three-hour run. A larger current, or a leak, extended the deposit of metal to the windows also. At the instant of fusion a good deal of gas was usually liberated, which was pumped away, but the pump could then be disconnected.

In the case of magnesium, the melted metal did not at first flow down about the copper terminal wire; the latter fused immediately and broke the circuit. It was necessary to melt about the wire a small quantity of metallic tin beforehand, in order to maintain contact; by this device the arc was made continuous. This introduced no difficulties, and the resulting spectra were free from the lines of tin.

One could not help admiring a material such as quartz which could tranquilly endure an ordeal of the sort that it was here put through. The terminal tubes dipped into water. The temperature inside them was probably extremely high; and, outside, within 2 or 3 mm of the water, one of the terminals was usually red hot. The tube proved to be fairly durable, and lasted for nearly a dozen runs with different metals. It was, however, found to be roughened internally after every long run, and the life of the tube was prolonged by re-fusion of the roughened parts each time in an oxy-hydrogen flame. The tube walls were also thickened where the action was greatest.

The column of luminous vapor obtained with this tube probably averaged at least 6 cm in length, and the spectra obtained by end-on observations throughout this column are, as one might expect, remarkable for the relative strength of faint lines, especially the outer members of the series. All lines are sharp, and the spectra are almost free from bands.

A noteworthy property of the calcium lamp (and probably with magnesium also) is its ability as an air-pump. The vapor absorbs gases with which it can react chemically. Oxygen and nitrogen are both quickly removed, and hydrogen apparently also to some extent. Argon is not affected. The result is that,

if there is a small leak in the connections, the lamp becomes a little reddish in color and numerous argon lines appear in the spectrum. An arc spectrum of argon is perhaps a novelty but it proved indistinguishable from the usual "red" discharge-tube spectrum.

EXPERIMENTS IN THE SCHUMANN REGION

It was noted in connection with the new lamps that the deposit of metal did not extend more than a few centimeters beyond the column of luminous vapor. It seemed to me likely, therefore, that any light belonging to the Schumann region would suffer very little absorption in passing from the source to the (nearly) empty space at the ends of the tube. Accordingly, the lamp was attached directly to a vacuum grating spectrograph, one of the quartz plates having been removed from the end of the lamp for this purpose. The light thus had no "window" to pass through, but went directly from the luminous vapor through the slit to the concave grating, and was returned to the Schumann plate, passing on its way through nothing but the *very* small amount of residual gas (mostly hydrogen) in the spectrograph. Lyman[1] has used a somewhat similar arrangement with hydrogen, but these experiments differ from his in that the vacuum in the spectrograph was higher, and the method is applicable to a number of metals. By its use, it should be possible to study several spectra to a limit considerably beyond what has already been attained. The experiments here described had to be discontinued almost at the beginning, but they went far enough to show that the method is a good one. The spectra of zinc and of calcium, and incidentally of hydrogen, were photographed to a limit somewhat beyond λ 1000.

The vacuum spectrograph was similar in general plan to that devised by Lyman. It consisted of a 1-meter concave grating (purchased through the generosity of the Rumford Fund Committee, for which I wish here to make grateful acknowledgment) mounted at a slight angle to the normal inside one end of a brass tube. Both ends of this tube were closed by brass caps, fitting

[1] *Astrophysical Journal*, **19**, 203, 1904.

the tube by ground conical joints, and readily removable. The cap on the end away from the grating carried a slit, outside of which sources of light could be sealed-on, and at one side of the slit was the photographic plate. The latter could be changed without disturbing the source, or the slit, by removing a flat plate, with ground surfaces, which formed the back of the cap.

A portion of the slit was covered with a small quartz plate, transparent to about λ 1600. If any ghosts of the type discovered by Lyman had been present in the region of still shorter wave-length, they would have been generated by strong lines of wave-length greater than this limit and would have been transmitted by the quartz. Since the grating gives practically no astigmatism in the first order with this arrangement, the false lines would have been longer than the real ones. None was found, and the method enables one to make sure that the new lines are all real.

The evacuation of the tube had to be done afresh for each plate inserted in the spectrograph, and the volume to be emptied was about 7 liters. There was a small discharge-tube connected to the spectrograph to indicate the state of the vacuum attained. With the pumps mentioned above, when everything was tight (not always the case!) green fluorescence appeared in the discharge-tube in less than three-quarters of an hour. The apparatus was washed out, generally more than three times, with hydrogen, each time exhausting the main tube to a pressure of less than a millimeter, and then letting in about 15 cc of hydrogen. After three washings, the spectrum of the discharge-tube showed nothing but hydrogen, with possibly a trace of mercury vapor from the pump. The entire operation of evacuating to green fluorescence in preparation for a run with the calcium lamp could be accomplished in about an hour. The self-evacuating power of the lamp maintained the vacuum afterward, and even carried it still higher. On one occasion, after an hour's run, no discharge would pass through the discharge-tube at all. This meant that the calcium lamp had exhausted the 7-liter spectrograph through a slit 5 mm by .04 mm to a much lower pressure than it started with, and this was accomplished in spite of the probable emission of gas from the walls of the brass tube during this time.

THE HYDROGEN SPECTRUM

Lyman has very recently[1] extended our knowledge of this spectrum to λ 905, and has found that the Ritz series behaves as though it were independent of the secondary spectrum. His results are here confirmed.

The preliminary discharge necessary to start the calcium lamp gives the secondary spectrum of hydrogen exactly as Lyman found it. This spectrum was also photographed by itself, for comparison. On all the plates taken with the lamps, a few of the strongest lines of this spectrum are faintly visible, probably on account of the preliminary discharge used to start the arc. On every vapor-lamp plate, however, with both calcium and zinc lamps, the hydrogen line λ 1216 is present, and always very strong relative to the lines of the secondary spectrum. This is the first line of the Ritz series; the second line λ 1026 is also to be seen on two plates, one of which is practically free from the secondary spectrum.

We have here, therefore, further evidence that the Ritz series lines are independent of the secondary spectrum, and behave toward it in the same way as does the ordinary series in the visible spectrum.

THE CALCIUM SPECTRUM

The spectrum of the calcium lamp in the Schumann region is almost the same as was obtained by Lyman[2] from a spark. The narrow pairs are the most prominent feature; other faint lines were obtained, as shown in Table I, and there are probably still more to be found, as the Schumann plates used were not very good; but the identification of new lines is not easy, and cannot be carried out until we know the spectra of several more elements in this region. There seem to be no strong calcium lines between λ 1300 and λ 900. None are to be expected, excepting the later ones of the principal series of pairs (the series to which H and K belong); of this series, one line may be in the list but there is no way of making sure of it. The shortest observed line was at λ 977.9,

[1] *Nature*, May 7, 1914.

[2] *Astrophysical Journal*, **35**, 341, 1912.

possibly due to Si, though its origin must be left an open question. There were indications of lines still farther out, but too uncertain to be worth publishing.

TABLE I

SPECTRUM OF THE CALCIUM LAMP

Intensities	λ Vacuum (Saunders)	λ Vacuum (Lyman)	Remarks
1	1850.9	1851.3	
0	——	1843.8	
6	Standard	1840.2	
5	——	1838.0	
1	——	1828.1	Mg
2	——	1815.0	
1	——	1807.8	
4	Standard	1555.1	
3	——	1553.5	
1	1533.8	1533.4	
2	1475.5	....	Probably Ca
2	Standard	1434.3	
1	——	1433.1	
1	1400.7	....	Probably Ca
0	——	1370.6	
0	——	1369.1	
1	1347.1	....	Probably Ca
0	1264.7	1264.5	? H
0	977.9	....	Probably not Ca

Table I shows, in the first column, the intensities estimated on a scale from 10 to 0; in the second column, the wave-lengths measured from certain lines assumed as standards; a dash indicates a line observed but not measured. In the third column are the wave-lengths given by Lyman for all the lines observed here; his list includes others in addition. There are some faint lines on the plates which are not included in the table, as the evidence is decidedly against their being due to calcium. The accuracy of my measurements should be about the same as Lyman's (i.e., within 0.3 A) in the region down to λ 1200, but the error may well be somewhat larger beyond.

THE ZINC SPECTRUM

In the attempt to push the limit of the spectrum farther out, the lamp was run with calcium in the negative terminal and zinc in the positive one. The spectrum obtained was that of zinc only,

but was underexposed. It shows, however, a few of the zinc lines found by Wolff,[1] the two hydrogen lines λ 1216 and λ 1026, a few belonging to the secondary hydrogen spectrum, and a few others, possibly zinc, the shortest of which is at λ 1099.6. Apparently, zinc gives no very strong lines far out in the spectrum, but a longer exposure would be interesting to try.

TABLE II

SPECTRUM OF THE ZINC LAMP

Intensities	λ Vacuum (Saunders)	λ Vacuum (Wolff)	Remarks
4	Standard	1589.7	
3	Standard	1457.6	
1	Standard	1404.2	
0	1394.1	....	? H
0	1364.0	....	Probably H
0	1345.5	....	Probably H
0	1342.6	....	Probably H
0	1334.1	....	Probably H
0	1104.2	....	? Zn
0	1102.6	....	? Zn
0	1099.6	....	? Zn

SUMMARY

1. Arc vapor lamps, of the sort used by Paschen with thallium, have been successfully run with Ca and Mg in red-hot quartz tubes at a high vacuum, and their spectra have been obtained over a wide range.

2. The spectra of the Ca and Zn lamps have been examined in the Schumann region without passing the light through any fluorite, or other absorbing material, and lines, probably metallic, have been obtained as far out as λ 977.9.

3. The Ritz series of hydrogen has been photographed as far out as λ 1026 in the spectra of these lamps, and it is practically independent of the secondary spectrum.

4. The Ca lamp acts like a chemical air-pump, removing the gases with which the calcium combines.

TÜBINGEN
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[1] *Annalen der Physik*, **42**, 825, 1913.

SOME SPECTRAL CRITERIA FOR THE DETERMINATION OF ABSOLUTE STELLAR MAGNITUDES[1]

By WALTER S. ADAMS AND ARNOLD KOHLSCHÜTTER

In the course of a study of the spectral classification of stars whose spectra have been photographed for radial velocity determinations some interesting peculiarities have been observed. The stars investigated are of two kinds: first, those of large proper motion with measured parallaxes; second, those of very small proper motion, and hence, in general, of great distance. The apparent magnitudes of the large proper motion, or nearer stars, are somewhat less on the average than those of the small proper motion stars, so that the difference in absolute magnitude must be very great between the two groups. The spectral types range from A to M.

The principal differences in the spectra of the two groups of stars are:

1. The continuous spectrum of the small proper motion stars is relatively fainter in the violet as compared with the red than is the spectrum of the large proper motion stars. The magnitude of this effect appears to depend on the spectral type, and increases with advancing type between F0 and K0.

2. The hydrogen lines are abnormally strong in a considerable number of the small proper motion stars. Thus six stars which show the well developed titanium oxide bands characteristic of type M have hydrogen lines which would place them in types G4 to G6, and many others which show the bands strongly would be classified under type K from their hydrogen lines. That the spectra of these stars are not composite is shown by their radial velocities. The hydrogen lines in the spectra of the large proper motion stars which show the titanium oxide bands are without exception very weak.

3. Certain other spectrum lines are weak in the large proper motion stars, and strong in the small proper motion stars, and

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 89.

conversely. It is with the possibility of applying this fact to the determination of absolute magnitudes that the results given in this communication mainly have to deal.

I. INTENSITY OF THE CONTINUOUS SPECTRUM

A comparison of the intensity of the continuous spectrum of several pairs of stars of small and of large proper motion photographed upon the same plate was made recently by one of us, and showed a marked weakening relatively in the violet region for a majority of the small proper motion stars.[1] With a view to supplementing these observations with the larger amount of material available in the radial velocity photographs we have calculated the densities at several points in the spectrum for a considerable number of these stars, and compared the resulting values for the stars of small with those of large proper motion.

The method employed, though by no means so accurate as a photometric measure of density, appeared to be as good as the character of the material would warrant. It is evident, of course, that in the case of an individual star the conditions of transparency under which the photograph was taken, the zenith distance, and some other factors might influence the result seriously. For the mean of a considerable number of photographs and stars, however, it would seem that these effects should counteract each other largely, or at least be similar for the two groups of stars under comparison. No photographs have been included which were taken at great zenith distances.

The plan adopted for the determination of the densities was as follows: A standard plate of α Tauri was first obtained, several spectra taken with different exposure times being placed side by side on the negative. The photograph of each star was then compared with this standard plate under a Hartmann spectrocomparator, and estimates were made of the intensity of the continuous spectrum relative to that of α Tauri at three selected points at the violet and four points at the red end of the spectrum. The points were selected in regions as free from lines as possible. The estimates were made in tenths of a unit between the α Tauri spectra. Thus

[1] *Mt. Wilson Contr.*, No. 78, *Astrophysical Journal*, **39**, 89, 1913.

1.5 indicates an intensity half-way between the first and second of the standard spectra. After the comparisons had been finished the α Tauri photograph was measured under a microphotometer, and the densities were calculated at the points of comparison. The results for all of the stars were then reduced to densities.

The values for the groups of stars are given in Table I. The densities for the three violet wave-lengths have been combined to form a mean at λ 4220, and similarly for the four wave-lengths near λ 4955.

TABLE I

	Number of Stars	Average μ	Average Type	Density at λ 4220	Density at λ 4955
A0–A9	15	0.″020	A2	0.30	0.32
	16	.13	A3	.29	.32
F0–F9	10	.012	F4	.25	.37
	23	.66	F6	.32	.37
G0–G4	8	.009	G3	.22	.41
	30	.64	G2	.33	.41
G5–G9	14	.011	G7	.25	.48
	22	.64	G7	.40	.48
K0–K4	24	.011	K2	.16	.44
	22	0.70	K1	0.31	0.44

The features of note in these results are:

a) The small proper motion stars of types F to K are decidedly weaker in the violet part of the spectrum than the large proper motion stars.

b) The difference is inappreciable for two groups of A-type stars for which the ratio of proper motions is 1:6.5.

c) The difference increases with advancing type from F to K, being twice as great for the latter. The ratio of proper motions for the groups of small and of large proper motion stars is nearly the same for the stars between F and K. Hence if interpreted in terms of distance the ratio of distances should be nearly the same, and it would appear that at least a part of the absorption in the violet part of the spectrum of the distant stars must be ascribed, not to scattering of light in space, but to conditions in the stellar atmospheres. In the case of the A-type stars the results are inconclusive, since the ratio of the proper motions shows that the negative result found may be due to the fact that the difference

of distance between the two groups of stars is insufficient to produce a measurable amount of scattering.

II. THE HYDROGEN LINES

The abnormal strength of the hydrogen lines in the spectra of certain of the small proper motion stars is of peculiar interest because of the possibility of selective absorption by hydrogen gas in interstellar space. The radial velocity affords a means of determining the origin of the additional absorption since it is highly improbable that the hydrogen in space would have the motion of the stars observed. Accordingly we have given especial attention to the determination of the radial velocities of these stars from the hydrogen lines as compared with other selected lines in the spectrum. The results obtained indicate that within the limits of error of measurement the hydrogen lines give essentially the same values as the other lines, and no differences have been found of an order to correspond to the abnormal intensity of the lines.

TABLE II

Star	Magnitude	Type	Metallic−H Lines	Computed Metallic−H Lines for Absorption	
				In Star	In Space
			km	km	km
Boss 539	5.6	G7p	−0.5	0.0	+13.4
707	5.6	G9p	+0.3	.0	+ 4.3
1309	5.8	G9p	+0.6	.0	−16.7
1560	5.3	G5p	+1.6	.0	+15.3
1846	5.7	G9p	+2.9	.0	−50.9
2020	5.8	G6p	+0.3	.0	−27.7
2144	6.2	G6p	+4.9	.0	+15.6
2378	6.3	G6p	+2.2	.0	+ 6.4
2612	5.6	G8p	−2.0	.0	+ 1.2
2915	6.0	G5p	+0.7	.0	−14.9
3867	6.0	G6p	−1.2	.0	−18.9
4159	5.6	G5p	−3.6	.0	−17.2
5125	5.6	G5p	+1.8	.0	+ 3.4
6089	5.4	G4p	−2.2	.0	−31.7
6145	6.0	F0p	−4.6	0.0	0.0

In Table II are collected the results for 15 stars which show abnormal strength of the hydrogen lines most prominently. All of the stars except Boss 6145 have the bands characteristic of

type M. The classification given is based on the hydrogen lines. The column designated "Metallic—H Lines" gives the values in kilometers of the differences in the velocities derived from about 12 selected metallic lines and those from H_γ and H_β; a small systematic correction is applied to the latter, due probably to the effect of blended lines. These differences would, of course, be zero if all of the hydrogen absorption occurred in the stellar atmospheres. If it all occurred in space the differences would be those given in the final column on the assumption that the absorbing gas is at rest in space. The quantities are derived by applying to the velocities of the stars obtained from the metallic lines the corrections to these velocities for the motion of the sun in space.

If any appreciable hydrogen absorption occurred in space the differences, Metallic—H Lines, should, of course, be intermediate between the quantities in the last two columns. When, however, we combine the values for all of the stars, assigning weights according to the numbers in the last column, we find that 98 per cent of the hydrogen absorption must occur in the stellar atmospheres, and that but 2 per cent can possibly be due to hydrogen gas in space. This amount is far below the limits of accuracy of the observations.

III. THE RELATION OF LINE INTENSITY TO ABSOLUTE MAGNITUDE

Systematic differences of intensity for certain lines between stars of large and stars of small proper motion soon became evident in the course of the study of the spectral classification of these stars. In order to secure an accurate system of classification as well as to investigate these differences the following method was adopted. Pairs of lines were selected not far from one another in the spectrum and of as nearly as possible the same intensity and character, and estimations were made of their relative intensities. For classification purposes a line decreasing in intensity with advancing type, such as a hydrogen line, was combined with a line increasing in intensity with advancing type, such as an ordinary metallic line. In addition to these pairs used for classification purposes several pairs were selected which included all lines suspected of systematic deviations in certain stars.

The estimations were made on an arbitrary scale extending from 1 to about 12, 1 being the smallest difference in intensity which could be detected. The method, therefore, is analogous to the *Stufenmethode* of Argelander used in estimations of variable stars; hence, for physiological reasons, our scale will be approximately proportional to the logarithm of the intensity differences of the two lines. In general three plates were used for each star, and the photographs of the large and the small proper motion stars were intermingled in order that systematic effects on the estimation scale might be avoided.

After all of the estimations had been completed the material was reduced uniformly, and the results were examined with two objects in view: first, to investigate the changes of the estimated intensity differences with the spectral type, and on this basis to form a classification depending on certain well defined criteria; second, after correcting for changes with type to investigate changes with absolute magnitude.

An examination of the pairs of lines used for estimation indicated that the following pairs showed the largest and most definite changes with type. The Harvard scale of classification has been followed closely.

F8–G6 stars: $\frac{4227}{H_\gamma}$; $\frac{4326}{H_\gamma}$; $\frac{H_\gamma}{4352}$; $\frac{H_\gamma}{4405}$; $\frac{H_\gamma}{4384}$

G6–K9 stars: $\frac{4326}{H_\gamma}$ (wt. $\frac{1}{2}$); $\frac{H_\gamma}{4352}$; $\frac{H_\gamma}{4405}$ (wt. $\frac{1}{2}$); $\frac{H_\beta}{4872}$; $\frac{H_\beta}{4958}$

These lines, accordingly, have been used to determine the type of each individual star, and since no systematic differences for the different lines have been found, the mean of the determinations from the five pairs has been used as the final result for the spectral type. This method of classification has proved most satisfactory in use, and shows good internal agreement. The mean error of one determination depending on three plates is ± 0.4 subdivision of the Harvard scale, equal, for example, to the interval from G5.0 to G5.4.

As soon as the spectral type of each star had been obtained in this way, the results for the remaining pairs of lines were examined

with a view to seeing whether all of them fell into agreement with the classification, or whether there were systematic differences for different groups of stars. For this purpose we constructed a normal curve for each pair of lines from the stars of rather low absolute luminosity, plotting as abscissae the spectral types, and as ordinates the estimations of intensity differences. Finally we formed for all of the stars the differences between our estimations of relative intensity and the values from the normal curve corresponding to the spectral type. These differences, combined into means for two separate groups, are shown in Table III.

At the head of each column of ratios is given the mean of the absolute magnitudes of the stars observed. Thus for the F8–G6 stars the mean of the absolute magnitudes of the small proper motion stars is -2.9, of the large proper motion stars, $+6.1$. Although the number of stars used in the estimate of the ratios of the different pairs of lines varies somewhat, the same mean magnitude, which was derived from all of the stars, is used throughout. The computation of the absolute magnitudes of the individual stars was made from the measured parallaxes where these were available. In the absence of such determinations, or when the parallax was very small or negative, the absolute magnitude was computed from the proper motion by aid of the parallax derived from the following formula:

$$\log \pi = -1.00 - 0.005m + 0.86 \log \mu$$

where m is the apparent magnitude and μ the proper motion. This formula is contained in an unpublished investigation by Kapteyn and Kohlschütter on the luminosity-curve of the K-type stars, and is based upon a discussion of the relation between proper motion and parallax for the K stars. The unit employed in the determination of absolute magnitudes is $0''.1$; that is, the absolute magnitude is the apparent magnitude of a star at a distance corresponding to a parallax of $0''.1$.

The number of stars used in each comparison in Table III is indicated by the figures in parentheses.

It is obvious from the method of derivation that the mean values in Table III for all the pairs of lines will be small in the case

TABLE III

Ratio	F8 to G6			G6 to K9		
	$M=-2.9$	$M=+6.1$	Remarks	$M=-2.7$	$M=+6.2$	Remarks
$\frac{4227}{H\gamma}$....	−0.2 (19)	−0.1 (47)	Class.	+1.9 (16)	+0.5 (19)	
$\frac{4326}{H\gamma}$....	0.0 (28)	0.0 (49)	Class.	−0.2 (52)	0.0 (45)	Class. (wt. ½)
$\frac{H\gamma}{4352}$....	−0.3 (28)	+0.1 (49)	Class.	+0.9 (52)	0.0 (45)	Class.
$\frac{H\gamma}{4405}$....	+0.4 (28)	−0.1 (49)	Class.	+0.1 (52)	0.0 (45)	Class. (wt. ½)
$\frac{H\gamma}{4384}$....	+0.6 (28)	0.0 (48)	Class.	+0.2 (51)	+0.2 (43)	
$\frac{H\beta}{4852}$....	−0.7 (13)	+0.2 (29)		−0.4 (48)	−0.1 (43)	Class.
$\frac{H\beta}{4958}$....	+0.1 (16)	−0.2 (28)		−0.9 (46)	0.0 (40)	Class.
$\frac{4337}{H\gamma}$....	+3.2 (28)	−0.4 (49)		+1.9 (52)	+0.3 (45)	
$\frac{4415}{4435}$....	−1.2 (28)	−0.2 (49)		−1.1 (52)	+0.1 (45)	
$\frac{4435}{4444}$....	−1.0 (28)	−0.1 (49)		−0.7 (52)	−0.2 (45)	
$\frac{4415}{4456}$....	+1.7 (28)	−0.1 (49)		+3.9 (52)	+0.2 (45)	
$\frac{4456}{4462}$....	−4.8 (28)	+0.4 (49)	Abs. mag.	−6.6 (52)	−0.3 (45)	Abs. mag.
$\frac{4456}{4495}$....	−3.6 (26)	+0.5 (36)	Abs. mag.	−5.2 (51)	−0.3 (45)	Abs. mag.
$\frac{4216}{4250}$....	+8.3 (21)	−0.4 (44)	Abs. mag.	+5.8 (31)	+0.2 (33)	Abs. mag.
$\frac{4250}{4255}$....	−0.5 (20)	−0.1 (45)		−1.3 (42)	−0.1 (38)	
$\frac{4271}{4275}$....	−1.2 (27)	0.0 (46)		−0.5 (48)	−0.3 (43)	
$\frac{4368}{4376}$....	−0.8 (28)	−0.3 (41)		+0.1 (52)	0.0 (41)	
$\frac{4395}{4415}$....	+4.1 (28)	−0.2 (41)	Abs. mag.	+3.4 (52)	+0.4 (40)	Abs. mag.
$\frac{4408}{4415}$....	+5.7 (28)	+0.1 (48)	Abs. mag.	+4.9 (52)	+0.5 (45)	Abs. mag.
$\frac{4495}{4497}$....	−3.4 (15)	−1.0 (4)		−2.8 (46)	−0.9 (14)	
$\frac{4462}{4495}$....	+1.2 (25)	+0.1 (33)		+1.7 (51)	+0.2 (45)	
$\frac{4415}{4535}$....	−2.3 (27)	0.0 (40)		−1.5 (52)	0.0 (45)	
$\frac{4325}{4326}$....	−4.0 (21)	+0.1 (46)		−5.1 (27)	+0.2 (37)	

of the stars of small absolute magnitude, and that the values for the pairs used for classification purposes will be small for stars of both small and large absolute magnitude. The most prominent cases of lines where systematic differences are seen to exist between the stars of high and of low luminosity are the following:

STARS OF HIGH LUMINOSITY

Strong	Weak
	4325 Sc
4216 Sr	4435 Ca
4395 Ti, V, Zr	4456 Ca
4408 V, Fe	4535 Ti

The Sr line at λ 4216 is an extremely prominent chromospheric line, and the same is true in less degree of the enhanced Ti line at λ 4395. The line at λ 4408 is a blend, and as given by Rowland consists of V and Fe. Some other element may perhaps contribute to the stellar line. All four of the lines which are relatively weak in the high luminosity stars are well known sun-spot lines, being greatly strengthened in the umbrae of spots.

The following five pairs of lines were selected from Table III as the basis for an investigation of the individual stars:

$$\frac{4216}{4250} \quad \frac{4395}{4415} \quad \frac{4408}{4415} \quad \frac{4456}{4462} \quad \frac{4456}{4495}$$

The results given in Table III, estimated value—normal value, for these five pairs of lines were combined into means. By assuming a linear relationship between these mean values D, and the absolute magnitude M, we then derived the formulae:

$$\text{F8–G6 stars: } M=+5.6-1.6\,D$$
$$\text{G6–K9 stars: } M=+6.8-1.8\,D$$

The difference between the two constant terms shows merely that the average magnitude of the stars used for the normal curve is 5.6 for the first group, and 6.8 for the second group. The agreement for the two groups of the coefficient of D indicates how well the same relationship holds throughout the whole range of spectral type from F8 to K9. For the very faintest stars, below absolute magnitude 7, the linear relationship does not seem to hold strictly but it has not seemed desirable for the present material to use a more complicated formula.

Tables IV and V show the absolute magnitudes computed from these formulae for 71 stars of types F8 to G6, and 91 stars of types G6 to K9. The spectral classification is that derived by the method already described and the parallax π is taken from *Groningen Publication*, No. 24. The first column of absolute magnitudes M contains the values calculated from the parallax or the proper motion, the latter being used wherever the measured parallax is less than $+0''.05$. The second column of absolute magnitudes contains the values determined from the intensities of the spectrum lines.

The average difference between the two sets of absolute magnitudes is slightly less than 1.6 magnitudes for the F8–G6 stars, and 1.5 magnitudes for the G6–K9 stars. In view of the uncertainties attaching to the determination of absolute magnitudes from proper motions, this difference is not excessive. There appears, therefore, to be considerable promise in the application of spectrum line criteria to the determination of absolute magnitudes and parallaxes.

SUMMARY

Including the results described here, we have found as a product of our investigations of the spectra of large and of small proper motion stars three phenomena which appear to have a distinct bearing upon the problem of the determination of the absolute magnitudes of stars.

1. The continuous spectrum of the small proper motion stars is decidedly less intense in the violet region relative to the red than the spectrum of the nearer and smaller stars. This effect appears to be a function of the spectral type, and so must be ascribed in part, at least, to conditions in the stellar atmospheres.

2. A considerable number of the small proper motion stars show hydrogen lines of abnormally great intensity. Measures of the radial velocity show the source of the additional absorption to be mainly, if not wholly, in the stars themselves.

3. Certain lines are strong in the spectra of the small proper motion stars, and others in the spectra of the large proper motion stars. The use of the relative intensities of these lines gives results

TABLE IV

F8–G6

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Brad. 3212	6.2	G6	0″.419	+0″.15	+7	+ 7
Boss 9	6.3	G4	0.006	....	−3	− 3
Boss 41	5.8	G2	0.002	....	−6	− 1
Pi. 0^h130	5.7	G5	1.390	+ .36	+8	+ 6
Pi. 0^h137	7.4	G1	0.790	+ .02	+7	+ 6
Boss 138	5.4	G6	0.007	....	−4	− 2
μ Cassiop.	5.3	G3	3.760	+ .11	+6	+10
Lal. 2450	8.1	G0	0.570	+ .02	+7	+ 8
Boss 349	5.5	G5	0.008	....	−4	− 2
Boss 355	5.7	G6	0.004	....	−5	− 5
Fed. 263	7.6	G4	0.750	+ .04	+7	+ 9
Lal. 3922–3	7.5	G0	0.510	− .04	+6	+ 4
Lal. 4141	6.9	G4	0.600	+ .03	+6	+ 6
Boss 619	5.6	F9	0.004	....	−5	− 1
Lal. 5490–6	6.7	G3	1.000	+ .06	+5	+ 6
Boss 977	5.6	G6	0.020	....	−2	− 6
Groom. 864	7.3	G0	0.730	+ .03	+6	+ 8
Boss 1131	6.8	G1	0.440	....	+5	+ 7
Groom. 884	7.1	F9	0.680	+ .09	+7	+ 7
Lal. 9091	7.0	F8	0.100	− .02	+3	+ 6
α Aurigae	0.2	G1	0.437	+ .07	−1	+ 1
λ Aurigae	4.8	G1	0.843	+ .11	+5	+ 7
Boss 1394	6.4	G5	0.002	....	−5	− 1
Boss 1441	5.9	G4	0.014	....	−2	− 1
Lal. 11196	6.5	G2	0.690	+ .06	+5	+ 5
Boss 1513	5.8	G5	0.007	....	−4	− 4
Boss 1704	6.5	G1	0.017	....	−1	− 1
W.B. 6^h1500	7.7	G3	0.570	+ .01	+6	+ 7
Lal. 13849	6.5	G6	0.520	+ .11	+7	+ 6
Boss 1873	6.5	G4	0.019	....	−1	− 2
28 Hev. Cam	6.5	G6	0.478	.00	+5	+ 5
Lal. 15565	6.9	G7	1.190	+ .09	+7	+ 7
Boss 2150	5.5	G3	0.006	....	−4	− 1
Pi. 7^h321	7.0	G3	0.807	+ .05	+5	+ 6
Boss 2236	6.3	G1	0.362	....	+4	+ 7
Lal. 16904	8.1	G3	0.440	− .04	+6	+ 6
Boss 2338	6.1	G3	0.020	....	−1	− 1
Lal. 19896	7.7	G0	0.470	− .05	+6	+ 5
Pi. 11^h32	7.3	G6	0.808	+ .01	+7	+ 8
Groom. 1822	7.9	G0	0.670	+ .02	+7	+ 2
Lal. 22585	6.4	G6	0.470	+ .12	+7	+ 4
Groom. 1855	7.4	G6	0.340	+ .06	+6	+ 8
W.B. 12^h69	7.3	G4	0.740	.00	+6	+ 5
Lal. 22908	7.5	G3	0.590	+ .09	+7	+ 6
Lal. 24414–6	6.9	G3	0.720	+ .01	+6	+ 5
Boss 3442	6.3	G6	0.008	....	−3	− 1
Pi. 13^h114	7.5	G6	0.940	+ .05	+6	+ 7
Lal. 27742 Br.	6.8	G4	0.680	.00	+6	+ 4
Lal. 27742 Ft.	7.6	G6	0.680	.00	+7	+ 7
Lal. 30694	6.8	G6	1.660	+ .07	+6	+ 7
Lal. 31132	6.7	G5	0.830	+0.14	+7	+ 7
Boss 4601	5.9	G6	0.009	....	−3	− 2

TABLE IV—*Continued*

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Boss 4763	6.2	G5	0".010	....	−3	−2
Groom. 2789N	6.6	G3	0.654	+0".02	+6	+7
Groom. 2789S	6.8	G2	0.630	+ .02	+6	+6
Boss 4910	6.3	G5	0.014	....	−2	+1
16 Cygni Pr.	6.3	G2	0.212	+ .16	+7	+9
16 Cygni Fol.	6.4	G2	0.220	+ .16	+7	+3
Lal. 38287	7.2	G6	0.600	+ .09	+7	+8
Pi. 20^h174	5.8	G2	0.317	+ .04	+3	+4
Groom. 3215	7.0	G6	0.480	+ .01	+5	+6
Fed. 3638	7.8	G4	0.700	− .02	+7	+6
Boss 5420	6.2	G6	0.012	....	−2	−1
Lac. 8777	6.5	G5	0.680	− .10	+6	+4
Lal. 43492	6.9	F9	0.840	+ .02	+6	+8
Boss 5769	6.1	G5	0.006	....	−4	0
Fed. 4371	7.5	G3	0.600	+ .06	+6	+8
Boss 5982	6.5	G0	0.015	....	−1	−1
B.D. 62°2244	7.3	G4	0.440	− .02	+6	+4
Pi. 23^h164	7.0	F8	0.620	− .05	+6	+7
Pi. 23^h267	6.2	G0	0.770	+0.01	+5	+1

for absolute magnitudes in satisfactory agreement with those derived from parallaxes and proper motions.

It seems very probable from physical considerations that the spectra of stars of quite different mass and size would differ considerably in certain respects even when the main spectral characteristics were the same. If the depth of the atmosphere for stars of similar spectral type is at all in proportion to the linear dimensions of the stars, we should expect the deeper reversing layers of the larger stars to produce certain modifications of the spectrum lines. Owing to the small scale of the stellar spectrum photographs, only the most marked changes could be distinguished, and among these the effect of the deep atmosphere upon the violet end of the spectrum should be especially prominent.

A case of somewhat similar nature is that found in observations of the center and the limb of the sun. The length of path through the solar atmosphere is much greater at the limb, and greater relatively for the lower and lower strata. On large-scale solar photographs the differences between the center and the limb spectra are very marked, but on very small-scale photographs, no doubt, only the most prominent differences could be observed.

TABLE V

G6–K9

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Boss 131.	5.7	K0	0″.019	. . . .	−2	−2
54 Piscium.	6.1	K1	0.595	+0″.14	+7	+5
Lal. 1045.	7.5	K3	0.810	+ .02	+7	+8
Boss 165.	6.2	G6	0.016	. . . .	−2	−3
Mayer 20.	5.8	K1	1.366	+ .17	+7	+10
Boss 209.	5.6	G6	0.017	. . . .	−2	−6
Lal. 1799.	8.0	K6	0.480	+ .06	+7	+6
Boss 267.	6.2	K1	0.004	. . . .	−4	−4
W.B. $1^{h}161$	8.0	G8	0.490	− .01	+6	+8
Lal. 2682.	7.8	K1	0.470	+ .01	+6	+5
Lal. 3022.	7.8	G7	0.500	+ .05	+6	+9
Boss 410.	5.9	G9	0.018	. . . .	−2	−3
Boss 420.	5.6	K1	0.009	. . . .	−3	−4
Boss 430.	6.1	K1	0.005	. . . .	−4	−3
Boss 432.	5.8	K0	0.180	. . . .	+2	−1
Boss 434.	6.0	G8	0.011	. . . .	−3	−2
α Arietis.	2.2	G8	0.239	+ .09	+2	−2
Boss 493.	6.2	K4	0.011	. . . .	−2	−2
Boss 539.	5.6	G7	0.006	. . . .	−4	−3
Boss 581.	5.9	G9	0.010	. . . .	−3	−3
Pi. $2^{h}123$	5.9	K6	2.320	+ .14	+7	+7
W.B. $2^{h}927$	8.2	G8	0.680	+ .10	+8	+8
W.B. $3^{h}113$	7.8	K0	0.620	+ .08	+7	+1
Boss 768.	5.6	G8	0.020	. . . .	−2	−4
Boss 832.	6.0	G7	0.002	. . . .	−6	0
γ Tauri.	3.9	G9	0.120	. . . .	0	−1
Boss 1014.	6.1	G7	0.011	. . . .	−2	−2
ε Tauri.	3.6	K0	0.120	. . . .	−1	0
θ Tauri.	4.0	G8	0.108	. . . .	+1	+1
α Tauri.	1.1	K3	0.203	+ .07	0	−1
Boss 1176.	6.4	K2	0.012	. . . .	−2	−2
W.B. $4^{h}1189$	6.5	K9	1.250	+ .30	+9	+5
Boss 1247.	5.8	K1	0.017	. . . .	−2	−2
Boss 1348.	6.0	K0	0.007	. . . .	−4	−4
Groom. 990.	8.1	K0	0.560	+ .03	+7	+11
Pi. $5^{h}146$	6.4	K0	0.510	+ .12	+7	+7
Lal. 10797–8	7.3	K2	0.720	+ .08	+7	+7
Boss 1444.	5.6	G8	0.009	. . . .	−3	−4
Boss 1608.	5.5	K2	0.011	. . . .	−3	−2
6 Lyncis.	5.9	G7	0.330	− .01	+4	+4
Boss 1632.	6.0	K2	0.007	. . . .	−4	−2
Boss 1643.	5.9	K6	0.019	. . . .	−2	−3
Lal. 13284–5	6.9	K9	0.570	+ .11	+7	+6
Lal. 13427.	8.2	K1	0.710	+ .05	+7	+7
Boss 1846.	5.7	K0	0.019	. . . .	−2	−4
Boss 1868.	5.9	K2	0.020	. . . .	−2	−2
Boss 2148.	6.4	K2	0.073	− .04	+1	−2
Boss 2220.	6.2	G8	0.011	. . . .	−2	−1
Pi. $8^{h}105$	6.0	K3	0.040	−0.03	0	−1
55 ρ^{1} Cancri	6.1	K0	0.541	+ .08	+6	+2
Boss 2449.	5.2	G6	0.008	. . . .	−4	+1
Boss 2455.	6.1	G6	0.010	. . . .	−3	−2

TABLE V—*Continued*

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Lal. 18286	7.3	K3	0″.520	+0″.07	+6	+8
Lal. 19022	8.2	K5	0.800	+ .06	+7	+6
Groom. 1596	8.2	G6	0.480	+ .07	+7	+6
Boss 2795	6.0	G7	0.016	....	−2	−2
Boss 2910	5.2	K2	0.113	+ .02	+1	−2
83 Leonis Br.	6.2	G8	0.743	+ .02	+6	+6
83 Leonis Ft.	7.6	K8	0.736	+ .02	+7	+5
Boss 3125	5.9	K1	0.012	....	−3	−2
Pi. 11^h218	6.8	G6	0.670	− .03	+6	+2
Boss 3406	6.0	G9	0.020	....	−2	−3
Lal. 26196	7.6	K1	0.680	+ .14	+8	+7
Boss 3810	5.6	G9	0.002	....	−6	−2
Lal. 27744	6.7	G7	1.380	+ .13	+7	+8
OΣ 298	7.9	K6	0.480	+ .05	+6	+7
W.B. 15^h720	6.8	K1	0.480	+ .05	+5	+6
Lal. 30024–6	7.0	K1	0.490	+ .09	+7	+9
Boss 4228	6.0	G8	0.020	....	−2	−6
Lal. 30699	7.8	G7	0.470	+ .05	+6	+4
Lal. 33439	6.7	K2	0.650	+ .04	+6	+8
Boss 4724	5.8	G8	0.006	....	−4	0
Groom. 2875	6.7	K1	0.660	− .05	+6	+8
Lal. 38383	7.2	K1	1.390	+ .03	+8	+10
Pi. 20^h23	7.3	G9	0.550	+ .08	+7	+9
Boss 5177	5.8	G8	0.020	....	−2	−3
Pi. 20^h29	5.7	G9	1.256	+ .01	+6	+6
Boss 5317	6.2	K1	0.012	....	−2	−2
η Cephei	3.6	K0	0.836	+ .10	+4	+3
Boss 5397	5.6	K4	0.005	....	−4	−4
Boss 5486	6.2	K0	0.020	....	−1	−2
Boss 5655	5.8	K1	0.006	....	−4	−2
Groom. 3689	8.1	G8	0.610	+ .04	+7	+6
Boss 5868	6.1	K4	0.016	....	−2	−4
Pi. 22^h214	6.5	G9	0.470	− .02	+5	+3
Lal. 45028	7.8	K2	0.500	+ .05	+6	+7
Brad. 3077	5.6	K7	2.105	+ .16	+7	+5
Lal. 45755	7.6	G8	0.670	+ .04	+7	+10
Boss 6016	5.9	G8	0.300	....	−5	−3
B.D. +58°2605	7.5	K1	1.080	+0.07	+7	+8
Boss 6123	5.8	K0	0.020	....	−2	−2
Boss 6176	5.8	G7	0.018	....	−2	−2

The difference, however, in the relative intensity of the violet portion of the continuous spectrum at center and limb as compared with the red portion, which is so marked a feature of the observations, would appear equally well on photographs taken with high and low dispersion.

MOUNT WILSON SOLAR OBSERVATORY
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THE SPECTROSCOPIC ORBIT OF RX HERCULIS DETERMINED FROM THREE PLATES WITH A NEW PHOTOMETRIC ORBIT AND ABSOLUTE DIMENSIONS[1]

By HARLOW SHAPLEY

The ordinary solution for the orbit of a spectroscopic binary involves six independent unknowns. These elements may be the period, an epoch, the longitude of periastron, the eccentricity of the orbit, the maximum apparent orbital velocity, and the radial velocity of the system with respect to the sun. Frequently the number of unknowns is reduced to four by the assumption of a circular orbit.

The complete solution for the elements of an eclipsing binary from its light-curve may involve as many as thirteen independent unknowns, but the precision of the photometric observations is seldom sufficient to permit the derivation of more than seven or eight of the quantities theoretically possible. The elements most generally obtained are the period, an epoch, the inclination, the relative light-emissions and dimensions of the two stars, the radius of the orbit, and occasionally the eccentricity and longitude of periastron.[2]

Solutions for orbits of double stars based upon variations in radial velocity, therefore, give four elements in common with solutions based on variations in apparent magnitude, and it is obvious that for any star which is both an eclipsing and a spectroscopic binary the existence of either solution greatly simplifies the derivation of the other, both from an observational and from a computational point of view. There is accordingly a particular

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 90.

[2] For some systems it is possible to determine also the ellipticity of the stars, the reflection effect, and the periastron effect, and to estimate the degree of darkening at the limb (*Astrophysical Journal*, **39**, 405, 1914). The apparent stellar magnitude at maximum is a thirteenth unknown, but except in systems of elliptical stars its determination is usually independent of the study of the light-curve.

interest and advantage in making photometric observations of certain spectroscopic binaries whose orbits are known,[1] and likewise in making spectroscopic observations of eclipsing binaries for which good light-curves exist or are possible; for not only is the problem of the second orbit in either case much simplified, but also the combination of the two sets of computed data affords knowledge of the absolute radii, masses, and densities of the component stars, and occasionally of the surface luminosities in terms of the sun.

Suppose the complete solution of a light-curve has given, among other quantities, four of the six unknowns generally derived from spectroscopic observations: P, the period; T, the epoch of principal minimum (or of node or periastron passage); e, the eccentricity; and ω, the longitude of periastron counted from the ascending node of the primary component. Then each determination of the radial velocity of the brighter star gives the linear relation

$$V_1=\gamma+K_1[\cos u+e\cos\omega] \tag{1}$$

between the two remaining unknowns—γ, the constant radial velocity of the center of mass of the system, and K_1, the semi-amplitude of the velocity variation. (u, the argument of the latitude, is readily obtained from tables when P, e, and ω are known.) If the lines in the spectrum of the second component are measurable, then

$$V_2=\gamma+K_2\,[\cos u+e\cos(\omega+\pi)] \tag{2}$$

and in both cases the last term in the brackets drops out for orbits circular or nearly so. We have then simply

$$V=\gamma+K\sin\theta \tag{3}$$

where θ is the phase angle from minimum. It is evident that a very few spectrograms made near greatest elongation, giving a large coefficient of K, will suffice, whether the orbit is circular or eccentric, to obtain γ and K with average precision, hence to determine completely the spectroscopic orbit. The following discussion shows what use we can make of the measures of a few lines on three plates

[1] See discussion of this phase of the combination by Stebbins, *Astrophysical Journal*, 34, 105, 1911.

of RX Herculis, an eclipsing binary the solution of whose light-curve eliminates four unknowns from the spectroscopic problem.

THE PHOTOMETRIC OBSERVATIONS

A new light-curve of RX Herculis is derived below from two series of observations, one made at Harvard and one at Princeton, neither of which has as yet been published. A long series of visual estimates of this star was obtained by Luizet during the years 1899–1904.[1] Especial care had been taken in making the observations, with the result that the light-elements derived from them are very accurate. The light-scale is not photometric, however, and it is not advisable to put great reliance upon the range of variation or upon the shape of the curve. The period given by Luizet is $0^{d}.889288$, and the range of variation is $0^{m}.63$, with the maximum at the seventh magnitude, approximately. The maximum light had been observed more than 300 times and there was no evidence of ellipticity or of a secondary minimum.[2]

The preliminary solution of Luizet's light-curve by the present writer was indeterminate. It was found that either the period must be double the given value with primary and secondary eclipses indistinguishable, or the period must be as given and the system supposed to consist of a small bright star with a faintly luminous, larger companion. The shape of the light-curve as derived from Luizet's measures contradicted the first supposition and the absence of a shallow secondary minimum contradicted the second. As the star is bright enough for spectroscopic work, it was considered of sufficient importance to justify a thorough photometric study, and accordingly it was put on the list of stars to be observed by the writer with the polarizing photometer of the Princeton University Observatory. The observations, which are published in detail elsewhere, are discussed briefly in the following paragraphs.

[1] The observations and light-curve are published in *Astronomische Nachrichten*, **168**, 283, 1905, and *Bulletin astronomique*, **22**, 232, 1905; see also *Astronomische Nachrichten*, **165**, 183, 1904; *Astronomical Journal*, **22**, 162, 1902; *Popular Astronomy*, **21**, 142, 1913.

[2] In conspicuous disagreement are the results deduced by Yendell from visual estimates, *Astronomical Journal*, **22**, 162, 1902.

The comparison star used in observing RX Herculis is a=B.D. +12°3546 ($7^{m}.7$). The observations were unusually difficult on account of the brightness of the two stars, and because many of the measures were made when the field was very low. Of the total of 133 sets (each of 16 comparisons) 63 were made during changing light. The 70 sets outside of eclipse, in agreement with Luizet's results, show no deviation from constant light. Hence the shorter period, which demands a secondary at the phase of 0.445 days, becomes doubtful. Moreover, an examination of the observations during changing light shows that the branches are not so steep as measured by Luizet and, therefore, that the double-period solution for the orbit is now suitable.

An investigation of the depths of alternate minima did not reveal definitely, so far as my observations were concerned, any difference larger than may be accounted for by errors of observation, but the data bearing on the point were rather weak. There was an indication that the minima at even epochs were slightly deeper, but it was not until the Harvard observations were examined that the difference was established beyond doubt. Consequently the photometric orbit already published is based upon an assumption of eclipses equally deep.[1] The existence of unequal minima shows at once that only one-half a revolution period elapses between successive eclipses. A third point in favor of the double value of the period is the spectroscopic evidence obtained at the Yerkes Observatory. The lines due to both components are visible on the plates made in 1905.[2] The recent measurement of these plates has furnished the data upon which the present spectroscopic orbit is based. They will be discussed on a later page.

A new mean epoch of heliocentric minimum, derived from the curve of the several partially observed eclipses, is J.D. 2419-658.5882, G.M.T. This time is eight minutes in advance of that predicted and, assuming that no secondary oscillations exist, furnishes a small correction to Luizet's period. The new light-elements are:

$$\text{Primary Minimum} = \text{J.D. } 2419658.5882 + 1^{d}.7785740 \text{ E.}$$

[1] *Astrophysical Journal*, 38, 163, 1913.

[2] *Ibid.*, 22, 215, 1905.

The weighted mean of the maximum light measures is $a-v=0^{m}.877 \pm 0^{m}.004$, the probable error of a single observation being $\pm 0^{m}.033$.

Through the kindness of Professor Pickering the proof sheets of a series of 127 observations of RX Herculis, made at Harvard by Professor Wendell during the years 1906–1910, have been furnished me in advance of publication in *Harvard Annals*, **69**, Part II. The measures were made with a polarizing photometer identical with the one used at Princeton, and as each set consists also of sixteen comparisons, the combination with equal weight of the observations of the two series is entirely justifiable. The maximum light, according to 23 observations by Wendell, is $7^{m}.07 \pm 0^{m}.006$, and the probable error of a single observation is slightly less than $\pm 0^{m}.03$. For combination with the Harvard results all the Princeton magnitudes were reduced so that the maximum magnitude should have the above value.

The preliminary plot of Wendell's observations confirmed the correction to Luizet's light-elements, but in addition showed an outstanding displacement of the minimum. This latter difficulty was found traceable to the chance adoption without alteration of the originally published initial epoch, which was referred to the Paris meridian, while the observation times are all referred to Greenwich. Two positive corrections have been applied, therefore, to the phases published at Harvard: one of ten minutes to reduce to Greenwich time, and the other increasing with the time from four to seven minutes to allow for the revision of the period.

With these adjustments the two sets of observations are found to be in extremely good agreement. The difference in the depths of successive minima of nearly a tenth of a magnitude is shown distinctly. In primary minimum there are 38 observations from Harvard and 41 from Princeton; in secondary, 66 observations from Harvard and 25 from Princeton. In the following tables of normal magnitudes the observations in the two minima are collected in order of phase into groups of five.[1] The phases refer to the middle epochs of the two minima. The third and fourth

[1] Half-weight is given to two Princeton observations made under very unfavorable conditions. The phase of one Harvard observation is arbitrarily altered because of the evident impossibility of the recorded time of observation.

columns give the number of observations contributed from Harvard and Princeton, respectively, to each normal. The last two columns contain the residuals from theoretical light-curves computed on the

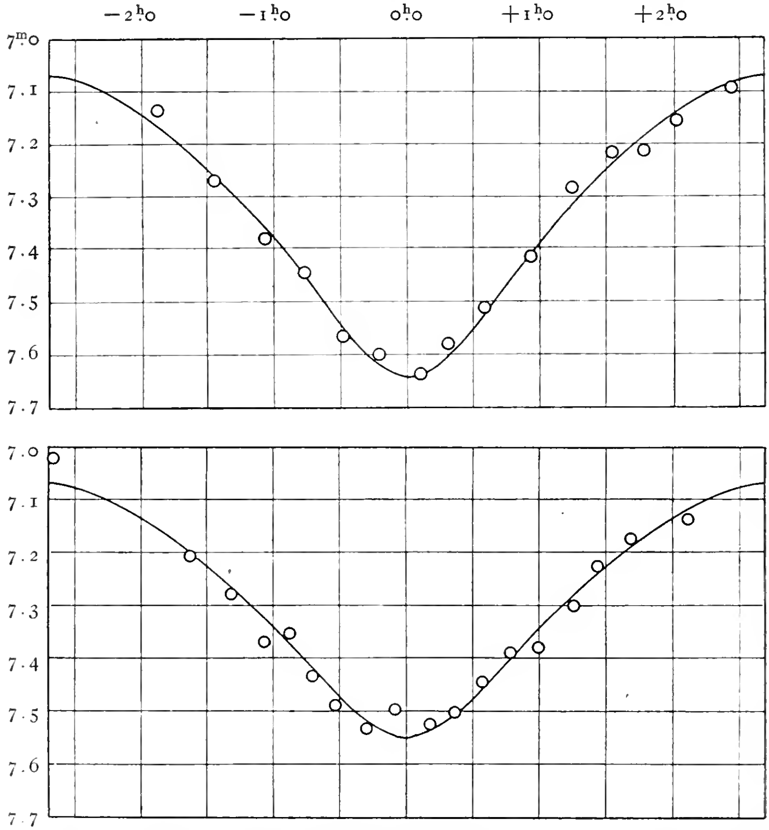

FIG. 1.—The mean light-curve of RX Herculis; primary minimum above, secondary minimum below.

two hypotheses of uniformly luminous disks and of disks completely darkened at the edge. In Fig. 1 are given the normal points and the computed uniform light-curves, with the primary minimum at the top of the diagram.

THE SPECTROSCOPIC OBSERVATIONS

The total available spectroscopic data for RX Herculis consist of four spectrograms, made with the Bruce spectrograph at the

Yerkes Observatory. Because of the faintness of the star and the nature of its spectrum, the investigation of its orbit has not hitherto

TABLE I

NORMAL MAGNITUDES OF RX HERCULIS NEAR PRIMARY MINIMUM

No.	Phase	No. Observations		Magnitude	O−C	
		Harvard	Princeton		Uniform	Darkened
1	$-1^h53^m.0$	3	2	7.136	$-0^m.03$	$-0^m.01$
2	−1 27.1	0	5	7.266	+ .01	+ .02
3	−1 4.6	2	3	7.380	+ .01	+ .01
4	−0 47.4	2	3	7.442	− .01	− .02
5	−0 29.8	3	2	7.564	+ .02	+ .01
6	−0 13.2	2	3	7.600	− .02	.00
7	+0 5.0	3	2	7.636	.00	+ .02
8	+0 18.0	2	3	7.578	− .03	− .02
9	+0 34.8	2	3	7.512	− .01	− .02
10	+0 55.4	3	2	7.414	.00	.00
11	+1 14.4	3	2	7.284	− .03	− .04
12	+1 32.0	4	1	7.216	− .03	− .01
13	+1 46.8	3	2	7.212	+ .03	+ .04
14	+2 0.6	2	3	7.154	+ .01	+ .03
15	+2 26.2	3	2	7.096	+ .01	+ .02
16	+2 58.5	1	3	7.058	−0.03	−0.03

TABLE II

NORMAL MAGNITUDES OF RX HERCULIS NEAR SECONDARY MINIMUM

No.	Phase	No. Observations		Magnitude	O−C	
		Harvard	Princeton		Uniform	Darkened
1	$-2^h40^m.0$	0	5	7.022	$-0^m.05$	$-0^m.05$
2	−1 37.8	3	2	7.206	+ .01	+ .03
3	−1 18.4	4	1	7.280	+ .01	+ .02
4	−1 3.8	4	1	7.370	+ .04	+ .03
5	−0 53.6	5	0	7.352	− .02	− .03
6	−0 42.8	3	2	7.436	+ .02	+ .01
7	−0 31.8	4	1	7.490	+ .02	+ .01
8	−0 18.0	4	1	7.536	+ .02	.00
9	−0 5.6	4	1	7.498	− .05	− .06
10	+0 10.8	4	1	7.522	− .01	− .03
11	+0 21.8	4	1	7.502	.00	− .02
12	+0 34.4	3	2	7.444	− .02	− .03
13	+0 47.2	4	1	7.394	− .01	− .02
14	+0 59.8	3	2	7.380	+ .03	+ .03
15	+1 14.8	4	1	7.302	+ .02	+ .03
16	+1 26.6	3	2	7.226	− .01	.00
17	+1 41.2	4	1	7.174	− .01	.00
18	+2 6.3	6	0	7.138	+0.02	+0.03

been attempted, and only a short note has been published in general description of the spectral lines.[1] At my request Professor Frost very kindly consented to measure the plates, as far as might be possible, and has recently communicated to me his results. Upon one of the plates the lines were too indefinite to permit a quantitative estimate of the velocity. For the other three the results are as tabulated in Table III. Concerning these measures Professor Frost writes:

> For the first two of the plates the results are clear, and have some fair degree of reliability, but I would not affirm that they were nearer than 10 km of the truth. . . . . I have re-examined the plates with a view of distinguishing between the components in intensity. The slightly less intensity of the broader component makes estimation difficult. On Plate 558 at λ 4481, broader component (toward red) showed slightly fainter than narrower component. The same remark refers to Plate 562. On Plate 839 the width and intensity of the components at λ 4481 were thought to be equal and the same for the other lines on that plate.

TABLE III

Plate No.	Date	G.M.T.	Primary Component		Secondary Component	
			Velocity	No. Lines	Velocity	No. Lines
558......	1905, July 16	16^h28^m	+84 km	2	−130 km	2
562......	1905, July 21	17 1	+42	3	−74	4
839......	1906, Sept. 8	15 25	−108	3	+75	2

On plate 839 the measure of one component of the line at λ 4481 gave a velocity of +188 km. The note is added:

> I cannot well account for the discrepancies in the positive values, but the very large positive displacement at λ 4481 is evident. The two other positive lines gave values of +69 and +82 km.

As this large positive value is in glaring disagreement with all the other measures on all the plates, there is nothing to do but neglect it completely.

THE PHOTOMETRIC ORBIT

Although a fairly good photometric orbit of RX Herculis is already available,[2] it was thought best to revise the solution because

[1] *Astrophysical Journal*, 22, 215, 1905.

[2] *Ibid.*, 38, 158, 1913.

of the additional photometric material from Harvard. The computations have followed the usual lines,[1] except that in certain details the spectroscopic data have influenced the choice between possible, ambiguous interpretations of the light-variations. The revised light-curve has shown definitely that the minimum at the even epoch is the deeper. The star approaching after that eclipse, according to the estimates of Professor Frost, is probably somewhat the fainter. At first glance there may appear to be a contradiction here. In general at the deeper minimum the star of greater surface intensity is eclipsed,[2] and almost invariably in practice the component with greater surface brightness has the greater total brightness.[3] If, however, the more intense star is small enough relative to its companion, it may have less than half the light and accordingly show fainter spectral lines. It is found from a study of the light-curve that RX Herculis is one of these rare exceptions and that the condition—fainter component smaller and with greater surface intensity—is not only possible but is required by the most probable solution.

Let us consider first the orbit from the standpoint of uniformly luminous disks. The range of variation at primary minimum is $0^{m}.57$ and at secondary $0^{m}.48$, corresponding to losses of light in terms of the total of $1-\lambda_p=0.408$ and $1-\lambda_s=0.357$, respectively.[4] The solution of the light-curve at the two minima gave $\chi\ (k,\ \alpha_0,\ \frac{1}{4})=2.01$, with an uncertainty of ± 0.01. With the above value of the losses of light this allows an indeterminateness of small range in the values of k and α_0 (the ratio of radii and fraction of eclipse) when computed from the relation $\alpha_0=1-\lambda_1+\dfrac{1-\lambda_2}{k^2}$. If we assume

[1] A detailed discussion of the method of solving for the orbital elements on the basis of two partial eclipses is to be given in *Contributions from the Princeton University Observatory*, No. 3, and an application is there made to the light-curve of RX Herculis.

[2] This does not necessarily hold if the orbit is eccentric and the inclination differs from 90°. For RX Herculis, however, we assume a circular orbit; later this assumption is proved.

[3] There are only three or four exceptions out of a hundred eclipsing binaries.

[4] The notation and formulae used in the present paper are essentially the same as those in former discussions of eclipsing binaries published in the *Astrophysical Journal* during the last two years.

that the large star is in front at primary minimum, k must lie between unity and 0.87; if we assume the small star to be in front at primary minimum, k must lie between unity and 0.98 or between 0.87 and 0.80, and for the last mentioned value the secondary minimum would be grazing total. The foregoing values of k are all possible without misrepresenting the curve; the best value, however, appears to be $k=0.90$ (small star eclipsed at primary) to which corresponds $\alpha_0=0.85$ and $\chi(k, \alpha_0, \frac{1}{4})=2.010$.

Suppose that from among the several possible values of the ratio of radii we choose $k=1.00$; then $\alpha_0=1-\lambda_p+1-\lambda_s=0.765$. The light of the star eclipsed at primary is then 53.3 per cent of the total, and the component approaching after primary eclipse would be fifteen-hundredths of a magnitude brighter than its companion, rather than fainter as indicated by the spectrograms. Obviously, then, the star in front at the primary eclipse must be the larger. If $k=r_2/r_1=0.936$, the components would be just equal in brightness. For the best values of the last paragraph ($k=0.90$, $\alpha_0=0.85$), however, the condition is attained that is in complete accordance with the light-curve and the spectrograms, for then the light of the star eclipsed at primary minimum is 48 per cent of the total; that is, the more intense surface, being the smaller, is approximately a tenth of a magnitude fainter. It is possible also that the difference in breadth of the spectral lines may be attributed to the inequalities of the components.

The solution for the elements on the darkened hypothesis gives results differing but little from those derived on the foregoing assumption. The computations have been much more difficult, however, because for unequal stars the theoretical light-curves at the two minima differ considerably from the uniform, and perceptibly from each other. The ranges of variation at the two minima were altered for this reason to $0^{m}.55$ and $0^{m}.49$, respectively. There was again a small indeterminateness in the solution for k and α_0', and therefore the same division of light was assumed as was derived before. For $L_2=0.48$, $L_1=0.52$, the ratio of the stars by chance came out the same. Their dimensions also are but little changed from the uniform values, chiefly because the computed duration of darkened eclipse is only three minutes longer

than for the uniform orbit. There is much less uncertainty in the photometric orbit of this star because of the indecision between the darkened and uniform hypotheses than is usually the case.

Light-curves have been computed for the two eclipses in both cases and the residuals from them are given in the last two columns of Tables I and II. The average deviation during changing light is $\pm 0\overset{m}{.}018$ for the uniform solution and $\pm 0\overset{m}{.}020$ for the darkened. The residuals are slightly smaller in primary than in secondary minimum. The representation of the observations in the minima is approximately of the same accuracy as during maximum light, and no attempt further to improve the accordance of theory and observation is justified. A shift of the epoch of secondary by a minute or two would reduce the average deviation slightly and would break up the apparently systematic nature of the run of residuals, but the adjustment would be within the possible errors of the observations and would not affect the computed elements.

TABLE IV

ELEMENTS OF THE PHOTOMETRIC ORBIT OF RX HERCULIS

	Uniform	Darkened
Ratio of radii, k	0.90	0.90
Fraction of eclipse, α_0, α_0',	0.85	0.828
$p(k, \alpha_0)$, $p(k, \alpha_0')$	−0.728	−0.691
$\sin^2 \theta'$	0.1411	0.1476
Semi-duration of eclipse, t'	2^h37^m	2^h40^m
Radius of brighter star, r_b	0.201	0.206
Radius of fainter star, r_f	0.181	0.185
Least apparent distance of centers, $\cos i$	0.069	0.078
Inclination of orbit plane, i	86°0	85°5
Light of brighter star, L_b	0.52	(0.52)
Light of fainter star, L_f	0.48	(0.48)
Ratio of surface intensities, $\frac{J_f}{J_b}$	1.14	1.14
"Equal-mass" density		
Bright star, $\bar{\rho}_b$	0.26	0.25
Faintstar, $\bar{\rho}_f$	0.36	0.34

The two sets of elements are given in Table IV. The epoch and period have been given on a preceding page. The ellipticity of the stars is negligible according to the maximum light-measures. The eccentricity of the orbit is discussed in the following paragraphs.

DISCUSSION OF ECCENTRICITY

In the solution for the elements of the photometric orbit the apparent equidistance, equal widths, and symmetry of the two minima have been accepted as sufficient proof that the orbit is circular. It may be of interest to see how great an uncertainty there is in this assumption. The two components of the eccentricity, $e \cos \omega$ and $e \sin \omega$, are determined independently of each other from the light-curve, the former with high precision from the displacement of the secondary minimum from the midpoint between the primaries, and the latter indirectly, and with less ease and precision, from the relative durations of the various phases of the two eclipses. Considering first the transverse component,

$$e \cos \omega = \frac{\pi}{P(1+\operatorname{cosec}^2 i)}\left(t_1 - t_2 - \frac{P}{2}\right) = 0.88 \times (\text{disp. in days}) \tag{4}$$

Inspecting the light-curve, we find that the combined uncertainty in the epochs of the two minima cannot exceed five minutes and is probably less than three. A change of epoch by as much as five minutes would nearly double the average value of the residuals. Hence $e \cos \omega$ is certainly less than ± 0.003.

To determine strictly the uncertainty in assuming $e \sin \omega = 0$, we should make independent solutions of the light-curve of each minimum, using only the depth of the other to make the derivation of the elements complete. Then, with close approximation,

$$e \sin \omega = \frac{r_1'' - r_1'}{r_1'(1.33 + 0.67 \cos \theta')} \tag{5}$$

where r_1' and r_1'' are the radii of the larger star derived from the solution of the primary and secondary minimum, respectively, and θ' is the phase angle of the end of eclipse. It is sufficient for the present purpose, however, to notice that when the inclination is 90° and $k=1$ (both of which conditions are approximately fulfilled in this and similar cases), the uncertainty in the radii is one-half that in θ'. Again referring to the light-curve, we find that the uncertainty in the adopted values for the semi-duration of eclipses is less than ± 4 minutes at the primary, and less than ± 5 minutes at the secondary; that is, since $t = P\theta/2\pi$, θ' must lie within

± 0.010 for one eclipse and ± 0.012 for the other of the values adopted. Hence $r_1' - r_1'' < \pm 0.011$, and since the radius of the larger star is approximately 0.20 we conclude from equation (5) that $e \sin \omega < \pm 0.03$. Combining this result with that obtained for $e \cos \omega$, the eccentricity is found to be not larger than 0.03. Its adopted value is zero and the probable error is estimated to be less than ± 0.02.

THE SPECTROSCOPIC ORBIT

The discussion up to this point has disposed of the elements P, T, e, and ω. It remains only to derive γ and K from the velocity variations by means of equation (3), and the spectroscopic orbit is complete. Using the light-elements given on a previous page, the phases of the middle of the exposure times have been computed. They are given in Table V, column 4, expressed in mean longitudes and referred to the preceding principal minimum. The primary star is taken to be the one eclipsed at secondary minimum.

TABLE V

SPECTROSCOPIC OBSERVATIONS OF RX HERCULIS

Plate No.	Julian Day and G.M.T.	Epoch	Phase	Primary Component		Secondary Component	
				Observed Velocity	O−C	Observed Velocity	O−C
558......	2417043.686	−1473	279°.2	−130 km	−7 km	+ 84 km	−2 km
562......	2417048.709	−1468	215.8	− 74	+6.5	+ 42	−1.5
839......	2417462.642	−1235	120.0	+ 75	+1.5	−108	+2.5

Plotting the observed velocities against the phases, we have Fig. 2. A preliminary solution shows that it is possible to assume the components equal in mass and be well within the probable accuracy of the observations. Then $K_1 = K_2$ and we have six equations for the determination of γ and K. Their solution gives:

$$\gamma = -18.5 \pm 1.4 \text{ km}$$
$$K = 106 \pm 1.7 \text{ km}$$

The residuals from the velocities computed with these elements are given in Table V, and, so far as they go, give ± 3.5 km for the probable error of the mean of the measures on one component on

one plate. This accuracy is even more than was hoped for, considering the faintness of the star and the nature of its spectrum. It is to be noted, however, that the plates were purposely made near times of greatest separation of the lines and troubles from blends were thereby avoided.

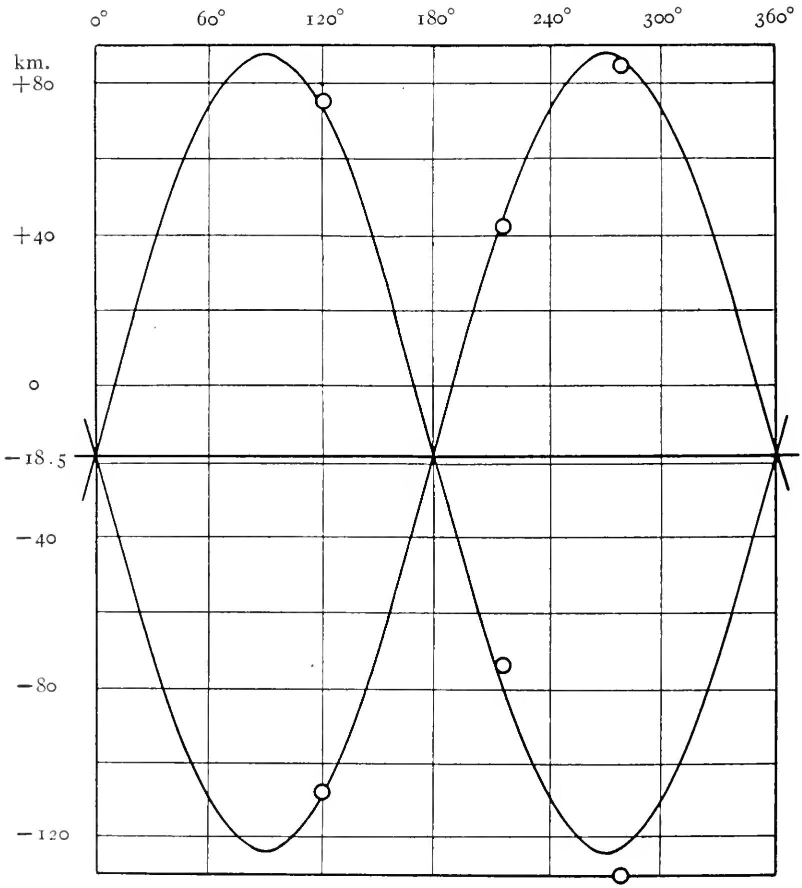

FIG. 2.—The velocity-curve of RX Herculis

It is evident from the velocity-curve that the residuals can still be reduced slightly by assuming the masses not equal. There is an indication that the secondary star, that is, the one with 48 per cent of the light but with the greater surface intensity, is a trifle the more

massive. But as is well known, the fainter companions of all spectroscopic doubles have smaller relative masses. This is without exception, as far as the writer knows. The above condition, then, would be in disagreement with the general rule. But it is at the same time true that, in all cases where such information is available, the star of least surface brightness is the one of smaller mass (for small total light and small surface luminosity go together in general). That would be in agreement with the suggested difference in mass of the components of RX Herculis, but the point is of little importance in this case of nearly equal stars. The masses are without doubt sensibly equivalent and will be so considered.

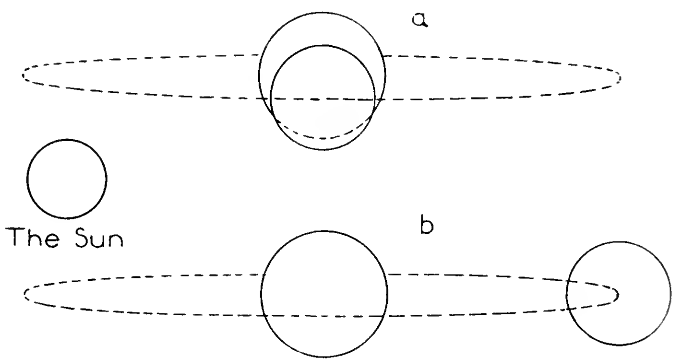

FIG. 3.—The system of RX Herculis compared with the sun

a, at secondary minimum
b, at greatest elongation

The following are the adopted elements of the spectroscopic orbits:

$P = 1.7785740$ days
$\mu = 202^{\circ}.4093$
$T =$ J.D. 2419658.5882 G.M.T. (primary minimum)
$e = 0.00 \pm 0.02$ (estimated)
$K_1 = K_2 = 106 \pm 1.7$ km
$\gamma = -18.5 \pm 1.4$ km
$a_1 \sin i = a_2 \sin i = 2{,}590{,}000$ km
$m_1 \sin^3 i = m_2 \sin^3 i = 0.88$

The probable errors of the apparent orbital radii and apparent masses are probably less than 2 per cent, but exact computation is not possible since the probable error of the adopted period has not been computed. The star is near the solar apex, and therefore the apparent motion of the center of mass of the system is evidently to be ascribed almost entirely to the motion of the sun. The mass of the system is smaller than we usually find for binaries of early spectral types.

ABSOLUTE DIMENSIONS

The spectroscopic and photometric elements may now be combined and we get the following comparison between RX Herculis and the sun (see Fig. 3):

	Un form	Darkened
Distance of centers in solar radii	7.47	7.47
Radius of brighter star in solar radii	1.50	1.54
Radius of fainter star in solar radii	1.35	1.38
Apparent distance of centers at middle of eclipse in solar radii	0.52	0.58
Mass of each star	0.89 ⊙	0.89 ⊙
Density of brighter star	0.26 ⊙	0.25 ⊙
Density of fainter star	0.36 ⊙	0.34 ⊙

From the description of the spectrum of RX Herculis in *Astrophysical Journal*, **22**, 215, it should evidently be classed as B9p, the anomalous character being the simultaneous presence of λ 4481 and the faint helium lines. If we knew the relation between the luminosity of its surface and that of a G-type star, we could at once deduce its parallax, since we know the stellar magnitude and the relative surface area of the star with respect to the sun. Or, vice versa, if the parallax were measured directly we could find the relation between the surface luminosities.

Assuming various values of the relative brightness of unit surface areas and expressing this quantity in stellar magnitudes (taking the solar surface as the standard), the computation for the parallax has been made, using the data derived for the brighter component (magnitude 7.78), with the results given in the following table:

Surface luminosity with respect to the sun........	$-3^{\mathrm{m}}.0$	$-2^{\mathrm{m}}.0$	$-1^{\mathrm{m}}.0$	$0^{\mathrm{m}}.0$
Corresponding brightness per unit surface area....	$15.8\odot$	$6.3\odot$	$2.5\odot$	$1.0\odot$
Absolute magnitude.......	$0^{\mathrm{m}}.82$	$1^{\mathrm{m}}.82$	$2^{\mathrm{m}}.82$	$3^{\mathrm{m}}.82$
Parallax..................	$0''.0041$	$0''.0064$	$0''.0102$	$0''.0162$
Total light of brighter star.	$37.3\odot$	$14.9\odot$	$5.9\odot$	$2.4\odot$

The four assumptions with regard to the surface luminosity correspond roughly to what we judge from other sources to be the relative intensities for spectral types B, A, F, and G, respectively. We should expect, then, that the parallax of RX Herculis is of the order of $0''.006$ and that the total light-emission of the system is thirty times that of the sun.

Among the faint eclipsing binaries similar to RX Herculis are the following, for which in a corresponding manner spectroscopic material of considerable value might be obtained without great difficulty:

	Magnitude	Spectrum
U Ophiuchi	5.7	B8
Y Cygni	6.9	A
Z Herculis	7.1	F
RR Centauri.....	7.4	F
W Ursae Majoris	7.9	G

SUMMARY

1. The solution for the elements of the spectroscopic orbit of a faint star is shown to be easily possible when only a few measures of the radial velocity have been obtained, provided that the system is also an eclipsing binary and provided that the period, the epoch of minimum, the eccentricity, and the longitude of periastron have been derived from the light-curve.

2. A new photometric orbit of RX Herculis has been computed from unpublished observations obtained at Harvard and Princeton (Table IV). Alternate minima are found to differ in depth by nearly a tenth of a magnitude. The stars are nearly equal in size and are sensibly spherical. Their surfaces are separated by three times the radius of the larger star.

3. From measures of the lines on three plates made with the Bruce spectrograph at the Yerkes Observatory, it has been possible to derive very satisfactory spectroscopic orbits of both components.

4. The combination of elements from the photometric and spectroscopic orbits gives the actual dimensions of the stars. (Such information is reliably known for only two other binary systems.) The mass is unusually low for an early-type star. The density is slightly above normal. The volume of both components is six times that of the sun.

5. With a reasonable assumption regarding the surface luminosity, the parallax of the system is found to be $0\overset{''}{.}006$. The total light of the system is likely more than thirty times that of the sun.

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ON THE DISTRIBUTION OF ECLIPSING VARIABLE STARS IN SPACE

By HENRY NORRIS RUSSELL and HARLOW SHAPLEY

When the elements of an eclipsing binary, and its observed magnitude and spectral type, are given, it is possible to make a very fair estimate of its distance. Since the photometric data give the relative brightness of the components, the stellar magnitude m which the brighter star of the system would appear to have if it stood alone can be found at once. If we can estimate the absolute magnitude M of this star, the parallax π may be found from the relation

$$M=m+5+5\log\pi \tag{1}$$

Kapteyn's definition of the absolute magnitude, corresponding to a parallax of 0."10, is here used, so that the sun's absolute magnitude is about 4.75. If the radius and surface brightness of the star under consideration, in terms of those of the sun, are R and J, we shall have

$$M=4.75-5\log R-\frac{5}{2}\log J \tag{2}$$

(The logarithms here, as elsewhere in this paper, are common logarithms with base 10.)

The elements of the eclipsing system give us also the ratio r of the radius of the brighter component to the mean distance of the

components.[1] If the mass of the system is 2μ times the sun's mass, and the period is P days, the mean distance, taking the sun's radius as unity, will be

$$5.29\ P^{\frac{2}{3}}\ \mu^{\frac{1}{3}}$$

and we may write

$$R = 5.29\ rP^{\frac{2}{3}}\ \mu^{\frac{1}{3}}$$

Setting for brevity

$$5.29\ rP^{\frac{2}{3}} = A \tag{3}$$

we find

$$M = 4.75 - 5\log A - \frac{5}{3}\log\mu - \frac{5}{2}\log J \tag{4}$$

The quantity A may be derived from the known elements of the eclipsing system.[2] The last two terms are usually unknown; but their probable values may be estimated if the spectral type of the star is known.

Consider first the surface brightness J. This is probably very closely related to the color-index of the star, and hence to its spectrum. There are two ways in which this relationship may be determined quantitatively with tolerable accuracy.

(*a*) On the assumption that the stars radiate like black bodies, the difference of surface brightness between any two stars (in stellar magnitudes) should be very nearly equal to the product obtained by multiplying their relative color-indices by the ratio of the mean effective photographic wave-length to the difference between the mean effective visual and photographic wave-lengths. From Parkhurst's data,[3] it appears that this ratio is about 3.8 in the case of the system of color-indices now in common use, based on extra-focal photographic work.

This method may, however, be expected to give somewhat too large values for the difference of surface brightness, for it is very

[1] This is called r_b in the table of elements (*Astrophysical Journal*, **38**, 162, 1913), to distinguish it from r_f, the radius of the fainter component. This distinction is unnecessary here.

[2] The density assigned to the brighter component of each system in the table just referred to is equal to A^{-3}.

[3] Russell, *Science*, N.S., **38**, 646, 1913.

probable that a part of the photographic faintness of the redder stars arises from increased intensity of both line and general absorption in the region of short wave-lengths.

(*b*) An attempt to determine the relative surface brightness of stars of the different spectral classes in quite a different way, by comparison of data obtained from visual and eclipsing binaries, has recently been described in some detail by one of the writers.[1] It gives results only for the whiter stars, but these should be of considerable accuracy.

As regards the mean masses of the stars of the various spectral classes, the principal data may be summarized briefly as follows:

(*c*) For 12 spectroscopic binaries of Class B, with both components bright, the mean mass of a system is 16 times that of the sun. There are good reasons to suppose that the methods of observation have in this case favored stars more massive than the average.

The mean mass of 8 visual binary systems of known parallax, and spectra A to F5, is 4.5 times that of the sun, and that of 10 systems of spectra F8 to K is 2.0. The few spectroscopic binaries of these classes whose actual masses can be determined give results of the same order of magnitude.

(*d*) A determination of the mean masses of binary stars of the various spectral classes, based on the data for more than 550 stars, on the assumption that the brighter components of these double stars are, on the average, equal in brightness to the mean of all stars of similar spectral type, is given in the paper already referred to. The brighter and fainter groups (giant and dwarf stars) among the redder types appear to be of different mass, and are treated separately.

The results of these various determinations are summarized in Table I. The second line gives the relative color-indices for the various spectral classes (taking for the moment the sun as a standard); the third, the values of the surface brightness in stellar magnitudes (i.e., of $-\frac{5}{2} \log J$) deduced from these by method (*a*); the fourth line, the independent results of method (*b*); and the

[1] *Popular Astronomy*, **22**, No. 6, June-July 1914.

fifth, the values adopted for the purposes of the present discussion. The next three lines give the values of $-\frac{5}{3}\log\mu$ derived

TABLE I

Spectrum...................	Bo	B5	Ao	Fo	Go	Ko	M
Color-index.................	−1.02	−0.88	−0.71	−0.41	0.00	+0.45	+0.91
$-\frac{5}{2}\log J$							
Method (a)...........	−3.9	−3.3	−2.7	−1.5	0.0	+1.7	+3.4
Method (b)...........	....	−3.2	−2.0	−1.1	0.0	....	>+2.5
Adopted..............	−3.6	−3.0	−2.3	−1.1	0.0	+1.2:	(+3.)
$-\frac{5}{3}\log\mu$							
Method (c)...........	−1.5		−0.6		0.0		....
Method (d). Giant stars	−0.8		−0.6	....	−0.8	−0.3	−0.3
Method (d). Dwarf stars	....	....	....	−0.1	+0.6	+0.9	+1.2
Adopted..............	−1.1	−0.8	−0.6	−0.3	0.0	0.0:	....
$4.75-\frac{5}{2}\log J-\frac{5}{3}\log\mu$							
Adopted..............	0.0	+0.9	+1.8	+3.3	+4.7	+5.9	....

from the discussions referred to under (c) and (d), and the following line the adopted values. Finally, the last line gives the adopted values of $4.75-\frac{5}{2}\log J-\frac{5}{3}\log\mu$ for each spectral class. If we call this quantity Q, then we have for any star, by (4),

$$M=Q-5\log A$$

With regard to the accuracy of the adopted values of Q, it seems probable, on the showing of the table, that they are usually within half a magnitude of the truth. Differences from the mean mass and surface brightness will cause larger deviations in the case of individual stars; but the probable magnitude of these deviations may be at least roughly estimated. For the 28 visual and spectroscopic binaries mentioned under (c) above, the differences between the individual masses, as derived from observation, and the means for the various groups correspond to an average deviation of ± 0.53 in $\frac{5}{3}\log\mu$. This includes very considerable effects of errors of observation, so that the real deviations must be decidedly smaller. The deviations of the surface brightness of individual stars from the mean for their spectral classes may be estimated

from the similar deviations in color-index. According to King,[1] the average observed deviation in color-index is ± 0.086 mag., including the effect of errors in both the visual and photographic magnitudes. Multiplying this by 3.8, we find ± 0.33 as the corresponding deviation in $\frac{5}{2} \log J$, which again, for the reasons just stated, is probably too great.

It would therefore appear that, for stars whose spectral class is accurately known, the average deviation from the mean values of Q can hardly exceed ± 0.50 mag.—which corresponds to an error of about ± 25 per cent in the estimated parallax. For faint stars, whose magnitudes have not been photometrically determined, and whose spectra can be only roughly classified, the errors may of course be much greater.

The stars whose parallaxes and distances are here estimated on the above principles are the 90 eclipsing variables for which elements have so far been computed. The elements of 86 of these have been published by one of us;[2] the others are added from later computations. Full details regarding all of these will appear in a forthcoming number of the *Contributions from the Princeton University Observatory*, and therefore only the essential facts are presented here.

Table II gives for these 90 stars (1) the name; (2) the magnitude m of the brighter component, obtained by correcting the observed magnitude of the system for the light of the fainter component. Magnitudes given in italics are photographic, and have been corrected for color-index in the ordinary way before the computations were made. Column (3) gives the spectral class, as determined at Harvard (except for the values in parentheses, which are rough estimates made in the manner explained below), and column (4) the quantity A derived from the elements of each system by means of equation (3). The elements computed on the assumption that the star-disks appear to be darkened at the limb have been used in all cases except that of X Carinae (for which they were not available); and in the case of ellipsoidal stars, the

[1] *Harvard Annals*, **59**, 175.

[2] Shapley, *Astrophysical Journal*, **38**, 162–168, 1913.

TABLE II

(1) Name	(2) m	(3) Sp.	(4) A	(5) λ	(6) β	(7) $1000\ \pi$	(8) R	(9) $R \cos \beta$	(10) $R \sin \beta$
U Cephei	6.9	A	2.15	91°	+19°	4″.4	750	710	+ 240
RZ Cassiopeiae	6.5	A	1.88	100	+ 9	6.0	540	530	+ 85
β Persei	2.3	B8	2.57	116	−14	26.2	125	120	− 30
RT Persei	10.8	F?	1.50	116	− 8	2.1	1580	1560	− 220
RW Tauri	8.1	B5	1.76	135	−17	2.1	1580	1510	− 460
β Aurigae	2.8	A	2.08	136	+12	30.0	109	106	+ 23
R Canis Majoris	5.6	F	1.93	198	− 0	18.2	180	180	− 0
V Puppis	4.7	B1	2.77	230	−10	4.6	720	710	− 125
S Cancri	8.1	A	2.28	175	+34	2.4	1360	1130	+ 760
S Velorum	8.2	A	2.11	238	+ 5	2.5	1300	1300	+ 110
W Ursae Majoris	8.7	G	0.79	125	+47	20.0	164	112	+ 120
ST Carinae	*9.5*	A	1.44	252	− 3	2.0	1600	1600	− 80
Z Draconis	10.5	(A5)	1.67	96	+45	1.6	2050	1450	+1450
SV Centauri	*9.1*	A	2.74	263	+ 1	1.2	2600	2600	+ 50
W Crucis	*9.0*	Gp	100.0	265	+ 4	0.19	17200	17000	+1200
RZ Centauri	*8.7*	A	3.70	272	− 2	1.1	2900	2900	− 100
SZ Centauri	*8.7*	A	3.85	278	+ 3	1.1	2960	2960	+ 160
RR Centauri	8.0	F	1.30	282	+ 3	8.7	375	375	+ 20
U Ophiuchi	6.3	B8	1.78	351	+20	6.3	520	490	+ 180
u Herculis	5.0	B3	2.45	24	+32	5.2	620	530	+ 330
RS Sagittarii	6.3	A	3.28	326	−10	3.8	860	850	− 150
V Serpentis	10.1	A	2.63	343	− 1	0.8	3900	3900	− 70
RX Herculis	7.8	B9	1.57	10	+ 9	3.6	900	890	+ 140
RX Draconis	10.9	F	1.33	56	−22	2.3	1420	1310	− 530
U Sagittae	6.5	B8	2.77	21	+ 2	3.6	900	900	+ 30
SW Cygni	9.2	A	2.42	49	+ 7	1.4	2400	2380	+ 290
UW Cygni	10.7	A?	2.37	48	+ 3	0.7	4700	4700	+ 250
ZZ Cygni	10.7	A?	1.40	51	+ 4	1.2	2800	2800	+ 200
W Delphini	9.5	A	2.56	30	−15	1.1	2850	2750	− 740
WZ Cygni	10.1	A	1.58	48	− 4	1.4	2360	2350	− 160
U Pegasi	9.9	F?	1.07	74	−45	4.6	710	500	− 500
RT Sculptoris	*9.7*	F	1.48	40	−87	4.0	820	40	− 820
RS Cephei	10.4	Ap	2.24	100	+23	0.9	3800	3450	+1480
ϵ Aurigae	4.0	F8p	116.0	130	+ 2	1.1	3000	3000	+ 100
TT Aurigae	9.1	A	1.88	135	+ 1	1.8	1800	1800	+ 30
Z Orionis	9.7	A	3.90	162	− 5	0.7	4900	4900	− 430
RW Monocerotis	9.0	A	1.78	171	+ 1	2.0	1640	1640	+ 30
RR Velorum	10.3	A	1.32	243	+13	1.5	2200	2150	+ 500
SS Carinae	*13.0*	(A)	2.15	257	− 2	0.26	12400	12400	− 430
SU Centauri	*8.8*	F2	4.11	254	+12	2.8	1180	1160	+ 250
SS Centauri	*8.7*	B9	2.07	273	− 2	1.8	1800	1800	− 60
δ Librae	4.9	A	3.59	316	+41	6.6	490	370	+ 320
U Coronae	7.7	A	2.44	17	+57	2.8	1190	650	+ 990
Z Herculis	7.7	F	1.49	9	+17	8.8	375	360	+ 110
RR Draconis	10.1	A2	1.39	60	+24	1.8	1800	1650	+ 730
RZ Ophiuchi	10.4	G8	10.1	6	+ 3	1.1	2860	2850	+ 150

TABLE II—*Continued*

(1) Name	(2) m	(3) Sp.	(4) A	(5) λ	(6) β	(7) $1000\,\pi$	(8) R	(9) $R \cos \beta$	(10) $R \sin \beta$
β Lyrae	3.9	B8	6.0	30°	+14°	5″.2	620	600	+ 140
U Scuti	9.8	A	1.94	349	− 8	1.3	2500	2450	− 350
Z Vulpeculae	8.1	A	2.48	27	+ 4	2.2	1480	1480	+ 100
WW Cygni	10.1	Ap	2.66	45	+ 5	0.8	3900	3900	+ 350
VW Cygni	10.5	A	2.57	40	− 1	0.7	4700	4700	− 80
RW Capricorni	9.5	A	2.74	354	−28	1.1	3100	2750	−1460
UZ Cygni	10.5	A	4.42	61	− 9	0.42	7900	7800	−1250
RT Lacertae	9.6	G5	4.45	61	−10	3.1	1050	1040	− 180
Y Piscium	9.0	A	2.28	62	−50	1.6	2050	1320	−1570
SY Andromedae	10.9	A?	2.37	84	−18	0.6	5200	4950	−1600
TV Cassiopeiae	7.4	B9	2.13	86	− 3	3.3	990	990	− 50
Z Persei	9.7	A	1.47	112	−15	1.8	1800	1740	− 470
RY Persei	8.3	A2	3.41	110	−10	1.7	1980	1950	− 340
ST Persei	10.0	A	2.09	116	−16	1.1	3000	2900	− 830
RV Persei	10.9	A	2.95	131	−11	0.5	6300	6200	−1210
RW Geminorum	10.0	A	2.65	155	+ 2	0.9	3750	3750	+ 130
Y Camelopardalis	9.9	(A)	2.44	105	+30	1.0	3300	2850	+1650
RU Cancri	10.4	(F)	1.42	169	+35	2.6	1240	1010	+ 710
RW Ursae Majoris	10.8	G?	1.42	110	+61	4.2	780	380	+ 680
TW Draconis	7.6	B9	1.92	66	+45	3.3	980	690	+ 690
RV Lyrae	11.8	A	1.56	33	+ 8	0.6	5200	5150	+ 720
RS Vulpeculae	8.0	A	3.61	24	+ 4	1.6	2050	2050	+ 140
TT Lyrae	9.6	A	2.43	42	+11	1.1	2900	2850	+ 550
SY Cygni	11.0	G5?	1.75	36	+ 3	4.2	800	800	+ 40
RR Delphini	10.9	A	1.99	27	−19	0.8	4300	4050	−1400
VV Cygni	13.0	(A5)	1.72	55	− 2	0.5	6800	6800	− 240
TT Andromedae	11.5	A	1.84	74	−13	0.6	5200	5100	−1180
SX Cassiopeiae	9.1	G3	13.8	84	− 7	1.1	2850	2800	− 350
RX Cassiopeiae	9.3	Ko	13.0	103	+ 9	1.6	2070	2050	+ 320
λ Tauri	3.9	B3	4.2	146	−28	5.2	620	550	− 290
RW Persei	*9.6*	A	2.27	126	− 4	1.2	2700	2700	− 190
RZ Aurigae	10.7	(A)	2.82	146	+ 3	0.6	5400	5400	+ 280
RR Puppis	9.9	A	2.01	223	− 7	1.2	2700	2700	− 330
X Carinae	8.6	A	2.45	242	−12	1.8	1800	1760	− 370
Y Leonis	9.5	A	1.47	171	+48	2.0	1650	1100	+1230
SW Centauri	*9.2*	A	1.67	265	+13	2.0	1650	1600	+ 370
SY Centauri	*10.1*	A	3.36	277	− 0	0.7	4950	4950	− 0
TX Herculis	8.7	A	1.07	34	+34	3.8	860	710	+ 480
SZ Herculis	9.8	(F)	1.23	25	+27	4.2	780	700	+ 370
RZ Scuti	7.6	B3	5.1	350	− 0	0.8	4100	4100	− 0
RZ Draconis	10.0	A	1.27	55	+26	1.8	1800	1620	+ 790
SX Sagittarii	*9.3*	A5	2.12	332	−14	2.4	1360	1310	− 330
RS Scuti	9.6	F	1.55	351	− 6	3.6	900	890	− 95
Y Cygni	7.6	A	1.83	45	− 7	3.8	860	850	− 105

shortest radius in the equatorial plane has been taken. In one or two indeterminate cases, the mean of the limiting possible values has been used in the calculations. This quantity A is the radius of the brighter component (taking the sun's radius as unity), divided by the cube root of the ratio of the mass of the system to twice the sun's mass. From the data of Table I it appears that a good approximation to the true radius may be obtained by increasing the tabular value by two-thirds of itself for spectrum B0, one-half for B5, one-third for A, and one-sixth for F. Columns (5) and (6) give the galactic longitudes and latitudes of the stars (taken in all but a few instances from the list in *Harvard Annals*, **56**, 187–191). The assumed position of the galactic pole is 12^h40^m, $+28°$. Column (7) gives the parallax computed by equations (1) and (4). To avoid the printing of needless ciphers, the values of $1000\,\pi$ are tabulated. Column (8) gives the corresponding distance R in light-years, and columns (9) and (10) the components of this distance, resolved parallel and perpendicular to the galactic plane passing through the sun.

The stars are arranged in four groups, according to the accuracy of their orbital elements, those in each group being in order of right-ascension.

In estimating the spectra of the seven stars which were too faint to be classified on the Harvard plates recourse was had to the fact that there is a marked correlation between the spectral type and the density (and hence the values of A). When A exceeded 2.0 the spectrum was assumed to be of Class A; when it was between 2.0 and 1.6, of Class A5; and when it was between 1.6 and 1.2, of Class F0. It should be borne in mind that the correlation here utilized, though fairly definite, is far from absolute, so that these estimates, and the numbers derived from them, are subject to considerable uncertainty.

The most conspicuous characteristic of the parallaxes estimated as in Table II is their extreme minuteness. The mean parallax of all the 90 stars comes out only 0ʺ.0033. Only four of them (β Aurigae, β Persei, W Ursae Majoris, and R Canis Majoris) have estimated parallaxes exceeding 0ʺ.01. The mean for the remaining 86 stars is 0ʺ.0023; 48 of these have estimated parallaxes

less than 0″002, and 19 of these are below 0″001. These small parallaxes are, however, confirmed by direct observation in the few cases in which observations have been made. The observed values, corrected for the probable parallax of the comparison stars, are given in Table III, with weights, assigned by the writer on the basis both of the probable errors found by the observers and of the general reliability of the methods of observation. The probable errors of the weighted means are derived from the residuals for the individual determinations in the ordinary way. The values estimated in the present paper are repeated for comparison.

TABLE III

Star	Observer	Parallax	Probable Error	Weight
β Persei	Chase	0″044	±0″025	2
	Russell	0.011	±0.025	2
	Flint	0.130	±0.026	1
Mean observed parallax		0.048	±0.019	......
Estimated parallax		0.026	......	......
β Aurigae	Flint (1st series)	0.000	±0.031	2
	Tikhoff	0.023	±0.019	4
	Flint (2d series)	0.034	±0.020	4
	Iewdokimow	0.150	±0.042	1
Mean observed parallax		0.034	±0.015	......
Parallax if a member of the Ursa Major group		0.019	......	......
Estimated parallax		0.030	......	......
Y Cygni	Slocum	0.005	±0.010	......
Estimated parallax		0.004	......	......

It would seem that direct observations for parallax are worth while, at present, only in the case of the four nearest systems, already mentioned; and even in these cases only the most precise methods, especially as regards freedom from systematic error, can be of any value.

The extremely small parallaxes derived for the general run of these variables may seem surprising, because they are less than the probable mean parallaxes of all the stars of similar magnitude. But evidence is gradually accumulating that the stars of the ninth magnitude and fainter are for the most part of the "later" spectral

classes, and, since the eclipsing variables are mainly of Class A, there is no reason to suppose them typical of the whole. They may very well be (as is assumed here) stars of greater actual luminosity, and hence remoter than the general run of those of similar visual magnitude.

It should, however, be expressly borne in mind that all the computations so far made depend upon the assumption that there is no loss of light by absorption in passing through interstellar space, even for thousands of years. If absorption of any kind exists, the estimated distances will be too great.

Except for the very remotest stars, the correction to the estimated parallaxes for absorption of light in space takes a remarkably simple form. Suppose that there exists a uniform absorption, amounting to d magnitudes for each distance of 32.6 light-years (corresponding to a parallax of 0".10). In the absence of absorption, the relation between the absolute and apparent magnitudes of any star would be

$$m = M - 5 - 5 \log \pi$$

but in the presence of absorption it will be

$$m = M - 5 - 5 \log \pi + \frac{d}{10\pi} \tag{5}$$

If now π' is the parallax estimated from the difference of the absolute and observed magnitudes, on the hypothesis that there is no absorption, we shall have

$$5 \log \pi' = 5 \log \pi - \frac{d}{10\pi}$$

whence

$$\pi' = \pi \cdot 10^{\frac{-d}{50\pi}} = \pi \cdot e^{-0.0461\frac{d}{\pi}} = \pi - 0.0461\, d + 0.00106\frac{d^2}{\pi} + \ldots \tag{6}$$

Hence, except for very small values of π, the correction for absorption can be very approximately made, by adding to the parallaxes computed on the assumption that space is transparent the quantity 0".0461 d. For example, King[1] has recently concluded from a study of the color-indices of stars of known parallax that the value

[1] *Harvard Annals*, **76**, 10, 1914.

of d (corresponding as above to a distance of 32.6 light-years) is 0.019 mag. for the visual rays, and 0.038 mag. for the photographic rays. If these values are accepted, the correction to be added to the larger parallaxes determined from observed visual magnitudes is $0''.0461 \times 0''.019$ or $0''.00088$, while for parallaxes determined from photographic magnitudes, it is $0''.0018$. For remote stars the correction is smaller, as is illustrated by Table IV, which is based upon the value $d=0.019$. It appears, therefore, that, with this value of d, the mere addition of $0''.0008$ to all the estimated parallaxes will accomplish the correction for absorption within $0''.0001$, except for a few of the remotest stars.

TABLE IV

$1000\,\pi$	$5 \log \pi$	$\frac{d}{10\,\pi}$	$5 \log \pi'$	$1000\,\pi'$	$1000\,(\pi-\pi')$
$10''.0$	-10.00	0.19	-10.19	$9''.16$	$+0''.84$
5.0	11.50	0.38	11.88	4.21	0.79
2.5	13.01	0.76	13.77	1.76	0.74
1.5	14.12	1.27	15.39	0.84	0.66
1.0	15.00	1.90	16.90	0.42	0.58
0.5	16.50	3.80	20.30	0.09	0.41

Other investigators, working upon the same assumption of a space-absorption varying inversely as the fourth power of the wave-length, have found smaller values of the coefficient d; for example, Kapteyn, for the visual rays, derives $d=0.0030$.[1] It is, however, not improbable that at least a part of any existing absorption arises from the presence in space of sensible particles of matter, whose influence is the same for all wave-lengths, and that the values of the absorption coefficient derived from the color-indices, on the assumption just mentioned are too small. King's value, 0.019, for d will therefore be employed in the present discussion as indicating the order of magnitude of the effect to be expected, if it really exists.

It appears then, that the introduction of corrections for such absorption would probably increase the mean parallax of the eclipsing variables here studied to little, if at all, above $0''.004$, or

[1] *Astrophysical Journal*, **30**, 399, 1909.

0."003, if the four nearest stars are rejected. The very great distances, exceeding 5000 light-years, which are computed above for certain stars, would, however, be greatly diminished. With $d=$ 0.019, the parallaxes and distances of the three remotest stars become: W Crucis, 0."00068, 4900 light-years; SS Carinae, 0."00078, 4200 light-years; UZ Cygni, 0."00102, 3200 light-years. It is noteworthy, too, that the estimates for the first two of these stars are based on photographic magnitudes. If King's value of d for the photographic rays, 0.038, is adopted for these stars, their estimated distances are reduced to 3100 and 2800 light-years; but, for reasons already stated, this may be going too far.

We may now consider the distribution of these eclipsing systems in space. Of the 90 stars, 64 lie between 0° and 180° of galactic longitude, and only 26 in the opposite hemisphere. This is obviously a result of the much more thorough study which has been given to the light-curves of northern stars, and has no cosmical meaning.

The distribution in galactic latitude is, however, significant. It is well known that the eclipsing variables are strongly concentrated toward the Milky Way. For the stars here investigated the distribution is as follows:

	Galactic Latitude					
	+90° to +30°	+30° to +10°	+10° to 0°	0° to −10°	−10° to −30°	−30° to −90°
Number..........	12	13	23	21	18	3

The deficiency near the southern galactic pole is doubtless due to the paucity of observations in this region, which would also explain the slight preponderance of northern latitudes (48 as against 42), but, even if allowance is made for this, the density of distribution in the heavens is some six times as great within 10° of the Galaxy as in the rest of the sky. When we pass from these apparent positions to the real co-ordinates in space, it appears at once that these stars are strongly concentrated near the galactic plane.

With the co-ordinates computed on the assumption that space is transparent, we find that 63 of the 90 stars are within 500 light-

years of the plane passing through the sun parallel to the galactic equator. Only 12 stars are more than 1000 light-years from this plane, and but three more than 1500 from it, the extreme distances being -1600 and $+1650$. Of the distances projected upon the galactic plane, however, 28 are less than 1000 light-years, 23 between 1000 and 2000, and 24 between 2000 and 4000, leaving 15 exceeding 4000 light-years.

This is in agreement with the accepted opinion that the region within which the stars are at all densely distributed extends much farther in the direction of the galactic plane than at right angles to it. The sun appears to be not far from the central plane of densest distribution. The mean value, with regard to sign, of $R \sin \beta$ is $+10$ light-years; but we have seen above that the observational preference for northern stars has discriminated against those in high negative galactic latitudes, there being only 3 between $-30°$ and $-90°$ against 12 between $+30°$ and $+90°$. The average distance of these 15 stars from the galactic plane is 820 light-years. If we assume that 9 similar southern stars have so far escaped the detailed observation necessary for the calculation of their orbits, but would be included if the whole heavens had been impartially studied, their inclusion would make the general mean for $R \sin \beta$ -65 light-years. It seems therefore probable that the sun is slightly, but sensibly, to the north of the median plane of the system of stars.

The average distance, regardless of sign, of the stars from the plane passing through the sun, is 436 light-years (which would be raised to 470 by the inclusion of additional southern stars as assumed above). The computed average distance from the galactic plane appears, however, to be to some degree a function of the distance projected on this plane, as is shown by Table V (in which

TABLE V

	Limits of $R \cos \beta$				
	0 to 1000	1000 to 2000	2000 to 4000	Over 4000	All Together
Number of stars	28	23	24	15	90
Mean of $R \cos \beta$	550	1470	2820	6550	2400
Mean of $R \sin \beta$ regardless of sign	256	485	444	685	436

all distances are given in light-years). The inclusion of more stars near the southern galactic pole (for most of which $R \cos \beta$ is relatively small) would raise the first of the quantities in the last line to about 370 light-years; but a decided progression would still exist. It does not appear very probable that the thickness of the region within which the stars lie actually increases with increasing distance from the sun; the alternative suggests itself that the distances of the remoter stars have been overestimated, as would be the case if their light suffered absorption in reaching us.

It seemed, therefore, desirable to recompute their positions, assuming a coefficient of absorption of 0.019 as discussed above. The resulting distribution may be summarized as in Table VI. The tendency toward increase of the computed thickness of the

TABLE VI

	Limits of $R \cos \beta$						
	0 to 500	500 to 1000	1000 to 1500	1500 to 2000	2000 to 2500	Over 2500	All Together
Number of stars........	14	23	17	18	9	9	90
Mean $R \cos \beta$..........	300	740	1230	1730	2280	3150	1350
Mean $R \sin \beta$ regardless of sign..............	218	318	270	298	246	302	280

starry region with increasing distance has now disappeared; but it would be premature to regard this as evidence of the reality of the absorption of light in space.

However this may be, it is evident that the extension of the region occupied by these stars is several times greater in all directions along the galactic plane than it is at right angles to it. In the latter direction, the limit of the region in which the stars are at all thickly sown is reached far within the distance at which they would become invisible in small telescopes; while along the galactic plane there is no evidence that our present studies have penetrated to the limit.

All these conclusions are in excellent agreement with those recently derived by Hertzsprung from a study of the short-period variables, on principles very similar to those of the present dis-

cussion.[1] He finds, from the parallactic motions of 13 of these Cepheid variables, which appear in Boss's *Preliminary General Catalogue*, that their mean absolute magnitude, on Kapteyn's scale, is -2.3 ± 0.5. Combining this with Miss Leavitt's discovery that among the variables in the Small Magellanic Cloud (whose periods and light-curves resemble those of the stars under consideration) the logarithm of the period increases with the mean brightness at the rate of 0.48 for each magnitude, and assuming the same law to hold for the isolated variables of short period, he deduces an expression for the mean absolute magnitude of such a variable which, on Kapteyn's scale, becomes

$$M = -2.3 - 2.1 \log \frac{P}{6.6}$$

when P is the period in days. Parallaxes and distances may then be computed for all the variables of this class whose periods and range of magnitude are known.

As no detailed list of parallaxes, etc., is given in Hertzsprung's paper, the writer has recomputed the parallaxes and co-ordinates in space for the 67 available stars found in the list in the *Harvard Annals*, **56**, 191 ff. (excluding as Hertzsprung does those with periods less than one day, which appear to form a class by themselves). This material is identical with Hertzsprung's (except that SZ Centauri has been excluded, since it is now known to be an eclipsing variable), and the only addition to his results has been the computation of distances, co-ordinates, etc., on the assumption that space-absorption exists, with the coefficient $d = 0.019$ (as above). The results of this discussion are strikingly similar to those derived above for the eclipsing variables. The short-period variables appear to be, on the average, about half as far away again, their computed mean distances being 4100 light-years if space is transparent, and 2040 with the assumed coefficient of absorption, as against 2400 or 1350 light-years for the eclipsing variables. This difference arises from the differences in the real luminosity of the two sets of stars, only 8 of the 90 eclipsing variables having estimated absolute magnitudes as bright as the faintest of the others.

[1] *Astronomische Nachrichten*, **196**, 205, 1913.

Since the search for variable stars has been carried down to about the same apparent magnitude in both cases, the difference in the mean distance is natural.

The short-period variables, however, are much more closely concentrated about the galactic plane than the others, their average distances from it being 260 light-years if there is no absorption, and 172 with the assumed absorption, while the corresponding values for the eclipsing variables are 436 and 280 light-years.

TABLE VII

A. Space Assumed Transparent

	Limits of $R \cos \beta$				
	0 to 2000	2000 to 3000	3000 to 6000	Over 6000	All Together
Number of stars	17	16	15	19	67
Mean $R \cos \beta$	1080	2500	4100	8100	4100
Mean $R \sin \beta$ regardless of sign*	158	214	285	368	260

B. Absorption with $d = 1.019$

	Limits of $R \cos \beta$					
	0 to 1000	1000 to 1500	1500 to 2000	2000 to 3000	Over 3000	All Together
Number of stars	11	8	19	14	15	67
Mean $R \cos \beta$	670	1220	1720	2440	3400	2040
Mean $R \sin \beta$ regardless of sign*	151	131	127	151	162	172

* These distances are measured from a plane passing 100 light-years south of the sun.

The median planes of the two sets of stars are apparently nearly coincident, if allowance is made for the effect of observational preference for northern eclipsing variables.[1] If space is transparent, the median plane for the eclipsing variables appears to be 65 light-years south of the sun, and that for the short-period variables 125 light-years south of the sun. The introduction of absorption reduces these distances to 40 and 75 light-years, respectively.

[1] No such allowance need be made for the stars of short period because the observational material required to furnish the data needed in the computations is very much less, and the necessary information is available for all parts of the heavens.

The same curious increase in the computed mean distance from the galactic plane with increasing distance from the sun is seen in Hertzsprung's figures, and the introduction of the hypothesis of absorption again removes it, as is shown by Table VII, A and B, similar to those already given.

The reappearance of this phenomenon with these quite independent data lends some support to the idea that it is really evidence of the absorption of light in space.

It may be desirable to add a table (VIII) showing in somewhat greater detail the distribution of the distances of the stars from the galactic plane.

TABLE VIII

	Distance from Galactic Plane in Light-Years											
	Over −1200	−1200 to −800	−800 to −600	−600 to −400	−400 to −200	−200 to 0	0 to +200	+200 to +400	+400 to +600	+600 to +800	+800 to +1200	Over +1200
A. No absorption												
Eclipsing variables . .	6	3	1	6	9	17	20	12	3	7	1	5
Short-period variables	....	1	1	6	13	13	17	11	2	3	....	....
B. $d=0.019$												
Eclipsing variables . .	....	3	2	7	10	20	26	10	7	0	5	....
Short-period variables	....	....	....	1	7	23	24	12	....	....	....	....

The results of the independent, though similar, investigations of Hertzsprung and of the writers may be summarized as follows:

A considerable majority of the eclipsing variables, and almost all those of short period, lie within a region bounded by two planes 1000 light-years apart, and parallel to the galactic equator, whose median plane passes about 100 light-years south of the sun. Almost all the rest of these stars lie within 500 light-years on each side of this region. Within this region, the stars extend in all directions to the limit of the present investigations. This limit, if space is transparent, must be at least 8000 light-years. The assumption of a plausible amount of absorption of light in space cuts this down to about 4000 light-years, and removes the anomaly of the apparent widening-out of the starry region with increasing distance from the sun.

Our "universe" of stars must be at least some thousands of light-years in diameter. The best available method of finding its limits (if these are not set by the absorption of light) is by the investigation of very faint variable stars of the Cepheid type. The rarity of variables among the fainter stars in the Milky Way[1] suggests that a limit exists, and is accessible to present means of investigation. It is of interest in this connection that Hertzsprung (assuming space transparent) has found a distance of 30,000 light-years for the Small Magellanic Cloud.[2] If at this distance, it must be about 1000 light-years in diameter, and a small affair compared with the star cloud of which the sun is one of the fainter members.

PRINCETON UNIVERSITY OBSERVATORY
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[1] D'Esterre, *M.N.*, **74**, 308, 537, 1914.

[2] *Loc. cit.* Misprinted as 3000.

AVOGADRO'S CONSTANT AND ATMOSPHERIC TRANSPARENCY[1]

By F. E. FOWLE

In an earlier number of the *Astrophysical Journal*[2] were derived values for the vertical transmissibility ($a_{a\lambda}$) of radiation through dry air above Mount Wilson together with certain correction factors ($a^{w}_{w\lambda}$) by which these could be altered for moist air containing w cm of precipitable water in the form of vapor. By means of Rayleigh's formula connecting the scattering of light passing through a gas with the number of molecules present, these dry-air coefficients were used to compute the number of molecules n_0 per cubic centimeter of a gas at 76 cm pressure and 0° Centigrade. This formula of Rayleigh's has been shown to be consistent with very different light-theories.[3] From n_0, N, Avogadro's constant, or the number of molecules per gram-molecule, directly follows. A gram-molecule is the amount of a substance equal to m grams where m is the molecular weight of the substance. The approximate agreement of these rather rough determinations of n_0 with the best value of this constant from other methods was used as an argument for the correctness of our determinations of the atmospheric losses in solar-constant computations.

It is perhaps worth while to redetermine with greater refinements the value of Avogadro's constant derived by this method. Mr. L. V. King,[4] regarding the process based upon molecular scat-

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] *Astrophysical Journal*, **38**, 392, 1913.

[3] Rayleigh, *Encyclopaedia Britannica*, 11th ed., XXV, 202 ("Sky"), 1911; Natanson, *Bull. intern. de l'Académie des sciences de Cracovie*, January 5, 1914.

[4] *Philosophical Transactions of the Royal Society of London*, **212** A, 375, 1913. *Nature*, **93**, 557, 1914. It is somewhat incorrect to state that our transmissibility coefficients rest "ultimately on the ratio of two galvanometer deflections . . . . quantities measurable to well within 1 per cent" (*loc. cit.*). Their accuracy depends rather on the constancy of the sky during about 2 hours for which our 6 observations furnish some indication. In those parts of the spectrum little affected by water vapor an accuracy considerably better than 1 per cent is generally obtainable.

tering as of considerable value as an independent method of determining this constant, has used our observed moist-air coefficients for this purpose but has proceeded along quite a different path to his end. It is proposed to offer a comparison of our procedures.

OBSERVATIONS

Briefly, the following is the method of deriving the dry-air transmission coefficients which will be used in my procedure. They were obtained for Mount Wilson at 30 different wave-lengths between $0.34\,\mu$ and $2.24\,\mu$. The logarithms of the observed transmission coefficients were plotted as ordinates against the corresponding quantities of precipitable atmospheric moisture as abscissae, and the best representative curves (which appeared to be right lines) were produced by a short extrapolation to zero of moisture. From the consideration of the slope of these right lines, factors expressing the effect of water vapor on the atmospheric transmission were determined for the 30 wave-lengths. The process and results are given in more detail in the paper cited.

In Table I are given the data thus obtained. In columns two and three will be found the zenith transmission coefficients for radiation through dry air above Mount Wilson ($a_{a\lambda}$), altitude 1730 m, barometer 62.3 cm, for 1910–1911 and 1913 respectively. During 1912 the variability from day to day in the amount of dust suspended in the atmosphere subsequent to the eruption of Mount Katmai[1] rendered the values then obtained useless for the present discussion. The column headed Δ contains the fractional deviations of the 1913 observations from the earlier mean and probably indicates the presence of a remnant of the suspended dust from the Katmai eruption causing still a loss of from 2 to 3 per cent; for, assuming $a_{d\lambda}$ the coefficient of transmission for this dust at the wave-length λ during 1913, then the fractional deviations are

$$\frac{a_{1910-1911}-a_{1913}}{a_{1910-1911}}=\frac{a_{a\lambda}-a_{a\lambda}a_{d\lambda}}{a_{a\lambda}}=1-a_{d\lambda}$$

or the amount of radiation lost because of the dust.

[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **3**, 211, 1913; *Smithsonian Miscellaneous Collections*, **60**, 1, 1913.

The observations of 1913 were excellent in quality throughout the spectrum, whereas those of 1910–1911 both in the infra-red and in the ultra-violet were less satisfactory. Accordingly for the first three ultra-violet places in the 1910–1911 column where field-light doubtless made the values of my earlier communication too

TABLE I

ATMOSPHERIC TRANSPARENCY AND n_0

WAVE-LENGTH λ	TRANSMISSION COEFFICIENTS: DRY AIR, $a_{a\lambda}$ 1910–1911	1913	Δ	WATER VAPOR, $a_{w\lambda}$ 1910–1911	1913	Mean	k	REFRACTION COEF. 1.000	n_0
0μ3504	(0.632)	0.614	(0.028)	0.917	0.933	0.925	0.459	3010	....
.3600	(.655)	.637	(.028)	.940	.948	.944	.423	3002	275×10^{17}
.3709	(.686)	.667	(.028)	.959	.923	.941	.377	2994	273
.3838	.713	.689	.033	.959	.929	.944	.338	2986	264
.3974	.752	.738	.019	.962	.962	.962	.285	2978	271
.4127	.783	.763	.026	.965	.938	.952	.245	2970	269
.4307	.808	.792	.020	.968	.944	.956	.213	2962	259
.4516	.840	.816	.029	.967	.955	.961	.174	2954	262
.4753	.863	.836	.031	.973	.951	.962	.147	2947	....
.5026	.885	.859	.029	.976	.957	.966	.122	2939	Mean=
.5348	.898	.873	.028	.980	.964	.972	.108	2931	268×10^{17}
.5742	.905	.877	.031	.974	.966	.970	.100	2924	....
.5980	.913	.885	.031	.978	.973	.975	.091	2921	....
.6238	.929	.897	.034	.977	.973	.975	.074	2918	....
.6530	.938	.916	.023	.987	.975	.981	.064	2915	....
.6858	.959	.935	.025	.985	.977	.981	.0419	2911	....
.7222	.970	.947	.024	.989	.977	.983	.0304	2907	....
.7644	.979	.956	.024	.985	.977	.981	.0212	2904	....
.8130	.980	.958	.022	.990	.981	.985	.0202	(2901)	....
.8634	.982	.965	.017	.989	.983	.986	.0182	......	....
0.9861	.987	.974	.012	.991	.989	.990	.0131	......	....
1.1474	.987	.973	.014	.988	.988	.988	.0131	......	....
1.3019	.986	.980	.006	.990	.984	.987	.0141	......	....
1.4520	.989	.984	.005	.988	.988	.988	.0111	......	....
1.6032	0.983	.989	−0.004	0.986	.984	.985	0.0171	......	....
1.738	....	.985	....	......	.989	.989	......	......	....
1.870	....	.984	....	......	.987	.987	......	......	....
2.000	....	0.983	....	......	0.986	0.986	......	......	....

high, new values have been computed from the excellent coefficients of the 1913 column assuming $\Delta=0.028$. Because of the less satisfactory character of the infra-red values especially for the observations of 1910–1911, the falling-off of the fractional deviations in the infra-red may not be real. The average of the 16

better values of column 4 gives 0.027 as the scattering due to the dust in the air above Mount Wilson during 1913.

The next three columns give the correcting factors ($a_{w\lambda}$) for 1 cm precipitable water in the form of atmospheric vapor for 1910–1911, 1913, and the mean, respectively. When the altitude of a station is such that dust is practically absent, say above 1000 meters, the transmissibility of radiation from a celestial body at a zenith distance z through the atmosphere when it contains w cm of precipitable water vapor and the barometer reads p cm. is[1]

$$\left\{(a_{a\lambda})^{p/76}(a_{w\lambda})^{w}\right\}^{sec\ z}$$

No account is taken by this formula of selective absorption.

Then follows a column containing values of the "coefficient of attenuation" k which are related to a by the equation $\log_e a_{a\lambda}=k$. The coefficients of refraction are from values determined for dry air by Kayser and Runge.[2]

AVOGADRO'S NUMBER

The determination of Avogadro's number from the observed scattering of radiation in passing through a gas is based upon Rayleigh's formula. If n_0 is the number of molecules in a cubic centimeter of a gas at 0° C. and 76 cm pressure, k as defined in the last paragraph, n the index of refraction, λ the wave-length in centimeters of the radiation, p the atmospheric pressure in centimeters and equal to 62.3 at Mount Wilson, H the height in centimeters of a homogeneous atmosphere at 0° C. producing a pressure at its base of 76 cm and equal to 7.99×10^5, then Rayleigh's formula may take the form:

$$k=\frac{32\pi^3(n-1)^2Hp}{3n_0\lambda^4 76}\ .$$

N, Avogadro's number, $=n_0\times 32$ (molecular weight oxygen) $\times 699.7$ (cubic centimeters of oxygen 0° C., 76 cm pressure, weight 1 gram) $=22390n_0$. If dust is present in appreciable amount in the atmos-

[1] See *Meteorologische Zeitschrift*, **31**, 270, 1914; *Monthly Weather Review*, **42**, 2, 1914. Comparisons are given in these communications of computations by this formula with actually observed transmissions for altitudes of 50, 90, 100, 1160, 1730, 1950, 3260, and 4420 meters.

[2] *Abhandlungen Akademie der Wissenschaften in Berlin*, 1893.

phere then $(k-d)$ must be equated to the above expression, k being the coefficient of attenuation belonging to the dust.

In the upper curve of Fig. 1 are plotted the values of $a_{a\lambda}$ (column 2 of the table) as ordinates against the corresponding wave-lengths as abscissas. In the lower curve the values of k (column 8) are

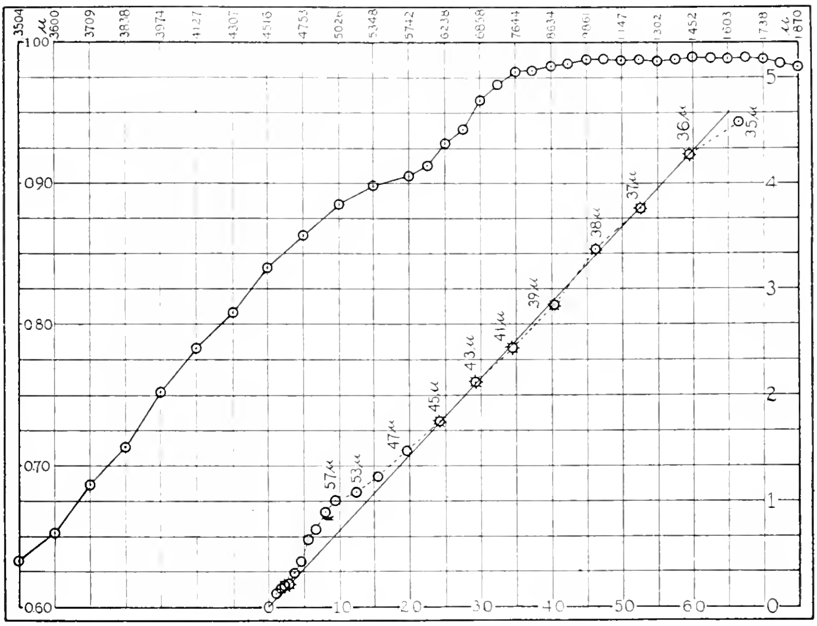

FIG. 1

Upper curve: ordinates, transmissibility of radiation through dry air vertically above Mount Wilson, 1910–1911, $(a_{a\lambda})$; abscissae, wave-lengths in μ

Lower curve: ordinates, values of k, where $\log_e a_{a\lambda}=k$; abscissae, $\lambda^{-4}\times 10^{-16}$

used as ordinates against those of λ^{-4} as abscissas. In the equation for k, were d zero and n constant, then the points of the latter plot should lie on a right line passing through the zero of co-ordinates. Mr. King makes plots similar to this last one except that the *observed* k's for *moist* air are used. Subsequently our procedures and interpretations diverge. He assumes that d is not zero and passes the best right line through the points from the ultra-violet up to about $0.610\,\mu$. Then he proceeds to draw a

steeper right line through the remaining points of longer wave-lengths. It is difficult to see the physical significance of this latter step or why the scattering due to the dust should suddenly and rapidly decrease in this part of the spectrum. Indeed he states "the precise cause of this well marked discontinuity furnishes a point which must be left for further investigation." The slant of the line for the violet end taken with a mean value of the index of refraction gives his value for n_0.

To me it seems the data admit of a very different interpretation. In my view the run of the dry-air coefficients is consistent with a practically dust-free air above Mount Wilson and the departures of the points between wave-lengths 0.47 μ and 0.76 μ from the representative right line are due to the selective absorption of the gases of our atmosphere. In this region Rowland gives some 440 atmospheric lines exclusive of those due to water vapor. When this region is passed the points naturally approach the right line again. Rayleigh's formula does not hold for selective absorption and therefore the points in this region should be omitted from the discussion.

Returning now to the table, in the last column are given the values of n_0 as computed separately for the upper 7 starred points in the lower plot. Their mean gives

$$n_0 = 2.68 \times 10^{19}, \qquad N = 6.00 \times 10^{23}$$

The values of the infra-red points where they next approach the representative right line have not so much weight, for here a change of 1 per cent in the observed values of the atmospheric transmission changes k, and therefore n_0 and N, by 100 per cent. The effect of a very small loss due to dust would therefore greatly influence values determined from the infra-red observations alone, whereas it would have comparatively little effect on the violet values. Attention has already been called to the fact that these infra-red points were not very satisfactorily determined during 1910–1911. However, using the red-end values of the greatest wave-lengths for which the indices of refraction are known, 0.76 μ and 0.81 μ, together with the seven values just used and the more complete equation containing d, a least-squares reduction gives

$$n_0 = (2.73 \pm 0.02) \times 10^{19}, \qquad N = (6.11 \pm 0.05) \times 10^{23} \qquad d = 0.005 \pm 0.002$$

The value of a corresponding to $d=0.005$ is 0.995, showing a very small loss due to dust during the years 1910–1911 as already inferred.

Since the first three values of the 1910–1911 column built up from the 1913 data may be open to some criticism, a least-squares reduction using the same wave-lengths and the more complete equation has been made for the 1911 data. The following values resulted:

$$n_0=(2.69\pm0.03)\times10^{19}, \quad N=(6.02\pm0.06)\times10^{23}, \quad d=0.026\pm0.003$$

The value of a corresponding to $d=0.026$ is 0.974 or a loss due to the dust in the air for 1911 of 0.026 corresponding very closely with the mean obtained from column 4 or 0.028.

SUMMARY

In the foregoing communication are given coefficients for the transmissibility of radiation of various wave-lengths through the dry air vertically above Mount Wilson together with factors and formula for computing the transmissibility for moist air for other zenith distances and altitudes where dust has become a negligible quantity (above 1000 meters). No account, however, is taken in the formula of selective absorption.

By means of Rayleigh's formula connecting the scattering of light passing through a gas with the number of molecules, these coefficients have been used to compute the number of molecules n_0 in a cubic centimeter of a gas at 76 cm pressure and 0° C. The merits of the present reductions over those of my earlier paper lie in using the index of refraction proper to each point in place of a mean value, in better values for k, and in a more accurate value of p, the value before used being taken at an appreciably higher altitude on the mountain than our observatory. The mean results give

$$n_0=(2.70\pm0.02)\times10^{19}$$

Avogadro's number, or the number of molecules per gram-molecule, corresponding to this value is

$$N=(6.05\pm0.04)\times10^{23}$$

The agreement of the above value for n_0, 2.70×10^{19}, with what is perhaps the best value from other methods, $2.705 \pm 0.005 \times 10^{19}$ (Millikan), must give weight to the accuracy of the estimation of the atmospheric losses in the determinations of the solar radiation by the Smithsonian Observatory.

A remnant of the volcanic dust from the eruption of Mount Katmai, Alaska, in 1912, scattering somewhat less than 3 per cent of the incident solar radiation, is indicated by the 1913 transmission coefficients. It is perhaps worth noting that, fine as this dust must be to have remained suspended in the upper air over a year, its scattering of radiation scarcely varies with the wave-length, at least between the limits $0.38\ \mu$ and $0.81\ \mu$.

Astrophysical Observatory
Smithsonian Institution
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NEW VARIABLES IN THE CENTER OF MESSIER 3[1]

By HARLOW SHAPLEY

In the investigations of the variable stars in the globular cluster Messier 3 by Professor Bailey at Harvard,[2] the central portion could not be studied successfully on his plates because the great density of stars made it impossible to distinguish individual images. Thus practically all of the region within one minute of arc of the center of the cluster has remained unexplored. Outside the central burned-out nucleus Bailey found 137 variables,[3] and for 110 of them has determined the periods and light-curves, finding that in all cases the periods are shorter than eighteen hours and that the variation is of the typical cluster type. Judging from the relation between the number of variables and the total number of stars at different distances from the center, Bailey estimated that there should be about twelve more variables in the central part of the cluster.

Photographs of the cluster made with the 80-foot-focus Cassegrain combination of the 60-inch reflector have a scale sufficiently large to permit a relatively easy examination of the individual brighter stars of the central region. Six plates have been obtained on three different nights, three with exposures of an hour each and three with exposures of fifteen minutes. The intercomparison of the plates in the Zeiss stereocomparator resulted in the discovery of more than 30 new variable stars. For several of these it is not possible to be absolutely certain of the variation, because either the ranges are small or the variation is observed on only two of the plates available. For 23 of the stars, however, there can be no doubt of the variability. It may be possible later to add a few more to this list of variables from among those suspected. Only two of the 23 are more than one minute of arc from the center of the cluster. The outer portions of the cluster were not systematically

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 91.

[2] *Harvard Annals*, **38**; *ibid.*, **78**, Part I.

[3] The later measurements of the plates at Harvard indicate that about five stars of the original list are probably not variable and that a few others are somewhat doubtful.

examined, for Bailey's thorough study no doubt makes that unnecessary.

In Table I the new variables are listed in order of right ascension. All but two of them are contained in von Zeipel's catalogue of Messier 3, which gives the positions of 1571 stars.[1] The numbers in the second column of Table I, and the right ascensions and declinations in the third and fourth columns are taken from that work. For No. 1 and No. 14 the positions were determined by reference to neighboring stars. In a large number of cases the catalogue position refers to the mean place of a double, triple, or multiple star which could not be resolved on the plates measured by von Zeipel and usually was not even suspected of being other than a single star. In such instances the notes following the table indicate which component is the variable.

The last two columns of Table I give, respectively, the maximum and minimum magnitudes observed on my plates. These numbers should not be taken as in any way indicating the total range. For many of the variables the length of the exposure no doubt minimizes the brightness at the sharp-pointed maxima; and many of them may be very close composite stars with one component varying through a large range, but without having conspicuous effect on the image of the composite mass. The magnitudes are based on estimates referred to Bailey's series of comparison stars as standards.

NOTES ON THE INDIVIDUAL STARS IN TABLE I

No. 1.—The new variable is the preceding companion of von Zeipel's No. 486, which is also Bailey's variable No. 29. Bailey partially resolved the double and considered at least one of the pair variable. Both components vary through a wide range.

No. 3.—Von Zeipel thought this was perhaps a double and made measures on the two ends of a nebulous streak as well as upon the middle. The star appears single on the Mount Wilson plates. (No star is seen in the neighboring catalogued position of No. 655.)

No. 4.—This is apparently a composite star, but it is the central nucleus that undergoes variation.

[1] *Annales de l'Observatoire de Paris*, **25**, F1–F101, 1908.

No. 5.—Von Zeipel's No. 676 is composed of five distinct stars, of which the variable is the brightest and most centrally located.

No. 7.—The variable is the brighter and southern component of a close double.

No. 9.—Von Zeipel's position refers to the center of mass of the image of a double. The variable precedes its faint companion and at minimum is equal to it in brightness.

TABLE I

New Variable Stars in Messier 3

No.	Von Zeipel's Number	α 1900	δ 1900	Magnitude	
				Maximum	Minimum
		13^h37^m	$28°$		
1	....	$30^s.12$	$51' 46''.5$	15.4	16.9
2	620	32.785	52 1.31	15.6	17.0
3	638	33.005	53 16.59	16.0	16.8
4	654	33.151	52 39.27	16.2	16.7
5	676	33.440	52 31.47	15.2	16.8
6	678	33.479	52 18.38	15.5	16.8
7	698	33.810	53 36.42	15.3	16.7
8	703	33.870	52 19.52	16.6	17.1
9	714	33.970	53 15.94	15.4	16.9
10	742	34.393	52 16.45	15.6	17.0
11	749	34.469	53 28.57	16.0	17.4
12	769	34.707	53 21.37	16.3	17.0
13	783	34.905	53 3.42	14.3	14.9
14	....	35.35	52 50.8	15.9	17.2
15	881	36.140	52 52.59	16.0	16.9
16	892	36.344	52 47.33	15.3	15.7
17	901	36.408	52 2.74	15.0	16.8
18	944	37.116	53 8.76	15.7	16.8
19	950	37.178	52 28.92	15.5	17.1
20	985	37.660	53 27.22	15.1	16.5
21	1052	38.665	53 1.13	16.5	17.4
22	1092	39.538	53 9.70	16.3	16.9
23	1193	42.194	51 59.27	15.7	16.8

No. 12.—The variable is the following component of a close double.

No. 16.—The variable is one of a group of stars whose images are partly superposed. The variation is very definite, though small; it would probably be measured much larger if the variable could be isolated.

No. 17.—The variable is the most southern member of a triplet.

No. 19.—Two or three faint companions precede the variable. Von Zeipel thought the image irregular in form.

No. 21.—The variable is the northern, following component of a double.

No. 23.—Von Zeipel's No. 1193 is a close double, two minutes of arc distant from the center of the cluster. The variable, which is the preceding

component, is brighter than its companion at maximum and fainter at minimum. It is not certain that the companion does not also undergo an appreciable variation.

It is to be noted that one-third of the new variables are equal to or fainter than $17^m.0$ at minimum. The faintest minima observed by Bailey were $16^m.9$, and he has suggested that the apparent absence of variables among the thousands of fainter stars is probably real. It is not possible from the data now available, beyond the evidence of the above table, either to support or to oppose this hypothesis, but in the next observing season it is proposed to secure plates that will definitely decide the matter. The light-curves and periods have of course not been determined for the new variables, but they are presumably analogous to those derived by Bailey for stars more distant from the center.

Table II contains the data relative to the stars suspected of light-variation. The arrangement is the same as in Table I, and the notes explain some of the individual cases.

TABLE II

STARS SUSPECTED OF VARIATION

No.	Von Zeipel's Number	α 1900	δ 1900	Magnitude	
				Maximum	Minimum
		13^h37^m	28°		
1	604	$32^s.497$	53′ 16″.29	16.5	16.8
2	624	32.806	52 42.25	....	....
3	....	32.97	52 52.1	16.0	16.5
4	666	33.323	53 14.79	16.3	16.6
5	702	33.866	52 28.34	15.5	15.8
6	705	33.878	53 30.93	15.5	16.0
7	712	33.952	53 4.89	15.7	16.5
8	720	34.119	52 30.95	....	....
9	894	36.362	53 52.23	....	....
10	1007	38.036	53 49.59	16.5	16.9
11	....	38.58	53 32.8	....	....
12	1106	39.988	52 38.22	17.2	17.6
13	1142	40.867	53 18.31	....	....

NOTES ON THE INDIVIDUAL STARS IN TABLE II

No. 2.—Von Zeipel's 624 is a triplet. It is difficult to tell which of the stars varies, but it is probably the following component.

No. 7.—The suspected variable is one or other of the two brightest stars in the composite mass catalogued as No. 712.

No. 8.—This star is apparently somewhat out of the position given by von Zeipel. It was at the limit of measurement on his plates. The variability is doubtful.

No. 9.—This extremely close double is Bailey's variable No. 2. It is almost certain that both components are variable. The preceding component varies through a range of a magnitude at least.

No. 10.—The suspected star is the faint preceding companion to von Zeipel's No. 1007, for which the position is given in the table.

No. 12.—A defective image may be the cause of the suspected variability.

No. 13.—The suspected star is the closest of the two faint preceding companions to von Zeipel's No. 1142, for which the position is given in the table.

Several variables of Bailey's list, which were found by him to be difficult of separation from surrounding stars or which for other reasons could not be studied or definitely proved variable, were examined on the Mount Wilson plates. The results are collected in the following notes. The numbers are those assigned in *Harvard Annals*, 78.

No. 2.—See the reference to this star in the note on No. 9 in Table II above. Only a very small range was observed at Harvard and no period was derived.

No. 4.—This relatively bright star was suspected of a small variation at Harvard. Mount Wilson plates show a variation from $14^{m}.0$ to $15^{m}.3$.

Nos. 8, 30, 73, 95, 98, 127, 129, and 130, all of which are stars concerning whose light-variability there is some question, show no appreciable variation on the Mount Wilson plates.

The suspected variability of No. 99 is definitely confirmed.

The variability of No. 122, doubted by Bailey, is established. The star is a close triplet and the variable is apparently the following, southern component.

Nos. 135 and 136.—Bailey writes concerning both of these stars, which are near the center of the cluster and difficult, that the variation, if genuine, is very small, though when first examined the stars seemed to vary. For the former the variation measured at Mount Wilson is from $15^{m}.3$ to $16^{m}.8$, and for the latter from $15^{m}.7$ to $16^{m}.6$.

MOUNT WILSON SOLAR OBSERVATORY
August 13, 1914

ON THE NATURE AND CAUSE OF CEPHEID VARIATION[1]

By HARLOW SHAPLEY

The purpose of the present discussion is an attempt to investigate the question of whether or not we should abandon the usually accepted double-star interpretation of Cepheid variation. In addition to the brief statement of some general considerations and correlations of the many well known characteristics of Cepheid and cluster variables, certain recently discovered properties of these stars are discussed in greater detail, because chiefly upon them are based the conclusions reached in this study.

It seems a misfortune, perhaps, for the progress of research on the causes of light-variation of the Cepheid type, that the oscillations of the spectral lines in nearly every case can be so readily attributed, by means of the Doppler principle, to elliptical motion in a binary system. The natural conclusion that all Cepheid variables are spectroscopic binaries has been the controlling and fundamental assumption in all the recently attempted interpretations of their light-variability, and the possibility of intrinsic light-fluctuations of a single star has received little attention.

From the very first there have been serious troubles with each new theory. Considered from the spectroscopic side alone, the Cepheids stand out as unexplainable anomalies. There are persistent peculiarities in the spectroscopic elements, such as the low value of the mass function, the universal absence of a secondary spectrum, and the minute apparent orbits. Practically the only thing they have in common with ordinary spectroscopic binaries is the definitely periodic oscillation of the spectral lines, which permits, with some well known conspicuous exceptions,[2] of

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 92. Read at the seventeenth meeting of the American Astronomical Society, August 1914.

[2] The irregularities in the velocity-curve of ζ Geminorum have been discussed by Campbell (*Astrophysical Journal*, **13**, 94, 1901), Russell (*Astrophysical Journal*, **15**, 260, 1902), and Plummer (*Monthly Notices*, **73**, 661, 1913). The deviations from purely elliptical motion in the case of W Sagittarii have been considered by Curtiss (*Lick Observatory Bulletins*, **3**, 36, 1904; *Astrophysical Journal*, **20**, 149, 1904); in the

interpretation as periodic orbital motion. Adding, then, to the spectroscopic abnormalities the curious relations between light-variation and radial motion, the difficulties in the way of all the proposed simple solutions seem insurmountable. Geometrical explanations of the light-variation fail completely, and little better can be said of the hypotheses that involve partly meteorological and partly orbital assumptions.

The writer can offer no complete explanation of Cepheid variability as a substitute for the existing theories that are shown to be more and more inadequate. At most, only the direction in which the real interpretation seems to lie can be pointed out, and an indication given of the strength of the observational data that would support the theory developed along the lines suggested. The principal results of a rather extensive investigation, further details of which it is hoped can be published in subsequent papers in the near future, are outlined in the following paragraphs. The main conclusion is that the Cepheid and cluster variables are not binary systems, and that the explanation of their light-changes can much more likely be found in a consideration of internal or surface pulsations of isolated stellar bodies.

THE ESSENTIAL IDENTITY OF CEPHEID AND CLUSTER VARIABLES

The subdivision of the short-period variables into the cluster type and the Cepheid type is an artificial one. This proposition scarcely needs proof, although the assumption of the essential similarity of the two groups is important in the following discussion. Practically all writers on the subject are more or less inclined to accept this view.[1] The definition of the cluster-type variable is,

case of Y Ophiuchi by Albrecht (*Lick Observatory Bulletins*, **4**, 134, 1907) and later by Zurhellen (*Astronomische Nachrichten*, **177**, 329, 1908) and Miss Udick (*Publications of the Allegheny Observatory*, **2**, 151, 1912); in the case of RT Aurigae by Duncan (*Lick Observatory Bulletins*, **5**, 120, 1909). For many Cepheids the total range of velocity variation is so small that secondary oscillations and other irregularities of considerable relative importance may easily be lost in the accidental errors (Curtiss, *op. cit.*, p. 39).

[1] See, for instance, Nijland, *Hemel en Dampkring*, April 1913; Williams, *Journal of the British Astronomical Association*, **23**, 134, 1912; Kiess, *Publications of the Astronomical Society of the Pacific*, **24**, 191, 1912, and **25**, 121, 1913; Townley, *Publications of the Astronomical Society of the Pacific*, **25**, 239, 1913.

in fact, by some merely "short-period Cepheid." Others, including Hartwig[1] and Kron,[2] have considered only those with rapidly decreasing brightness and constant light at minimum as "antalgol" or cluster-type variables. Kron calls the shortest-period variable known a Cepheid,[3] and Hertzsprung[4] designates as Cepheids only those variables whose periods are greater than a day. The writer proposes to adopt, merely as a convenience, the latter practice, arbitrarily calling the Cepheids of periods less than a day cluster-type variables; for there is at present no evidence of real difference between the two classes in the nature or probable causes of the light and velocity variations.[5] Hertzsprung[6] calls attention to the maxima in the frequency-curve of the periods at twelve hours and at seven days, and notes also that the longer-period Cepheids are

[1] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **37**, 284, 1902.

[2] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **22**, Pt. III, 53, 1912. See also Newcomb-Engelmann, *Populäre Astronomie*, 5th ed., p. 623, Leipzig, 1914.

[3] XX Cygni, period $3^h 14^m$.

[4] *Astronomische Nachrichten*, **192**, 262, 1912.

[5] It is hardly necessary to remark that Cepheids and Geminids are physically identical. The latter term merely signifies that the rise and decline of brightness require approximately equal intervals of time. There are numerous types of variation intermediate between the chosen typical curves of ζ Geminorum and δ Cephei. A significant feature that has not been pointed out explicitly heretofore is that the smaller the ratio of interval of increasing light to interval of decreasing light the greater the eccentricity. Orbits of Geminids therefore have smaller eccentricities. This really amounts to observing that the light-curves and velocity-curves of all classes of Cepheids are generally identical in form. See the study by Luizet, "Les Céphéides considérées comme étoiles doubles," *Annales de l'Université de Lyon*, Nouvelle Série, I, Fascicule 33, 67–148, 1912.

Some Geminid curves, however, permit of hypothetical interpretations that cannot be applied to the more typical Cepheid. Russell has found (*Popular Astronomy*, **22**, 142, 1914) that, if the time of rise to maximum is greater than one-fourth the period, the light-variations may be interpreted as the rotation of a spotted body, but such an explanation is otherwise untenable. The writer has shown (*Laws Observatory Bulletin*, **2**, 71, 1913; *Astronomische Nachrichten*, **194**, 353, 1913) that for certain symmetrical curves of the Geminid type the light-variations may be due entirely to the rotation of a single ellipsoidal star. This explanation is a possible and plausible one, but for SZ Tauri, one of the stars suitable for such an interpretation, Haynes has found a typical Cepheid velocity variation (*Lick Observatory Bulletins*, **8**, 85, 1914).

[6] *Astronomische Nachrichten*, **179**, 376, 1909; **192**, 262, 1912; **196**, 205, 1913. Chandler reached some of the same conclusions twenty-five years ago (*Astronomical Journal*, **9**, 1, 1889).

in the galaxy, while the shorter-period Cepheids or cluster variables are apparently distributed more at random over the sky. Making the reasonable assumption that the data, though rather meager, are sufficient, nevertheless, to establish the reality of both phenomena, these conditions do not impeach the hypothesis that the light and velocity variations of the long- and short-period Cepheids are attributable to the same causes, and that the only modifications necessary in an explanation of one, to make it applicable to the other, are those depending on the length of the periods and other gradative characteristics, such as differences of spectral type and relative speed of light-change at corresponding phases. Among the several arguments that tend to prove the inherent similarity of the two groups of Cepheids, the following are the most important.

a) For RR Lyrae, period 13.6 hours, which is commonly classified as a cluster-type variable, the spectroscopic orbit by Kiess[1] resembles in all details the peculiar orbits characteristic of the longer-period Cepheids. The light-curve is typical of cluster variables in all its properties.[2]

b) From the photometric standpoint, Graff and Bottlinger[3] have found no essential differences between light-curves of cluster and Cepheid types, and insist on the artificiality of the division into two classes. Very few, if any, of the cluster-type variables have rigorously constant light at minimum phase, as Plummer,[4] among others, has shown. In fact, it was partly for this reason that Hartwig abandoned, in the *Vierteljahrsschrift* catalogue, the former term "antalgol" and the former distinction between cluster and Cepheid variables.[5]

c) Russell's harmonic analyses of the mean light-curves of typical cluster variables and typical Cepheids indicate the necessity of analogous interpretations of the two.[6]

d) An unpublished investigation by the writer of the relation between the periods and spectral types of all variables shows the

[1] *Lick Observatory Bulletins*, **7**, 140, 1913.

[2] Kiess, *Publications of the Astronomical Society of the Pacific*, **24**, 189, 1912.

[3] *Astronomische Nachrichten*, **196**, 113, 1913. [4] *Monthly Notices*, **73**, 657, 1913.

[5] *Vierteljahrsschrift der Astronomischen Gessellschaft*, **48**, 287, 1913.

[6] An abstract is printed in *Popular Astronomy*, **22**, 142, 1914.

existence of a continuous property from the longest-period Cepheids to the shortest-period cluster variables.

e) The shift of the maximum intensity in the spectra toward the violet with increasing light is a property common to both classes.[1]

IRREGULARITIES IN THE LIGHT-ELEMENTS

Starting, then, with the apparently well-grounded assumption that the reasoning relative to the nature of the cluster-type variation applies equally well to Cepheid variation, the first argument presented against the binary character of Cepheids deals with the irregular oscillations in the photometric period. In a paper presented to the American Astronomical Society at its last meeting, the writer reported on the oscillations in the periods of several cluster-type stars.[2] The study of such irregularities must necessarily, for the present at least, be confined to photometric observations, for the faintness of the stars, and the consequent length of spectroscopic exposure would conceal irregular oscillations in the velocity measurements.[3] The investigation, as is also obvious, must succeed first with the stars of shortest period, for with them the light-change is of sufficient rapidity to permit the determination of points on the steep ascending branch of the light-curve with high precision.

Further observations of SW Andromedae, made since the last report, have confirmed the previous results, showing that the time of the rise to maximum light varies from the mean predicted time by ten or fifteen minutes within the short interval of two or three days, but evidently without exhibiting regular periodicity. The uncertainty of the determination does not exceed three or four minutes. The similar oscillations in the light-curve of RR Lyrae

[1] For the Cepheids see the work of Albrecht (*Lick Observatory Bulletins*, 4, 131, 1907) and of other Lick observers. For the cluster-type variables see the work of Kiess, referred to above, and the indirect determinations of the maximum intensity shifts presented in a later section of the present paper.

[2] An abstract is printed in *Popular Astronomy*, **22**, 144, 1914.

[3] The average length of exposure on RR Lyrae with a one-prism spectrograph attached to the 36-inch refractor of the Lick Observatory was more than two hours—nearly one-sixth of the entire period. RR Lyrae is the brightest cluster variable known (excepting β Cephei).

were apparently periodic throughout the interval of two or three years covered by the earlier series of Harvard observations,[1] but the later work at Harvard[2] and at the Lick Observatory[3] shows a different amplitude and perhaps no periodicity at all. The Lick and Harvard observations were made with visual photometers. In a recent letter Professor Hertzsprung writes that he also finds irregularities from night to night in photographic observations of the star.

The oscillation in the time of brightening to maximum seems to be a pretty general characteristic, though possibly not universal. It is shown definitely for several other stars besides those mentioned above,[4] and perhaps most noticeably in the case of XX Cygni, which is discussed in the next section. The most remarkable feature of the oscillation, which from night to night is conspicuously large in some cases, is that a change in the mean period of the light-variation has been recorded for only two stars, and these changes are relatively minute.[5] If the observed oscillations were definitely periodic, it would perhaps be possible to attribute them in some kind of a binary system to orbital changes, such as the rotation of the line of apsides. But the sudden and unpredictable changes in the light-variation, very likely accompanied by analogous oscilla-

[1] *Harvard Annals*, **69**, Pt. I, 45, 1909.

[2] The manuscript of these observations was kindly sent to the writer by Professor Pickering; more recently the work has been published in *Harvard Annals*, **69**, Pt. II, 124, 1913.

[3] *Lick Observatory Bulletins*, **7**, 141, 142, 1913.

[4] *Popular Astronomy*, **22**, 144, 1914.

[5] According to Kron the mean period of XX Cygni is decreasing by about a tenth of a second a year (*op. cit.*, p. 47). Roberts finds that the mean period of S Arae is decreasing by four-hundredths of a second a year (*Astrophysical Journal*, **33**, 200, 1911). The long-accepted secular change in the period of δ Cephei, first established with some uncertainty by Chandler (*Astronomical Journal*, **13**, 101, 1893) and later maintained by Nijland (*Astronomische Nachrichten*, **161**, 229, 1903), has recently been completely rejected by Luizet in his monograph on the light-variations (*Annales de l'Université de Lyon*, Nouvelle Série, Fascicule 33). Belopolsky, however, finds an oscillation in the spectroscopic period (*Mitt. Pulk.*, **3**, 63, 1909). W. J. S. Lockyer has found (Dissertation, Göttingen, 1896) that, while the mean period of η Aquilae is constant, there is an oscillation in the epoch of maximum through an amplitude of ten hours. Hellerich (Dissertation, Berlin, 1913) has studied the periods of ten Cepheids and finds no necessity of second-order terms.

tions in the velocity-curve, introduce another difficulty into the binary system theory.

IRREGULAR CHANGES IN THE LIGHT-CURVES

The second argument against the double-star explanation of Cepheid variation lies in the continually changing form of the light-curves from one maximum to the next. Again we will consider mainly the cluster-type stars. For the Cepheids the length of the period prohibits, in general, continuous observations throughout successive epochs, and in the long run the irregularities smooth out in a mean light-curve. Curtiss[1] has noticed, however, that the light-curve of W Sagittarii, period 7.6 days, changes shape with the time, and similar results are suggested for other Cepheids by various observers.[2] For the stars with periods less than a day, however, many long series of observations have been made, and although the work was rarely if ever undertaken for the purpose of seeking short-period changes in the form of the light-curve, nevertheless irregularities have often been found. Many observers have noticed that the errors are larger in observing short-period variables than in any other class. Roberts[3] and Innes[4] were suspicious of the large deviations in their measures of S Arae. Sperra[5] concluded from an extensive treatment of his visual observations on SW Draconis and SU Draconis that the shapes and durations of both maxima and minima varied from night to night. This was, I believe, the first and only serious attempt that has been made to question the supposed clocklike precision of short-period variation. Plummer and Martin[6] are disinclined to accept Sperra's results without further proof, for the photographic observations at Dunsink (exposure times from thirty minutes to an hour) do not confirm such irregularities (though they do show remarkable irregularities,

[1] *Lick Observatory Bulletins*, **3**, 168, 1905.

[2] For instance, the light-curve of δ Cephei, as pointed out by Luizet (*op. cit.*, pp. 58–60).

[3] *Astrophysical Journal*, **33**, 201, 1911.

[4] *Annals of the Cape Observatory*, **9**, 126B, 1903.

[5] *Astronomische Nachrichten*, **184**, 241–252, 1910.

[6] *Monthly Notices*, **73**, 440, 1913.

supposedly permanent, in the mean curves). The proof is now at hand, however, and with amazing clearness in the observations of XX Cygni, published at Potsdam.[1] The three-hour period of this star is obviously a great advantage in the study of the changing form of the light-curve. If the variations of SW Andromedae and RR Lyrae referred to above could have been followed regularly throughout their entire periods, there is little doubt that the oscillations in the time of the rise to maximum would have been found to constitute only a part of the irregularities.

In his discussion of nearly three thousand observations of XX Cygni, Kron finds a small secular change in the mean period, but does not consider real the deviations from the mean curves. The observations of ten observers are included. Both Schwab and Guthnick considered the irregularity of form a real phenomenon, and the former notes that, while the maximum and minimum magnitudes remain sensibly constant, the times between the ascent and descent past magnitude 11.2 vary from 35 minutes to an hour.[2] A study of the observations shows the same phenomenon in the work of all the observers.[3] The various forms of light-curve cannot be attributed to night errors. The differences between the curves at different epochs is distinctly larger than the errors of the observations. This is particularly true for Kron's photometric work. The average deviation of his measures from a normal curve, based on his own observations, is much larger than that of measures on the comparison stars, and probably more than twice as large as the average deviation of the observations from separate nightly curves. There appears, however, to be no definite periodicity in the changing shape of the curve; as a rule sharp maxima follow each other

[1] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **22**, Pt. III, 1912.

[2] *Astronomische Nachrichten*, **170**, 369, 1906.

[3] This and other points relative to the anomalies of the light-variation will be discussed more fully in a later communication. Contrary to all other experience with Cepheid variables the photographic range measured by Parkhurst and Jordan (*Astrophysical Journal*, **23**, 84, 1906) is less than the visual range. To examine this question more closely a series of simultaneous photographic and photovisual observations has recently been made with the 60-inch reflector. This will furnish a definitive color-curve, as well as serve as a control on the secular change in the period. (Parkhurst and Jordan also suspected oscillations in the period; *op. cit.*, p. 86.)

for a few days, to be succeeded by an intermediate type and then by a series of relatively wide, flat-topped maxima. Sometimes the extreme change in form occurs on successive nights. In the variations of the light-curve, as in the oscillations of the time of the rise to maximum light, the regularity and continuity of the phenomena that would be demanded by an orbital explanation is apparently lacking.

CHANGES IN COLOR AND SPECTRAL TYPE

A third argument against the binary interpretation of Cepheids is the difficulty such theories would have in explaining the periodic change of the spectral type, though it must be admitted that to a certain extent Duncan's hypothesis,[1] if otherwise acceptable, could account for spectral changes through the medium of atmospheric absorption. The evidences of the change of spectral type with changing light, though not well known nor generally recognized, are decisive and important. Schwarzschild,[2] Wirtz,[3] and more particularly Wilkens[4] have demonstrated for Cepheids of longer period that the range of light-variation is greater in the photographic than in the visual part of the spectrum. The photographic work of Martin and Plummer[5] suggests similar results for cluster-type variables, while the recent simultaneous photographic and photovisual observations by Mr. Seares and the writer at

[1] *Lick Observatory Bulletins*, **5**, 91, 1909; *Publications of the Astronomical Society of the Pacific*, **21**, 123, 1909.

[2] *Publikationen der v. Kuffnerschen Sternwarte*, **5**, C100, 1900. The photographic range of η Aquilae is found to be double the visual range. More recently Kohlschütter has repeated the photographic work and finds that the color-curve has an amplitude of four-tenths of a magnitude (*Astronomische Nachrichten*, **183**, 265, 1910).

[3] *Astronomische Nachrichten*, **154**, 327, 1901. Wirtz measures the photographic ranges of δ Cephei and ζ Geminorum.

[4] *Astronomische Nachrichten*, **172**, 316, 1906. An average value of 1.6 is found by Wilkens for the ratio of photographic to visual range for the Cepheid variables SU Cygni, X Cygni, T Vulpeculae, S Sagittae, and U Vulpeculae. The visual ranges, it should be remarked, are collected by him from various sources and can hardly be considered homogeneous or reliable. The results, however, are qualitatively dependable, but more work along this line is desired. The Cepheids mentioned in the three last notes are of spectral types F to K5, with an average very close to the solar type.

[5] SU Draconis, *Monthly Notices*, **73**, 166, 1912; SW Draconis, *ibid.*, **73**, 440, 1913; Cygni, *ibid.*, **74**, 225, 1914.

Mount Wilson establish the fact definitely.[1] The shift of the maximum intensity in the spectra of Cepheids, discovered by Albrecht,[2] has been confirmed by Kiess[3] and other Lick observers. These two factors—the greater photographic range and the shift of the maximum intensity—would suggest as an underlying and common cause a change in the spectral type. Albrecht and Duncan[4] have observed that Wright's spectrograms of η Aquilae suggest a later type of spectrum at minimum than at maximum. The Harvard classification of TT Aquilae[5] at maximum is G, at minimum, K.

For the cluster-type variables there is more direct evidence of distinct and continuous change. At the writer's request Miss Cannon has examined some of the Harvard spectrograms of certain cluster variables. For RR Lyrae no definite change was recorded on the plates examined, and similarly for XZ Cygni, but the spectrum, when faint, was extremely uncertain. For SW Andromedae the spectrum was of type A at maximum and clearly of a redder type at minimum. The most conclusive results, however, are obtained from the series of spectrograms taken by Mr. Pease[6] in July of this year with the 60-inch reflector of the Mount Wilson Observatory. The variable RS Boötis, period $9^{h}.1$, shows a continuous change of spectral type from F0 at minimum to B8 at maximum. One consequence of this result is that hereafter the classification of all Cepheid and cluster-type spectra must be made with due specification of the corresponding phase of light-variation.[7]

[1] A report on this work was presented at the meeting of the American Astronomical Society at Evanston, Illinois, August 25–28, 1914.

[2] *Lick Observatory Bulletins*, **4**, 131, 1907.

[3] *Ibid.*, **7**, 140, 1913.

[4] *Ibid.*, **5**, 93, 1909.

[5] *Harvard Annals*, **55**, 285, 1909.

[6] A report on this work was presented at the meeting of the American Astronomical Society at Evanston, Illinois, August 25–28, 1914.

[7] Miss Clerke writes: "The spectrum [of δ Cephei] is of the solar type, and does not change with the brightness" (*Problems in Astrophysics*, p. 320, London, 1903). This, however, should not discourage new attempts to classify the spectrum of the type-star and of other longer-period Cepheids at various phases of their light-changes. It is possible, of course, that the changes in color index and shifts of maximum intensity are not generally accompanied in the longer-period Cepheids by those changes in the absorption lines that are necessary to give a different spectral classification under the present system, in which the absorption lines receive much attention and the background intensities but little. But in the early study of the spectrum of δ Cephei, published by Belopolsky in *Bulletin*, No. 3 of the Imperial Academy of Sciences of

Another difficulty in the spectral changes, that must not be overlooked in attempting a complete explanation of the Cepheid phenomena, is a peculiarity observed by Albrecht[1] on his plates of Y Ophiuchi and T Vulpeculae. Various lines showed large irregular shifts, which are not progressive with the phase of the star in its light-period.

CONCERNING EXISTING HYPOTHESES

The fourth principal argument against the binary interpretation of Cepheids is the inadequacy of all the existing double-star hypotheses. To many this is not only the best argument but is sufficient in itself.[2] A detailed criticism of these attempted explanations is unnecessary, for this has been generously provided by the proposers of the theory themselves, as well as by others, including Campbell,[3] Plummer,[4] Brunt,[5] Kiess,[6] and Ludendorff.[7] There is

St. Petersburg, 1894, the variations in the relative intensities of several lines are suggested and certain deviations from the solar spectrum are explicitly pointed out on many spectrograms. The question is one to be answered definitely by future researches. (A very recent study of the changes in the spectra of δ Cephei and ζ Geminorum has been made at St. Petersburg by Lohmann, but the paper is not yet available to the writer.)

[1] *Lick Observatory Bulletins*, 4, 131–132, 1907.

[2] There has been a growing and but half-concealed discontent with the double-star explanations of Cepheids. Ludendorff writes (*Astronomische Nachrichten*, **193**, 304, 1912): "Freilich kann man sich aus verschiedenen Gründen des Eindrucks kaum erwehren, dass die in den Spektren der δ Cephei-Sterne beobachteten periodischen Linienverschiebungen nicht durch Radialbewegungen der Sterne, sondern durch irgendwelche andere Ursachen hervorgerufen werden." On the other hand, Paddock interprets Ludendorff's data in a manner favorable to Duncan's double-star hypothesis (*Publications of the Astronomical Society of the Pacific*, **25**, 180, 1913).

Plummer considers, in a recent paper on the nature of the Doppler principle when based on the Ritz theory of light, the possibility of getting around the difficulties presented by the velocity variations of certain variables (obviously Cepheids are meant) by abandoning the binary interpretation altogether, but he is led rather to abandon the Ritz theory (*Monthly Notices*, **74**, 660, 1914). Until the velocity variation of RR Lyrae was discovered he was inclined to suggest that cluster-type variation might be "a prominence effect on a large scale" (*ibid.*, **73**, 658, 1913).

[3] *Stellar Motions*, pp. 305 ff., New Haven, 1913; *Lick Observatory Bulletins*, **6**, 51, 1910.

[4] See various papers cited above.

[5] *Observatory*, **36**, 59, 1913.

[6] *Publications of the Astronomical Society of the Pacific*, **24**, 186, 1912; see also other papers cited above.

[7] *Astronomische Nachrichten*, **184**, 384, 1910.

one point, however, that has not been considered, which is of prime importance in the discussion of Cepheid phenomena. Russell[1] and Hertzsprung[2] have independently shown that the Cepheids are stars of small peculiar motions and small parallaxes, and hence of great absolute brightness. The former finds a mean absolute magnitude of -2.4 and the latter of -2.3, that is, the average Cepheid (the spectrum is of solar type) is nearly 700 times as bright as the sun. It is reasonable to assume that the Cepheids and the sun have a comparable surface brightness. The average Cepheid, then, has a volume between fifteen and twenty thousand times as great as that of the sun.

Interpreted as spectroscopic binaries these giant stars move in orbits whose apparent radii average less than one-tenth the radii of the stars themselves.[3] In order that the radii of the real orbits may greatly exceed those of the apparent orbits, the inclinations must be very small, a condition which cannot be supposed to exist generally for Cepheid orbits. The difficulty in applying the hypotheses of Eddie,[4] Loud,[5] Duncan,[6] and Roberts[7] is therefore immediately apparent. Moreover, if the mass of the average Cepheid is admitted to be as much as five times the solar mass, the density is still astonishingly low—hardly three ten-thousandths that of the sun. Considering the low average value of the mass function[8] derived from the orbits of the Cepheids, and taking a random distribution of the orbital inclinations, the non-luminous second body, to which Duncan's theory assigns the extensive atmosphere that must envelope the giant primary, has about one-tenth of the mass and therefore must move with an average apparent

[1] *Science*, N.S., **37**, 652, 1913.

[2] *Zeitschrift für wissenschaftliche Photographie*, **5**, 107, 1907; *Astronomische Nachrichten*, **196**, 201, 1913.

[3] The average value of $a \sin i$ for 15 Cepheids is 1,116,000 km. The greatest value is 2,000,000 km, and the least is 45,000 km.

[4] *Astrophysical Journal*, **3**, 227, 1896. [5] *Ibid.*, **26**, 369, 1907.

[6] *Lick Observatory Bulletins*, **5**, 91, 1909.

[7] *Astrophysical Journal*, **33**, 197, 1911; *Monthly Notices*, **66**, 329, 1906.

[8] The average value of $\frac{m_1 \sin^3 i}{(m_1+m)^2}$ for 15 Cepheids is 0.0025. The greatest value is 0.0058, and the least is 0.00001 for Polaris.

orbital velocity of about 200 km a second. Remembering the size of the primary star compared with its orbit, we know that the mass of the secondary must be still smaller and the velocity higher to separate the stars.

A SUGGESTED EXPLANATION OF CEPHEID VARIATION

In the face of all these difficulties, it seems appropriate to abandon completely the attempts to interpret Cepheids on the basis of a binary-star assumption. It has been shown by Russell[1] that the light-variations cannot be explained satisfactorily by the uniform rotation of a single spotted star; the light-change must be intrinsic, and not just apparent. The explanation that appears to promise the simplest solution of most, if not all, of the Cepheid phenomena is founded on the rather vague conception of periodic pulsations in the masses of isolated stars. The vagueness of the hypothesis lies chiefly in our lack of knowledge of the internal structure of stellar bodies, and not in the difficulty of explaining the observed facts if once we assume the stars to be ideally gaseous figures of equilibrium. Moulton[2] has considered the matter of explaining certain types of stellar variation from this point of view, but his conclusions are scarcely applicable to Cepheid variables in the light of our present knowledge of their peculiar properties.[3] According to him, the light-change should be due to the heat generated by the oscillation of a spherical star from an oblate to a prolate form, there being a maximum of light-emission every time the star passes through its mean spherical figure. The period of velocity variation, then, should be double that of the light-change,[4] and

[1] *Popular Astronomy*, **22**, 142, 1914. See the footnote on a preceding page relative to this work. Russell's investigation, then, opposes the explanation suggested by Hellerich (Dissertation, Berlin, 1913).

[2] *Astrophysical Journal*, **29**, 257, 1909.

[3] The hypothesis also is not obviously applicable as a complete or even partial explanation of the variation of elliptical eclipsing binaries, considering the present state of the orbital theory. The presence of the secondary spectrum, the very large periodic shift of the spectrum lines, and many other factors are almost unimpeachable proofs of the binary character of eclipsing variables. Very little if any effect on the light-variation can be attributed in any of the systems studied to pulsations, or even to tidal disturbances.

[4] That is, if the shift of the spectrum lines is to be attributed to a radial motion of the source.

this, of course, does not conform with known conditions.[1] It is to this phenomenon of pulsating stellar masses, however, that the writer would ascribe the light and velocity variation of Cepheid and cluster variables, and the theoretical work of Moulton,[2] Jeans,[3] Emden,[4] and others[5] on the properties of gaseous spheres already justifies the conclusion that such oscillations are both possible and probable. They might arise, as Moulton suggests, from the collision with masses of only planetary dimensions, from the near approach of two stars, or in other ways.[6]

Without any pretense of explaining clearly or fully on this hypothesis all the properties of Cepheid variation that have given the double-star theories such hopeless difficulty, a few points favoring the pulsation suggestion will be summarily stated. There will exist originally, as the result of the initial disturbance, a great number of oscillations with different periods. The character of these various vibrations will depend on the nature of the stellar structure. For the ideal homogeneous fluid mass investigated by Kelvin,[7] and for the polytropic gaseous sphere defined and studied by Emden,[8] the period of vibration of each type is independent of the volume and mass and depends only on the mean density and the order of the harmonic term defining the oscillation.[9] For any given

[1] It is well to keep in mind, however, the secondary maxima in the light-curves of η Aquilae and similar variables. Therein perhaps is a visible trace of the secondary heating of each oscillation period.

[2] *Op. cit.*

[3] *Philosophical Transactions of the Royal Society of London*, **199 A**, 1, 1902; *ibid.*, **201 A**, 157, 1903; *ibid.*, **213 A**, 457, 1914.

[4] *Gaskugeln*, Leipzig, 1907.

[5] The numerous papers by Ritter, *Wiedemanns Annalen*, **5–20**, 1878–1883, are of fundamental importance in this problem. His consideration of vibrational variable stars has been briefly discussed by Moulton (*op. cit.*).

[6] Perhaps then we should expect to find great numbers of these variables in condensed regions, such as the globular clusters and the Magellanic clouds.

[7] *Mathematical and Physical Papers*, **3**, 384, 1890. [8] *Op. cit.*, pp. 13, 37, 448 ff.

[9] The vibration period in seconds is for the former

$$\tau = 2\pi\left(\frac{2m+1}{2m(m-1)}\right)^{\frac{1}{2}}\left(\frac{R}{G}\right)^{\frac{1}{2}}$$

and for the latter

$$\tau = 2\pi\left(\frac{R}{mg}\right)^{\frac{1}{2}}$$

where m is the order of the spherical harmonic defining a given oscillation, R is the radius, and g, the surface gravity, is proportional to radius times mean density.

mean density the most important oscillation is that corresponding to the second-order harmonic. Its period is the longest, its amplitude the greatest, and it may persist with inappreciable change in period almost indefinitely, while the oscillations of higher order are more rapidly destroyed by friction.

If, then, we attribute the principal light-change in a Cepheid variable to this principal oscillation, and if we are willing to adopt Emden's polytropic gaseous sphere as a stellar model, we can compute at once the density of each individual variable. Obtained in this way the densities are probably of the right order of magnitude, whatever function of the radius, within reasonable limits, the density is assumed to be; and for an incompressible homogeneous fluid they would be only 2.5 times as large. The densities in terms of the sun for all the Cepheids of known periods and spectra[1] have been derived in this manner, with the results given in Table I.[2]

[1] It is impossible to say, of course, whether the spectra listed were obtained near maximum or near minimum. Errors in the grouping, for the brighter Cepheids at least, will probably balance in the means.

[2] The data for this computation have been derived mainly from *Harvard Annals*, **56**, 191–195, 1912. In the table of "short-period variables" given in that publication are included many stars that are now known to be eclipsing binaries. It is very likely that further study will show that others are not Cepheids, but in the means in Table I are included all variables not known to be eclipsing stars.

The densities in the table are computed for the mean periods of each class. If the means of the individual densities were taken one badly discordant period would greatly distort the result.

A striking fact shown by Table I is the progressive relation between spectrum and period, and hence, between spectrum and density. "The redder the tint, the longer the period" was observed by Chandler in 1888, but referred mainly to long-period variables (*Astronomical Journal*, **8**, 137, 1888). He called attention to the importance of this correlation to variable star hypotheses. Campbell noticed that "the length of periods seems to increase with the spectral types, but the relationship is not strongly marked" (*Lick Observatory Bulletins*, **6**, 51, 1910).

Russell's theory of the order of stellar evolution is supported by the results of Table I when we remember that the Cepheids are giant stars. For it is obviously fair to conclude from the progression of the densities that, if the present Cepheid hypothesis approximates the truth, the order of evolution is in the direction from type M to type B. The Cepheids then are young stars. Perhaps the long-period redder variables of the Mira Ceti type, with spectra of types M*a*,....M*d*1, M*d*2,....M*d*10, N, etc., are still earlier in their evolution. Are they merely Cepheids of very long period? In many respects they appear to be—the light-curves are similar, the shape and time of successive maxima often oscillate around probably rigorously constant mean values, the spectra change progressively throughout the light-period, and finally

The extremely low densities for the Cepheids of the redder spectral types until recently might have thrown serious doubt on the hypothesis that demands such abnormally low values. But now for two reasons we are ready to accept as possible these supposedly impossible densities. In the first place, as Hertzsprung has pointed out[1] in his proof that the Cepheids of solar spectral type are giant stars, the average mean density must be of the order of 6×10^{-5}, if the masses are comparable with that of the sun. They may be larger, but from our knowledge of stellar masses in general we are inclined to believe that they are not more than ten times that of the sun,[2] which is sufficient to prove the point. In the second place, the densities of several long-period eclipsing binaries of types G and K are now available for comparison.[3] For instance: RX Cassiopeiae, type Ko, mean density 5×10^{-4}; W Crucis, type Gp, mean density 3×10^{-6}; SX Cassiopeiae, type G3, mean density 5×10^{-4}; RZ Ophiuchi,[4] type F8, density of one component 10^{-3}.

TABLE I

Type	No. Stars	Mean Period	Mean Density
M	3	33	0.000006
K	9	18	0.000020
G	31	11	0.000056
F	31	6	0.000200
A	9	0.4	0.04
B*	1	0.19	0.2

* β Cephei is properly to be assigned to the Cepheid type. Notwithstanding the very short period ($4^{h}.5$), the preliminary orbit by Frost (*Astrophysical Journal*, **24**, 259, 1906) is typical of the Cepheids. Guthnick's discovery (*Astronomische Nachrichten*, **196**, 357, 1913) of the light-variation and its nature (a typical cluster-variable curve of small amplitude) further supports this classification, and the harmony of the hypothetical density with that for the other groups of Cepheids is probably a third favorable argument. The presence of a second spectrum, however, is suspected on certain plates, according to the preliminary announcement (*Astrophysical Journal*, **24**, 261, 1906).

they are apparently stars of great absolute luminosity. Long ago, however, Chandler gave several good reasons for distinctly separating "long period" and "short period" variables (*Astronomical Journal*, **9**, 1, 1889), and to a certain extent these distinctions are still to be maintained.

[1] *Astronomische Nachrichten*, **196**, 203, 1913 (footnote).

[2] Russell, *Nature*, **93**, 283, 1914; *Popular Astronomy*, **22**, 294, 1914. Ludendorff, *Astronomische Nachrichten*, **189**, 151, 1911.

[3] *Contributions from the Princeton University Observatory*, No. 3, 1914.

[4] The spectrum has been reclassified recently by Miss Cannon at the writer's request. The components are of nearly equal brightness (visually), but it is very likely that the photographic spectrum classified is that of the smaller component which has high relative surface brightness, and density as given above. The density of the fainter component is 2×10^{-5} (*Astronomische Nachrichten*, **194**, 225, 1913).

For the A- and B-type spectra, the Cepheid densities above are, of course, entirely normal compared with eclipsing star densities.[1]

As previously stated, the Cepheids without doubt are enormously large. Their small observed velocity variations, even if attributed altogether to motion in the line of sight and not at all to pressure-shifts, are not larger than might arise from a radial oscillation through but a small fraction of their mean diameters.[2] In the central mass of the star the period of the supposed pulsation should, of course, be perfectly regular, but its effect need by no means be regular on the radiating surface.

We may suppose that, because of the internal vibration, the photosphere of the star is periodically scattered or broken through by the rush of hotter gases from the interior. Maximum light and maximum velocity of approach would obviously be approximately synchronous, and their coincidence would naturally be independent of the direction of the observer in space. Ludendorff's correlation of range of light and range of velocity is highly significant in this connection.[3] The essentially harmonic nature of the oscillation at the surface of the star would easily lend itself to interpretation as elliptic motion, though non-elliptic motion need not be unexpected, nor the anomalous behavior of certain spectral lines. In stars in which the initial disturbance is of recent origin, the presence of secondary oscillations could be expected, which would affect the light as well as the velocity.[4]

It should be noted as an important factor in the explanation of Cepheid variation, that a change in the spectrum of a given radiating surface from one type to the next will change the visual brightness of that surface by approximately one stellar magnitude,

[1] *Astrophysical Journal*, **38**, 173, 1913.

[2] The radial motions observed in sun-spots are suggestive in this connection.

[3] *Astronomische Nachrichten*, **193**, 301, 1913. He finds that $2K=47.3A$ with close approximation, where A is the amplitude of magnitude variation and $2K$ is the total range of velocity variation in kilometers.

[4] By means of a detailed periodogram analysis of the light-curve of the famous irregular variable SS Cygni, Gibb has recently found a prominent underlying periodicity of 40.86 days (*Monthly Notices*, **74**, 678, 1914). The spectrum of SS Cygni is extremely peculiar in that it varies through a number of different types, occasionally showing bright lines and at other times an apparently continuous spectrum (*Harvard Annals*, **56**, 211, 1912).

and the color range by four-tenths of a magnitude. These quantities correspond very closely to what is observed in cluster variables, and suggest that, if desired, it is unnecessary to go farther for the explanation of the light-variation than to suppose that a surface of approximately constant area progressively changes its spectral type as the result of a periodic flow and ebb of heat.[1] That the light-change should be of a more explosive character for the cluster variables than for the longer-period Cepheids would be expected because of their higher mean densities.

Various other details suggesting the possibility of the above interpretation could be cited, but this sketch of the pulsation argument will suffice for the present, since the purpose of the paper is not so much to advance an alternative theory as to question the validity of the spectroscopic binary hypothesis.

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[1] Cf. the hypothesis proposed by Schwarzschild as an alternative explanation of the change by six-tenths of a magnitude in the color index of η Aquilae (*Publikationen der v. Kuffnerschen Sternwarte*, **5**, C125, 1900).

THE RELATION BETWEEN THE WOLF-RAYET STARS AND THE PLANETARY NEBULAE[1]

By W. H. WRIGHT

The position of the Wolf-Rayet stars in the scheme of stellar classification has long been a matter of speculation by astronomers. In 1890, and again in 1891, Pickering drew attention to certain points of resemblance between their spectra and those of the nebulae, and classified both as belonging to a fifth type of stellar spectrum.[2] At about the same time (1890) Keeler, who was then engaged in his visual investigation of nebular spectra, independently announced his conviction that the nuclei of the planetary nebulae are closely related to the Wolf-Rayet stars.[3] Keeler's conclusion was based on spectroscopic evidence, but he does not make a categorical citation of the facts upon which his conclusion rests, and I have looked rather carefully through his published writings for light on this matter. There are, indeed, observations which might be regarded as in harmony with this view, but one is led by the inclusion of "other bright line stars" with those of the Wolf-Rayet type to hazard a guess that the deductions were perhaps of a general character. Still Keeler's belief in a close relation is so definitely asserted, not only in one, but in several publications, that few who knew that shrewd investigator's rare ability to say exactly what he meant can doubt that he was firmly convinced that some close relationship exists.

Four years later, in 1894, Campbell published the most exhaustive account of the spectra of the Wolf-Rayet stars that has yet appeared.[4] He observed all of the then known stars of this type which are within the reach of the instruments at Mount Hamilton. His observations were both photographic and visual, and covered

[1] Read before the Evanston meeting of the American Astronomical Society, August 1914.

[2] *Astronomische Nachrichten*, **127**, 1, 1891.

[3] *Publ. A.S.P.*, **2**, 279, 1890; also *Publ. Lick Observatory*, **3**, 227–228.

[4] *Astronomy and Astrophysics*, **13**, 448, 1894.

the spectrum from λ 4100 to H_{α}. He added numerous lines to the list of known ones, and, as most of these are of unknown origin, left the spectra even more enigmatic than he had found them. At about this time the same investigator made an equally comprehensive study of the spectra of the nebulae.[1] His conclusions as to a relationship between these two classes of spectra are therefore of the highest interest and, as expressed in his own words,[2] are as follows:

Aside from the hydrogen, D_3, and 4472 lines, the nebular spectrum presents a few interesting coincidences with the Wolf-Rayet spectrum. The lines at 5412 and 4687 observed in several nebulae were found with more or less prominence in nearly all the Wolf-Rayet stars; but the lines at 4389, 4067, and 4026 were found in only a few of the stars, and it is not certain that the last two lines occupy the same positions in the nebulae and stars, though they probably do. The prominent star lines 5813, 5693, 5593, 5472, 4652, 4636, 4541, 4509, 4442 were specially searched for in the nebulae and not found. Likewise, the nebular lines 5751, 5007, 4959, 4715, 4364 were most carefully looked for in the stars, and no trace of them is visible. I think we must say that if any relation exists between these spectra, it is not clearly established and its nature is not apparent. . . . .

In conclusion, I think we can say, from the foregoing observations, that the spectra of the Wolf-Rayet stars are not closely related to any other known type. They appear to have several points in common with the nebular and Orion-type spectra; but the last two appear to be much more closely related to each other than to the Wolf-Rayet spectra. It is therefore difficult to place these stars between the nebulae and Orion stars. They certainly do not come *after* the Orion stars, and one does not like to place them *before* the nebulae. We can probably say that the bright lines are chromospheric, owing their origin to very extensive and highly heated atmospheres, but showing very little relation, in constitution and physical condition, to that of our own sun. For the present, at least, this type of spectrum must be considered as distinct from every other known type, just as the nebular spectrum is distinct, and like the nebular spectrum containing lines whose origin cannot now be assigned.

I think that no one will question the justice of Campbell's conclusions, in the light of the great mass of evidence which he had brought to bear upon the subject. They seemed to set at rest the earlier notions that the connection between the two classes of objects was a close one.

[1] *Ibid.*, **13**, 384–494, 1894.

[2] *Ibid.*, **13**, 474, 1894.

Among the most interesting of the stars observed by Campbell is B.D.$+30°3639$. This he found to be surrounded by a glowing hydrogen envelope, and it is with this object that the present communication has, in part, to deal. It is hardly necessary to recount here the details of the discovery, as the remarkable observation on which it is based is familiar to everyone interested in the subject. This star is naturally an object of interest by reason of its peculiar appendage, and has perhaps been observed more than any other of its class. Palmer and Stebbins found the well-known nebular line λ 3727 in its spectrum,[1] but do not appear to have appreciated the significance of the discovery. The line was later reobserved by Wolf.[2] More recently Merrill has found the nebular lines $\lambda\lambda$ 6548 and 6583 in the spectrum of the star.[3]

A question at once suggested by the finding of nebular lines in the spectrum of this star is: Do they belong to the star proper or to the surrounding gaseous envelope? A careful examination of the material furnished by the foregoing observations discloses no evidence on this point.

The writer became actively interested in the problem through the following circumstance. In making a long exposure on the planetary nebula Struve 6=N.G.C. 6572 with a spectrograph attached to the 36-inch refractor, the following of the telescope was sufficiently good to isolate the spectrum of the nucleus as a narrow band of continuous spectrum cutting across the centers of the bright nebular lines, and in this band the line λ 4686 appeared, not as a narrow line, but as a band some 17 A wide.[4] The line in question does not appear in the outlying parts of the nebula. This appearance was so suggestive that the spectrum of the nucleus was of a Wolf-Rayet character that another exposure of 14 hours, running through two consecutive nights, was undertaken. The second exposure confirmed the first and showed in addition to other nucleus lines a faint broad emission band at λ 4057. The investi-

[1] *Lick Observatory Bulletin*, **2**, 53, 1902.

[2] *Sitz. Heidelberger Akad. Wiss.*, **14**, 1913.

[3] *Lick Observatory Bulletin*, **7**, 129, 1913.

[4] The nucleus of this nebula is not a stellar point, but consists of a thickening of the nebulosity close to the center. Nebular nuclei differ in this respect, some, for instance that of N.G.C. 6826, present in the telescope the appearance of stars.

gation was extended to the visible region by a two-night exposure with a bathed plate. The spectrogram shows a faint broad nucleus band at λ 5807. All the plates show numerous narrow lines extending to the outer limits of the nebula, and a few hazy ones restricted to the vicinity of the center. The three bands already mentioned, λλ 4057, 4687, and 5807, are the only really broad ones which are recorded. They are confined very closely to the center of the nebula. The most characteristic Wolf-Rayet bands are probably λλ 4057, 4687, 5694, and 5813. The agreement with the nucleus lines is in the first two cases exact. The third line is not shown in the nucleus spectrum, though a longer exposure might bring it out. The fourth band, λ 5813, varies in position in different stars from λ 5804 to λ 5817. It is in fact composite in the Wolf-Rayet star B.D.$+30°$3639, with components at λλ 5801, 5812, 5828. The nucleus line λ 5807, though uncertain, on account of the faintness of the image, is probably due to the blending of the first two components. The nucleus also exhibits the Wolf-Rayet bands λ 4633 and λ 4649 as hazy bright lines.

It will probably be admitted that the presence of the five bands λλ 4057, 4633, 4649, 4687, and 5807 establishes the nucleus of the nebula as a Wolf-Rayet star.

After settling this point it naturally became a matter of interest to inquire whether the so-called hydrogen envelope surrounding the Wolf-Rayet star B.D.$+30°$3639 is not in reality a true planetary nebula. If this were found to be so, the relation between the two classes of objects would appear to be quite a definite one. Observations were begun on the two red lines near H_α to determine whether or not they extend out into the nebula. The slit was adjusted to the focus of the large refractor for this region and the following was done as carefully as possible. The three lines λ 6548, H_α, and λ 6584 are sensibly of the same length, and correspond to an envelope diameter of 7 seconds of arc. Campbell's visual estimate of the diameter of the H_β image is 5 seconds. Since a visual measure of such a faint object might be too small, while the photographic record, on account of imperfect following and other disturbing factors must be too large, the agreement between the two results is satisfactory.

Exposures, each of two nights' duration, were then made covering respectively the visible and photographic regions of the spectrum. These plates show all of the brighter nebular lines, except $\lambda\lambda$ 3869, 4363, and the helium lines, to be present as narrow bright lines running out into the surrounding envelope. The observed lines include $\lambda\lambda$ 3726, 3729, 4069,[1] 4076, 4959 (N_2), 5007 (N_1), 5755, 6303,[2] 6548, 6583, and 6730 in addition to the 11 hydrogen lines from H_λ to H_α inclusive. This array is probably sufficient to give the so-called hydrogen envelope the status of a planetary nebula. A number of these lines will perhaps not be recognized, at once, as nebular radiations. They are all, however, in a list, which will shortly be published, of nebular lines observed by the writer. N_1 and N_2 are very faint and their lengths are not easily determined. They are undoubtedly much shorter than the hydrogen lines.

In two cases we have then a planetary nebula with a Wolf-Rayet nucleus. That this is not an exceptional condition is attested by a number of isolated observations made by members of the Lick Observatory staff, but heretofore unpublished. The observers have kindly accorded me permission to refer to their results in advance of their own announcements. A brief summary of these observations follows:

N.G.C. 6826: This nebula has a large, round, fairly bright disk. The nucleus is very bright. The line λ 4686 was found by Merrill to be localized as a hazy dot in the spectrum of the nucleus, and Dr. Paddock strongly suspects the well-known Wolf-Rayet band at λ 4650. The writer has also found the characteristic band λ 4057 in this nucleus spectrum. The visible spectrum has not been photographed. The nucleus is undoubtedly a Wolf-Rayet star.

[1] Since this paper was prepared my attention has been called to the fact that this line was observed as an envelope line by Adams about the year 1910 (*Science*, **32**, 882, 1910).

[2] This does not agree exactly with the nebular wave-length, which is λ 6301. In this object the line seems to broaden where it crosses the stellar spectrum, and may therefore be displaced to the red at this point in common with the stellar bands. In this region of the spectrum the prismatic dispersion is of course weak and the linear equivalent of the discrepancy is small.

N.G.C. Index 418 = S.D.M. $-12°1172$: This is a particularly interesting nebula. According to Campbell the disks corresponding to N_1 and N_2 are smaller than the H_β disk.[1] The nucleus is a ninth-magnitude star. Paddock has found $\lambda\lambda$ 4686 and 4650 to be localized in the nucleus. The lines are rather narrow, and while the Wolf-Rayet character is not so pronounced as in other cases, the spectrum exhibits a strong tendency in that direction. λ 4057 is not present.

N.G.C. 40: This is a dim nebula with a central star. Paddock has found most of the stellar light to be concentrated in the Wolf-Rayet band at λ 4650.

These constitute all of the nebular nuclei of which the Lick Observatory has spectrograms. In every case the object is undoubtedly to be classified as a Wolf-Rayet star.

There seems to be no escaping the conclusion that the nuclei of planetary nebulae not only are closely related to Wolf-Rayet stars, as was first suggested by Keeler, but that in many cases they are such stars. Whether the converse proposition is true—that is, that those stars are in general the nuclei of planetary nebulae—cannot by any means be considered as having been established. One is in fact tempted to risk a guess that it is not. However, it is possible that further observations will shed some light on this, as well as on other aspects of the problem.

The observations already secured not only demonstrate the relationship which has been indicated between the nebulae and the stars, but are strongly suggestive of the nature of certain steps in the progression from one form to the other. Attention has already been called to the fact that the helium lines are absent from the nebulous envelope of the star B.D. $+30°3639$. They are, however, well represented in the star proper, where they occur as broad bands displaced an angstrom or two to the red of their normal positions. The lines N_1 and N_2 are probably shorter, as well as fainter, than the hydrogen lines. If we assume that by some process of evolution the nebula is condensing into the star, it is apparent that the order of settling is helium, "nebulium," hydrogen. There is considerable additional evidence favoring

[1] *Astronomy and Astrophysics*, 13, 494, 1894.

the view that this is a usual line of development, and the theory finds confirmation in the relative sizes of the disks in the planetary nebula S.D.M. $-12°1172$. More observations are required, however, before the validity of this rule can be considered to be established.[1]

The foregoing is only a preliminary account of the observations already secured; a more detailed one will appear later. The need of a more complete investigation both of the Wolf-Rayet stars and of the planetary nebulae in the vicinity of their nuclei is indicated. Although the faintness of the objects renders the work both precarious and tedious the writer hopes to extend the range of the observations during the coming year.

Mount Hamilton
August 1914

NOTE

Further observation strengthens the conclusion that the relationship between the Wolf-Rayet, or Class O stars, and the nuclei of the planetary nebulae is a very close one. The extra series of dark lines found by Pickering in the spectrum of ζ Puppis are of rather common occurrence, as absorption lines, in the spectra of the nuclei; in fact the resemblance to the spectrum of ζ Puppis is, in many cases, very strong. In at least one instance, the nucleus of the beautiful planetary in Geminorum, N.G.C. 2392 = G.C. 1535, the hydrogen and helium lines, as well as the members of the Pickering series, are dark. The spectrum is that of a Class Oe star. Hydrogen is represented by comparatively weak emission lines in the nebula proper.

November 1914

[1] Strictly speaking, evidence of this nature, even if amply confirmed, could only be taken to *prove* the settling of the lines, or say the conditions favorable to radiations of a certain character. For instance the line λ 4686 becomes localized in the nucleus before either the hydrogen series or the helium lines, yet it probably belongs to one or the other of the elements in question, perhaps to hydrogen. The phenomenon is of course analogous to the local occurrence of lines in terrestrial sources.

ON SYSTEMATIC ERRORS OF STELLAR RADIAL VELOCITIES

By SEBASTIAN ALBRECHT

Ever since it was found that the wave-lengths of numerous lines in stellar spectra vary progressively as a function of the spectral types of stars,[1] the writer has, on various occasions, suggested that it is extremely likely that a large share of the systematic differences in radial velocity results is due to the wave-lengths used in the reductions. This applies to the relative radial velocities for the different types as determined at any one observatory, and especially to the systematic differences between radial velocities for each type considered separately as determined at the different observatories. The recent publication by Küstner[2] of a second list of radial velocities and the wave-lengths upon which they are based makes possible an approximate test of an effect of the second kind referred to above.

The basis of this test will be a comparison, separately for each stellar spectral type, of Küstner's wave-lengths with the writer's preliminary wave-lengths as published in *Boletín No. 1* of the Córdoba Observatory. Use will also be made of the systematic differences which Küstner finds between his radial velocities and those of the Lick Observatory.

Table I is self-explanatory. For present purposes it is inadvisable to assign different weights to individual lines in forming the means at the bottom of this table, nor is it necessary to make a detailed study of each line (or blend), as must be done later in a definitive discussion of this problem. Such study would possibly have resulted in the exclusion of a few lines from the table, principally on account of differences in dispersion.[3] The large number

[1] *Lick Observatory Bulletin*, No. 106; *Astrophysical Journal*, **24**, 333, 1906.

[2] *Astronomische Nachrichten*, No. 4750, 409, 1914.

[3] The majority of the Lick radial velocities are based upon plates taken with three-prism spectrographs, with a considerable range in the dispersions used. The Bonn spectrograms are also taken with a three-prism spectrograph but with smaller dispersion. In the Bonn second series the dispersion is only about one-third that of the Lick plates, while in the Bonn first series the dispersion was double that of the second series.

TABLE I

Differences in Wave-Length (Albrecht−Bonn)

λ \ Type	F5	G	G5	K	K5	Ma	Mb
	A						
4236.1	−.021	−.021	−.021	−.021	−.021	−.021	−.021
42.7	−.070	....	....	....	....	....	....
45.4	+.008	+.008	+.008	+.008	+.008	+.008	+.008
46.9	−.077	−.034	+.019	+.013	+.006	+.001	−.002
50.2	+.006	+.006	+.006	+.006	+.006	+.006	+.006
50.9	+.005	+.005	+.005	+.005	+.005	+.005	+.005
54.5	−.006	+.032	+.030	+.028	....	....	....
58.4	....	....	....	−.006	+.006	+.008	+.008
60.6	+.113	+.128	+.152	+.182	....	....	....
74.9	+.002	−.005	−.009	−.010	....	....	....
86.1	....	....	....	−.024	+.039	+.047	+.067
88.1	−.008	+.013	+.027	+.036	(+.181)*	....	....
93.2	....	−.016	+.001	+.018	+.036	....	....
94.2	+.007	+.007	+.007	+.007	+.007	+.007	+.007
4313.0	+.049	+.043	+.088	+.083	+.100	+.099	+.098
14.3	+.011	+.019	+.059	+.051	+.034	....	....
15.1	+.056	+.046	+.034	+.021	+.007	−.005	−.010
18.8	+.030	+.044	+.056	+.062	+.062	+.057	+.054
21.0	−.008	+.005	+.028	.000	−.028	....	....
34.0	....	+.007	+.033	+.050	+.061	+.066	+.068
39.7	....	+.043	+.043	+.043	+.043	+.043	+.043
40.6	−.001	−.002	−.070	−.082	(−.191	−.220	−.240) †
44.5	−.050	−.024	.000	+.021	+.038	+.052	+.057
52.9	−.014	−.003	+.009	+.025	+.042	+.057	+.066
66.7	....	+.090	+.048	+.005	−.045	....	....
76.1	−.006	−.006	+.084	+.084	+.084	+.084	+.084
79.3	....	+.023	+.023	+.023	+.023	+.023	+.023
83.7	+.051	+.051	+.051	+.051	+.051	+.051	+.051
91.9	....	....	−.006	+.017	+.045	+.075	+.092
95.2	−.076	−.065	−.054	−.044	−.031	−.015	−.005
4401.6	+.063	+.063	+.063	+.063	+.063	+.063	....
04.9	+.022	+.022	+.042	+.052	+.072	+.072	....
12.2	+.021	+.060	−.015	+.017	+.044	+.062	+.069
25.6	−.145	−.100	....	+.022	+.047	+.057	....
27.4	−.031	−.037	+.008	+.002	−.002	−.006	−.008
59.3	−.048	−.033	−.026	−.023	−.022	....	....
64.7	−.083	−.019	+.025	+.055	+.078	....	....
66.7	−.027	−.021	−.007	+.009	+.018	+.019	+.019
68.6	−.009	.000	+.014	+.035	+.055	+.068	+.073
69.5	−.084	−.050	−.017	+.013	+.043	....	....
73.0	+.080	+.022	+.020	+.040	−.010	−.061	−.085
82.4	....	−.026	−.025	−.027	−.048	−.081	−.105
94.7	+.044	+.044	+.044	+.044	+.044	....	....
4522.8	−.003	+.011	−.015	−.020	+.024	−.052	....
28.8	−.016	.000	+.010	+.015	+.016	+.017	+.017
31.2	....	−.010	−.007	+.003	+.034	+.065	+.084
34.1	+.020	+.016	+.009	+.005	+.001	....	....
49.7	−.044	−.027	−.013	+.003	+.017	+.030	+.036

*Omitted because it is used only 7 times by Küstner and therefore would influence these results unduly if included.

† Omitted because it is used only 4 times by Küstner and therefore would influence these results unduly if included.

TABLE I—*Continued*

λ \ Type	F5	G	G5	K	K5	Ma	Mb
	A						
4565.7	....	....	−.010	−.016	−.023	−.029	−.031
80.3	....	....	....	−.005	+.005	....	....
86.1	....	−.007	+.050	+.001	+.025	+.035	+.038
4603.1	....	....	....	....	+.016	....	....
	A						
Means	−.0063	+.0067	+.0174	+.0188	+.0225	+.0260	+.0260
No. of lines	38	45	46	50	46	35	31
Equivalent difference in radial velocity	km +0.43	−0.46	−1.19	−1.29	−1.60	−1.77	−1.77

of lines used (53) gives ample reliability to the main result, namely, that the systematic differences in the radial velocities of Lick and Bonn are largely (and perhaps nearly entirely) due to differences in the wave-lengths which were employed for the lines used in the reductions.

Directly below the means of $\Delta\lambda$ in Table I are given the equivalent systematic differences in the radial velocities, in the sense (Albrecht−Bonn), which would result from the use of the two sets of wave-lengths respectively.

TABLE II

SYSTEMATIC DIFFERENCES IN RADIAL VELOCITIES

Type	(*a*) Küstner's Observed (Lick−Bonn)	(*b*) Küstner's Adopted (Lick−Bonn)	(*c*) Ludendorff, for Bonn First Series (Lick−Bonn)	(*d*) Due to Wave-Lengths Used (from Table I) (Albrecht−Bonn)	No. of Lines on Which (*d*) Is Based
	km	km	km	km	
F5	+0.23	0.0	0.0	+0.43	38
G	+0.15	0.0	−0.6	−0.46	45
G5	−1.06	−1.0		−1.19	46
K	−1.42	−1.4	−1.1	−1.29	50
K5	−2.30	−2.3		−1.60	46
M	−2.78	−2.8	−2.4	−1.77	35

Table II gives a comparison of: (*a*) Küstner's observed mean differences (Lick−Bonn); (*b*) Küstner's adopted differences (Lick−Bonn); and (*d*) the differences (Albrecht−Bonn) with the number of lines upon which the values in (*d*) are based.

The similarity between columns (*a*), (*b*), and (*d*) seems quite convincing. In fact, from types F5 to K they are nearly identical. The smaller value of (Albrecht—Bonn) in the M type is due without much doubt to one or more of the following causes: (1) the list of lines in Table I[1] does not constitute the complete list upon which the radial velocities are based; (2) for the reduction of each individual spectrogram only a small fraction of the total list of lines is employed, and it is therefore practically certain that the different individual lines have been used very unequal numbers of times in both the Bonn and Lick results; (3) though the writer's wave-lengths—upon which the comparison rests—are based entirely upon measures of Lick spectrograms, there may readily be a small systematic difference between this system and the average for all the reductions of M-type stars in the Lick results.

In column (*c*) of Table II have been added the results of Ludendorff,[2] who has found similar systematic differences between Lick and Bonn for Küstner's first list of radial velocities. Whether the results for the first list may be looked upon as additional evidence, supplementary to the more definite evidence from the second list, cannot be decided with certainty without recourse to the wave-lengths used in the reduction of the first list, which were not published. However, it seems likely that the accordance is not accidental, but that the systems of wave-lengths of the two lists are approximately the same in a systematic sense, for the ordinary method of determining relative wave-lengths in the different spectral types by means of residuals leaves *the system as a whole* nearly unchanged.

The question as to the proper distribution of these systematic differences between the two observatories need not be considered here in detail. Küstner finds that his check plates (of type between G and G5) give a systematic correction of -1.06 km, which must be very close to the exact amount required to bring the Bonn radial velocities for this type into coincidence with those from the Lick. As far as the evidence in Table II goes, it indicates that each of

[1] Only my list of lines published in the article referred to is being used in this comparison.

[2] *Astronomische Nachrichten*, No. 4705, 1, 1913.

these two observatories has succeeded in large measure in freeing its radial velocities for stars of the sun (G+) type from systematic error.

The comparison of Küstner's radial velocities with those of the Lick Observatory refers only to types F5 to M. For the earlier type stars attention has already been called, in former notes,[1] to the great uncertainty in the radial velocities on account of the presence in them of systematic errors of (at present) unknown amount, due to the considerable uncertainties as to the correct wave-lengths to be used for the lines which must be employed in determining these radial velocities. In the first of the two references quoted it was shown that the only two available good determinations with high dispersion of the wave-lengths of the three important silicon lines λ 4552.7, λ 4567.9, and λ 4574.9 differ from each other systematically by 0.090 A, corresponding to a systematic difference in the radial velocity obtained from them of 6.0 km. In the laboratory spectra the lines are generally somewhat diffuse and seem to be sensitive to the atmosphere surrounding the spark. As far as the writer is aware, no further laboratory investigations of these lines have been made, and he wishes again to call attention to the importance of, and the need for, a definitive study of them in the laboratory, under a variety of different conditions.

The systematic errors in stellar radial velocities which are discussed above are independent of, and in addition to, other and at present almost entirely unknown systematic errors for which the source of error lies chiefly in the stars themselves, such as pressure and radial motions in the atmospheres. In this connection it may be well to refer to Campbell's[2] inference, based on the recent work of Evershed on the sun, of rapid convection (radial motions) in stellar atmospheres as the source of the systematic differences which he found in the radial velocities for the different stellar spectral types, and which he designated by the letter K.

[1] *Publications of the Astronomical and Astrophysical Society of America*, 2, 71, 1911; *ibid.*, 73.

[2] *Lick Observatory Bulletin*, No. 257, p. 82, 1914.

In Table III are given approximate values of Campbell's K-term.[1] These are seen to be of the same order of magnitude as the quantities in Table II, which latter have been shown to be due to differences in the assumed wave-lengths. Nothing in Table III is in itself contradictory to the extreme view that the K-term may have its source entirely in the adopted wave-lengths. On the other hand, Campbell's suggestion involves some difficulties, though these may not be insurmountable. Perhaps the most serious difficulty, in our present state of knowledge, would be to explain, on the basis of convectional currents in the atmospheres, the large value of the K-term in the M-type (see Table III). It

TABLE III

Type	Campbell's K-Term
B	+4.0±km
A	+1.0
F	+0.1
G	±0.0
K	+2.0
M	+4.0

does not seem likely that in the M-type stars, which are presumably quite viscous and sluggish in comparison with the B-type stars, the radial convection in the atmosphere should be as rapid as in the B-type stars. It might be well, however, to bear in mind that the final reductions are likely to alter quite materially the values of the K-term in the different types, and that this difficulty may then disappear. That radial movements do occur in the stellar atmospheres is the direct inference from the recent work on the sun. Besides Evershed's work, St. John has shown radial motion for Ca vapor[2] and Perot and Lindstreedt[3] for H, Mg, and Na. The extent to which these radial movements are involved in the stars is a problem for the future.

As the writer has pointed out before, up to the present no adequate attempt has been made by the use of carefully studied systems of wave-lengths to avoid introducing in the radial velocities

[1] *Op. cit.*, No. 196, p. 127, 1911.

[2] *Astrophysical Journal*, **32**, 36, 1910.

[3] *Comptes rendus*, **152**, 1367, 1911; **154**, 326, 1912.

just such systematic errors as would result in a K-term. Until this has been done we cannot escape the inference that at least part of this term, and possibly the major part, may be due to systematic errors of this kind. The problem is complicated in some respects by the fact that a large percentage[1] of the lines in stellar spectra have different wave-lengths in the different spectral types. In other respects, however, the fact that these variations are *progressive* as a function of the spectral type places in our hands an additional tool. Those lines which are of constant wave-length over a considerable range of spectral type promise to be the most important connecting links between the types, though additional adjustment may also be required.

In another connection the writer has been engaged in the determination of standards of wave-length for each stellar spectral type. These systems must be upon a fundamental basis as far as possible, and though—perhaps contrary to the general impression among astrophysicists—the laboratory wave-lengths are at present not as satisfactory for this purpose as could be desired, the writer believes that his systems will be sufficiently close approximations for present needs. For this reason it seems best to defer to a later date an attempt to separate from the K-term the portion which is due to the use of erroneous wave-lengths (erroneous for each type considered separately) from the portion that may be due to causes within the stars themselves.

SUMMARY

Stellar radial velocities are known to contain systematic errors. These may be separated into two divisions: (*a*) systematic differences between the results for the same stars as observed at different observatories, and (*b*) systematic differences dependent upon the stellar spectral type, which were found by Campbell and designated by the letter K. The former must clearly be due to causes within

[1] In *Córdoba Observatory Bulletin*, No. 1, in the region of spectrum covered and for types A to M, an average of one line per 3.5 angstroms was found for which the wave-length varies as a function of the stellar spectral type. In the B-types the problem is more difficult, but here also several lines were found in 1907 and 1911 (not yet published) which seem definitely to vary in wave-length as we proceed from the early to the late B-types.

the control of the observer, i.e., the instruments and the methods of measurement and reduction of the spectrograms. The latter are due in part to these same causes, though they may—and probably do—depend also upon causes within the stars themselves.

Owing to the great perfection of modern stellar spectrographs, errors from instrumental sources must be nearly, if not entirely, negligible. The writer has previously shown: (1) that a large percentage of the spectrum lines vary in wave-length progressively as a function of the stellar spectral type; (2) that for many of the lines—and these must be used in determining the stellar radial velocities—the laboratory values of the wave-lengths are not sufficiently accurate to meet the requirements of the radial-velocity observer. This leads unavoidably to the inference that the systematic differences in the radial velocities under division (*a*) above are due in major part, if not entirely, to the adoption by different observers of different wave-lengths for the same lines. A discussion of the systematic differences between the radial velocities as determined at the Lick and Bonn observatories shows this to be true for these two observatories.

From (1) and (2) in the preceding paragraph it is evident that the systematic errors K, dependent upon stellar type, will require revision. At present it is not feasible to separate from the K-term the portion that is due to the use of erroneous wave-lengths from the portion that may be due to causes within the stars themselves. In another connection the writer has been engaged in the determination of standards of wave-length for each stellar spectral type. When these are completed it will be possible to attempt this separation.

Dudley Observatory
Albany, N.Y.
September 24, 1914

Minor Contributions and Notes

A SIMPLE METHOD FOR DETERMINING THE AMOUNT OF LIGHT LOST IN A STELLAR SPECTROGRAPH

The proportion of light lost in a spectrograph may be determined photographically by comparing the exposure time necessary to produce an ordinary trail at the focus, of the same length, width, and density as that of the spectrum, with the exposure time of the spectrum, for the same star.

Owing to the great disparity in the times required to produce a trail and a spectrum of the same star with the same telescope, the trail should be very much widened and this difference of width allowed for. Usually bright stars are to be preferred for such experiments as the foregoing.

As the spectrum will not be of uniform density, its average should be determined by measures at a number of points.

It is hardly necessary to point out that the same emulsion of dry plates should be used for both spectrum and ordinary trail and that care should be exercised that the development is exactly alike, either by development together in the same bath or by means of standard squares from a reliable light-source, or, better still, by making exposures on the same plate where possible.

Comparisons may be made for different dry plates and for different parts of the spectrum.

It would also be of value to determine the loss for different widths of slit and qualities of "seeing." This latter would be of especial interest in the case of very large telescopes and would, I think, disclose great losses due solely to the spreading of the disk over a comparatively large area by atmospheric disturbances and by the bodily wanderings of the image from the slit due to instrumental and atmospheric causes.

To make certain of getting all of the starlight through the spectrograph for the case in which this condition is desired, the slit should be opened very wide.

The simple comparison of loss of light for different slit-widths would be of interest as compared with the total light falling on the slit-plates, and should be made as a part of such an investigation as suggested.

Although especially developed for spectrographs used in stellar work, the above method is applicable to determining the loss where surfaces are dealt with.

This method is also suitable for use with all forms of slitless instruments.

C. D. PERRINE

OBSERVATORIO NACIONAL ARGENTINO
CÓRDOBA
May 16, 1914

Reviews

Die veränderlichen Sterne. Erster Band: Geschichtlich-Technischer Teil. Von Johann Georg Hagen, S.J. Erste Lieferung: "Die Ausrüstung des Beobachters." Freiburg: Herdersche Verlagshandlung, 1913. 4to, pp. xv+152. M. 10.

The pressing need of a comprehensive work on variable stars has been long felt, but the accelerated speed of discovery and the diversity of methods used in the last few years have made the need even more urgent. No branch of astronomy is so attractive to amateurs and none has been so dependent on amateurs for observational data, but the available variable star material has been scattered among periodicals and fragmentary publications largely inaccessible even to the average professional astronomer, and beyond the reach of most amateurs. If the author had done nothing more than to collect the bibliography given on pp. ix–xv, he would be entitled to our gratitude; but in addition he has assorted, arranged, and condensed this rich material with a skill and ripe judgment which could come only from a lifetime devoted largely to this branch of astronomy, and evidenced by his previously published works, for example, the unrivaled *Atlas Stellarum Variabilium.*

The old epoch of exclusively visual observations ended with the application of photographic and spectroscopic methods, especially to the discovery of variable stars; and while there is no immediate prospect of the eye being superseded by the prism and plate, it is no longer the only or even the main reliance. It is therefore fitting that the history of visual methods and results should be written at this time. The present work is therefore distinguished by the demand, its timeliness, and the quality of its execution.

The whole work is planned to appear in two volumes, of which the second, dealing with the physical explanation of the phenomena of variability, will be written by Father J. Stein. The present "historical-technical" volume is to appear in the following four parts: (I) "The Equipment of the Observer"; (II) "Methods of Observation"; (III) "Reduction of the Observation"; (IV) "Elements of the Light-Changes." The forthcoming publication of the great catalogue of variable stars, which has taken so much of the time of the Commission of the Astronomische Gesellschaft for the past thirteen years, will find

a fitting supplement in this more general work, though Part IV will apparently overlap the Gesellschaft catalogue. Taken together, these two works will cover the variable star field with great thoroughness, and will seem to secure for the Germans the lead in the literature of this branch of the science.

The present work is intended as a "source book," for which each reference has been verified with great care, and given (usually in its original language) in sufficient detail to make the meaning clear. For this reason the author does not consider the work a proper subject for translation, but believes rather that it should serve as a basis for manuals in the different languages.

Only one who has tried to keep track of the literature scattered through so many periodicals can fully appreciate the wealth of material collected in the introduction and five chapters, as evidenced by a bare list of subjects: the beginnings and meaning of the theory of variation; methods of observation; history of the study of variables; publication of observations and results; the visible signs of variation; classification, number, and distribution of variables; catalogue, nomenclature, and alphabetical lists; instruments, charts, ephemerides, and directions for their use; observing program for known, recently discovered, and suspected variables; co-operation in observation.

Without attempting to mention all the notable features of the work, the following points deserve special mention: (1) the table on p. 56 comparing seven of the principal classifications proposed or in use; (2) the table on p. 62 showing the distribution with reference to the galaxy of the faint variables, the bright variables, the Algol-type, and the Novae; (3) the table on p. 68 giving the principal catalogues of variables; (4) the comparison of different systems of nomenclature given on p. 84; (5) the advice given in chap. v on the preparation of an observing program, so much needed in view of the recent enormous increase in number of known variables.

The point of view of the author is mainly historic, as shown by the statement on p. 5: "Visual observations without the aid of photometers or colorimeters will have especial attention, as they form the historic methods and have not yet been comprehensively treated." The few criticisms which might be made have to do mainly with astrophysical subjects.

1. The statement on p. 6 in regard to the greater range of the photographic, as compared with the visual light-curves of certain variables is correct; but that implies a change in color, i.e., the variable is redder

near minimum. This is inconsistent with the statement on p. 30 that stars varying in color have not yet been discovered.

2. On p. 143, referring to "short-period" variables, the author says "their spectra appear to lie chiefly between the types Oe5 and B9"; but the table on p. 56 shows that most "short-period" variables are of the ζ Geminorum or δ Cephei types, of which the spectra are usually of Harvard Class G. It seems probable that the author means to refer to the Algol-type variables.

3. The statement on p. 34 that most modern Novae have been identified with previously existing stars seems open to question.

4. On p. 41 the author states that "small departures [from regularity of period] up to 10 per cent occur with all variables." Does this apply to the Algol and δ Cephei types?

5. The statement is made on pp. 56–57 that "knowledge of the inner mechanics of variables is disclosed only by spectrum analysis." The discovery of color-changes might be added to the sources of such knowledge.

6. The omission of the highly valuable observation of Yendell from the list on p. xv should be noticed.

7. The lack of an index will doubtless be supplied in the later parts of the volume.

The best proof of the usefulness of a book is the use that is made of it. This book has already been drawn upon as a source by Kempf in his fifth edition of Newcomb-Engelmann's *Populäre Astronomie*, and by Hartwig in his "1914 Ephemeris of Variable Stars" in the *Vierteljahrsschrift*.

The format, 9×12 inches, is inconveniently large and is not required by any tabular matter in this part, but may be needed in later parts. The type and presswork are extremely good, and the price, $2.70 net, is low. American purchasers can secure copies from Herder's St. Louis branch, at 17 South Broadway.

J. A. PARKHURST

YERKES OBSERVATORY
WILLIAMS BAY, WIS.

Beyond the Atom. By JOHN COX. Cambridge Press, 1913. 12mo, pp. i+146. $0.40.

This is a charming little primer on radio-activity written by one who is at the same time a physicist, a philosopher, and a literary artist. The two most delightful chapters are the first, which deals with the growth

of physical ideas from 1800 to 1890, and the last, which presents the necessarily vague and impressionistic picture of the "new atom" and its relations to the physical world. The material between these two chapters is divided into seven others and represents largely an abridgment and restatement in most readable form of the content of Rutherford's *Radio-Active Substances and Their Radiations.* I know of no better short treatment of the subject for the non-technical reader. It is both reliable and interesting. The only defect is a certain lack of up-to-dateness shown in the inclusion of some old values of constants and in the omission, though published in 1913, of the epoch-making work of Laue and his co-workers, published in 1912. In view, however, of the rapidity with which new points of view chase one another across the stage of modern physics, it is scarcely possible even for the actors to get a wholly correct view of the present situation, and a *book* must necessarily depict an act which is past rather than the one which is passing.

R. A. Millikan

University of Chicago
May 11, 1914

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

Where unusual expense is involved in the publication of an article, either on account of length, tabular matter, or illustrations, arrangements are made whereby the expense is shared by the author or by the institution which he represents.

Authors will please carefully follow the style of this JOURNAL in regard to footnotes and references to journals and society publications.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

If a request is sent *with the manuscript,* one hundred reprint copies of each major article, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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ERRATA

Vol. **39**, May 1914, in Professor Grabowski's article "On the Theoretical Photometry of Diffuse Reflection":

Page 299, line 20, *cancel* diffuse scattering or.

" 299, " 25, *for* This value is, *read* The value of J for this direction is.

Page 302, line 15, *for* The, *read* Thus, the.

" 303, lines 7, and 20, *for* $\sqrt{1-s^2\cdot a^2}$ *read* $\sqrt{1-s^2\alpha^2}$.

" 303, line 8, *for* illumination, *read* the emanating radiation.

" 305, " 1, *for* the first, *read* also the first.

" 305, " 30, on the right margin add the designation (b) of the equation.

Page 306, lines 15 and 27, *for* normals to the mirror, *read* normals of mirrors.

Page 306, line 26, *after* element *add* of the body.

Vol. **39**, May 1914, in the article by Adams and Kohlschütter, on "The Radial Velocities of One Hundred Stars with Measured Parallaxes":

Page 349, line 15, *for* 0".01, *read* 0".10.

Vol. **39**, June 1914, in Professor Barnard's article on "Visual Observations of Halley's Comet in 1910":

Opposite page 380, Plate VIII, middle photograph, *for* $18^h.5$, *read* $20^h.1$.

Vol. **40**, July 1914, in Professor Plaskett's article on "Improvements in the Optical System of the Stellar Spectograph":

Page 133, heading of Table IV, *for* exponents, *read* exposures.

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